

MANAGE-WB: The Mitigation, Adaptation and New Technologies Applied General Equilibrium Model of the World Bank

Model documentation and user guide
(under review)¹

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¹ This documentation is based on the documentation of the MANAGE v2f and MANAGE-WB v3 with updates to equations and model structure.

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1. Introduction

Overview

MANAGE-WB has been developed and used by a network of CGE modelers to support World Bank teams and clients in macroeconomic analyses of a wide range of topics. In recent years, it has been extended in a modular fashion to better capture emissions and their abatement, climate change damages and related adaptation measures as well as features of energy markets to support macroeconomic analysis in Country Climate and Development Reports as well as Long Term Greening Strategy documents. It takes the standard assumptions found in most single country CGE models: aggregate firms minimize costs under constant return-to-scale technologies, households maximize utility, economic agents own the production factors and supply them to firms, all agents are price takers in perfectly competitive markets for products and production factors. In addition to these standard features, the MANAGE-WB model includes a completely flexible functional specifications that allow analysts to easily implement an appropriate nesting structure for the production functions and factor supplies. Similarly, sub-nests under the top-level final demand functions allow to flexibly depict the substitutions between single products and product bundles.

MANAGE-WB is a recursive-dynamic model running in yearly steps, using by and large the neo-classical growth specifications – i.e., labor growth is exogenous and capital accumulation derives from savings/investment decisions. Optional, a version with forward looking behavior over multiple periods is available which comprises endogenous stock changes and bases savings and investment decision on expected returns. Savings and investments are endogenously determined in bond markets where the interest rate on government debt and returns on private investment clears supply and demand of loanable funds.

The model allows for a wide range of productivity assumptions that include autonomous improvements, for instance, in energy efficiency, that can differ across agents and energy carriers. The model also features levers to integrate results from sectoral models such as energy, transport or biophysical ones. These levers comprise of endogenous CES twist parameters and endogenous productivity and demand shifters. Most parameters are defined as variables to enable high flexibility in such closure swaps. Finally, the model can incorporate an on-demand vintage structure for capital that allows for putty/semi-putty assumptions about sluggish mobility of installed capital.

MANAGE-WB is a template model which can be calibrated to any Social Accounting Matrices (SAM) which adheres to a set of conventions on how to represent the economic structure. Accordingly, the number of sectors and products or the tax details differ across projects. A larger set of pre-defined closure rules ease a project specific model application. The model is implemented in GAMS (General Algebraic Modeling System), a higher-level Algebraic Modeling Language and uses the GGIG (GAMS Graphical User Interface Generator) framework for its Graphical User Interface and reporting tools. It can be solved as a Mixed-Complementarity Problem (MCP) to capture regime switches, such as emission taxes becoming zero when the emission cap is not binding. Routines allow estimating a balanced SAM using country specific data when available, the GTAP database or a mix of both. The code of MANAGE-WB is structured to ease the use of one model installation simultaneously in multiple projects.

MANAGE-WB in a nutshell

The MANAGE-WB model is a recursive dynamic computable general equilibrium (CGE) model. For each year, a model is solved to define a simultaneous static equilibrium in all commodity and factor markets. The equilibrium reflects certain dynamics such as population growth and capital accumulation between two consecutive years, or changes in productivity. Agents have perfect foresight for the current period but are myopic in the sense that they do not base their decisions on expectations of the future. Product and factor markets are competitive. Each static equilibrium relies on a relatively standard set of equation specifications in CGE modeling as discussed next.

Production is depicted by several aggregated firms (or sectors) which minimize costs under constant-returns-to-scale-technologies. These are modeled by trees of nested constant-elasticity-of-substitution (CES) functions to capture the substitutions and complements across the different inputs such as capital, labor and intermediates. Based on this flexible production nesting, energy is usually assumed to be a near-complement with capital in the short-run but a substitute in the long-run. Thus, rising energy prices tend to lead to rising production costs in the short-run when substitution is weak, but a long-run response would lead to energy-saving technologies that dampen the cost-push factor. This feature of the model is embodied in an optional vintage capital structure that captures the semi-putty/putty relations across inputs with more elastic long-run behavior as compared to the short-run.

The model allows for both multi-input and multi-output production. Multi-input allows, for instance, that electricity is produced by multiple power generation activities, including thermal, hydro, solar and other renewable forms of electricity production. Multi-output allows for a single activity to produce more than one product, for instance, oil seed crushing outputting both vegetable oils and oil cakes (for feed).

Labor and capital income are largely allocated to households with pass-through accounts to enterprises, which can also save and transfer income to the government to depict (partially) state owned enterprises. Government revenue is derived from a detailed list of both direct and indirect taxes as well as factor income due to the ownership on enterprises and transfers. Household demand is modeled using the constant-differences-in-elasticity (CDE) demand function, which is the standard utility function used in the GTAP model. Final demanders consume commodity bundles (indexed by k) composed of shares of supplied goods or CES composite thereof, based on a completely flexible nested CES approach. For instance, a part of fuel demand can be combined with demand for cars to define a transport service demand bundle while the remainder of fuel demand, related to heating, could be defined with other products as a housing service bundle.

Goods are evaluated at basic prices with tax wedges. The model can incorporate trade and transport margins which add additional wedges between basic and end-user prices. These trade and transport margins are differentiated across transport nodes-farm/factory gate to domestic markets and the border (for exports) and from port to end-user (for imports).

Import demand is driven by the ubiquitous Armington assumption which considers quality differences between the imported and domestic origin. The level of the related CES (often called Armington) elasticity determines the degree of substitutability across these two regions of origin. Domestic production is analogously distributed to exports and domestic sales using a constant-elasticity-of-transformation (CET) function. The ability of producers to switch between domestic and foreign markets is determined by the

level of the CET elasticity. Optional additive CES and CET functions can be employed to ensure aggregation in physical volumes. The model allows for imports and exports to assume perfect substitution and transformation respectively, with domestic production, to implement the law-of-one price instead of the CES or CET case.

Market equilibrium for domestically produced goods sold domestically is assumed through market-clearing prices. By default, the small open economy assumption implies exogenous export and import prices independent of export and import levels. Alternatively, the model can use iso-elastic functions to implement export demand and import supply schedules to render the terms-of-trade endogenous in certain markets.

The model assumes market-clearing wages in the labor markets with the possibility of an upward sloping labor supply schedule and sluggish mobility of labor across multiple nested labor market segments (such as agricultural versus non-agricultural sectors). An alternative that assumes fully elastic labor supply at fixed prices is also possible.

In dynamic simulations using the vintage mechanism, *new* capital is generated by current year investments and allocated across sectors to equalize the rate of return. *Old* capital, considering depreciation, remains in use in its original sector unless the sector is in decline – i.e., it does not attract any *new* capital. In this case, only the old vintage is active and some of the *old* capital is sold based on an upward sloping supply schedule depending on returns-to-capital in such sectors. Returns to capital hence drop in declining sectors below the economy-wide average. The *old* capital in expanding sectors earns the same rate of return as the *new* capital. This vintage mechanism can also be removed and capital movement across sectors can then be depicted by sluggish or perfect factor mobility.

The dynamics of MANAGE-WB are composed of three elements. Population and labor stock growth are exogenous where the latter is often equated to the growth of the working-age population. The aggregate capital stock grows according to the overall level of total savings (enterprises, households, public and foreign), considering depreciation and shares of non-productive investments. Investments are equal to savings net of new government debt. The third component relies on productivity assumptions where usually labor productivity is endogenous during baseline generation to replicate a given growth path of GDP. Additional endogenous specific productivity shifters or update of share parameters are often employed to superimpose specific developments in baseline construction or counterfactuals, such as changes in the energy mix.

2. Model specifications

MANAGE-WB is a template model such that the coding of the model is independent of the dimensionality of the SAM and other functional dimensions of the data. Model equations and data transformations for benchmarking, dynamics, solving for a new equilibrium and reporting provide a structural template which reflects the concepts and methodology underlying the model. The actual equation and data transformation instances used in a project reflect the data detail as defined by the main sets and subsets of the model as shown in Table 2.1, and implement project specific model parameters. A well-documented and structured EXCEL workbook comprises all project specific data, parameters, sets and maps and

provides a bridge between the specific project and the generic model template. Additional coding for a specific project implements the project specific shocks.

The input SAM has a dimensionality of $is \times is$ and most of the remaining sets and subsets derive from is . Sectors have three classifications: a , i , and k , which are (production) activities, marketed commodities and consumed commodity bundles, respectively. In a traditional model the three sets are identical. In the MANAGE-WB model, with its multi-input multi-output production structure, output from activities (a) is combined with imports to supply (or 'produce') commodities (i). This allows, for example, to have multiple activities produce a single commodity (for example, electricity) and to have single activities produce multiple commodities (e.g., the sugar production sector produces sugar, ethanol, rum and even power). In addition, MANAGE-WB allows for commodities (and/or bundles) in final demand (indexed by k) to differ from marketed commodities (i). A consumer-based 'make' or transition matrix maps supplied commodities in shares to multi-level demand nests. This allows for more realistic demand behavior, for instance to model demand for energy carriers. Household demand for transportation services can be a combination of demand for automobiles and (a part of the) demand for fuel. If the price of fuel goes up, the combined demand for fuel/autos would decline. It also allows for specific treatment of the demand for fuel and intra-fuel substitutability. A remaining part of the fuel demand (heating, cooking) could then be combined with (shares of) other goods and services into a housing service bundle.

Table 2.1: Core sets

Set	Description
<i>is</i>	Full set of SAM Accounts
<i>aa(is)</i>	Armington agents—includes all production activities and final demand
<i>a(aa)</i>	Production activities
<i>oa(aa)</i>	Other Armington agents—mostly final demand accounts
<i>fd(oa)</i>	Final demand accounts (excludes the trade and transport margin accounts)
<i>h(oa)</i>	Household accounts
<i>f(oa)</i>	Other final demand accounts
<i>i(is)</i>	Commodities
<i>e(i)</i>	Energy commodities
<i>k</i>	Consumed commodity bundles
<i>inst(is)</i>	Institutions (for transfers)
<i>fp(is)</i>	Factors of production
<i>l(fp)</i>	Labor types
<i>lnd(fp)</i>	Land types
<i>cap(fp)</i>	Capital types
<i>natres(fp)</i>	Natural resources
<i>v</i>	Vintages (Old and New)

The next sections of the document describe the different block or modules of the model using the rather traditional circular flow scheme of economics – i.e., starting with production and factor incomes, income distribution, demand, trade and macro closures. At the end, there is a discussion on the model dynamics.

Production block

Production functions

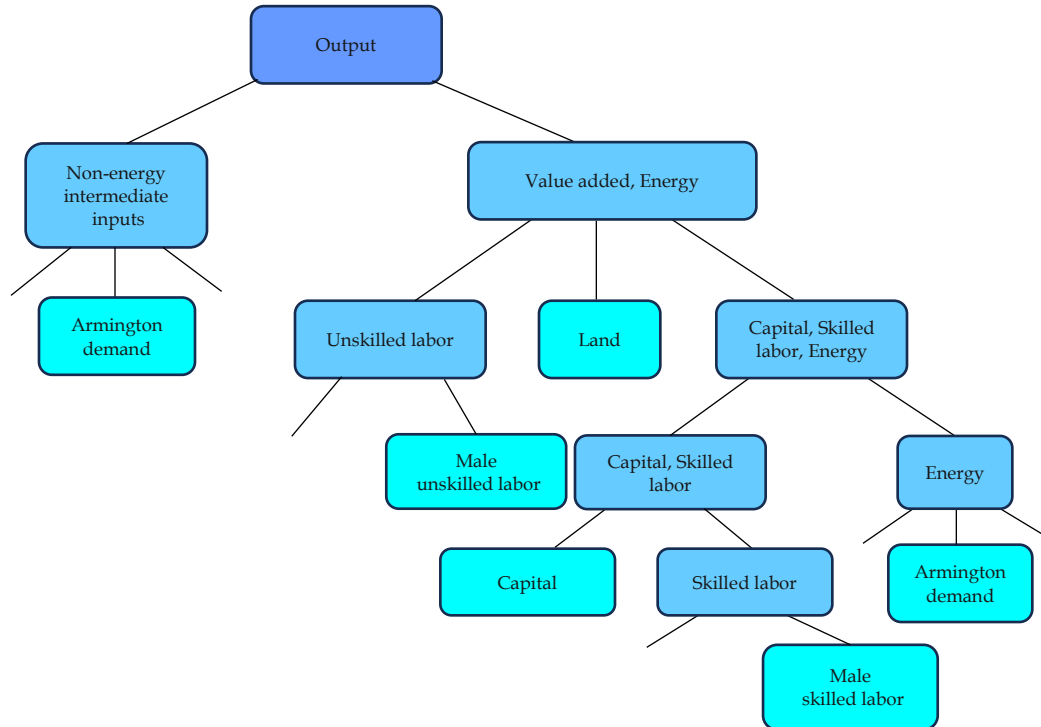
All of the equations related to production are indexed by a for production activities and v for capital vintages (*old* and *new* where *new* uses capital equipment newly installed at the beginning of the period while *old* uses capital greater than a year old)². Note that the list of produced commodities i is not identical to the list of activities. The MANAGE-WB production structure is a set of nested (i.e., multiple layers), potentially activity specific, constant-elasticity-of-substitution (CES) functions, an example is depicted in Figure 2.1. The CES structures are depicted in the model code by a handful of equations, only, which stay untouched if the nesting structure or its parameterization is changed, this approach is called “flexible nesting”.

The purpose of the nested CES structures is to replicate the substitution and complementary relations across various inputs (intermediate and factors) to production. A substitution elasticity of 1 implies a Cobb-Douglas technology where cost shares are fixed, while a zero elasticity implies a Leontief relation with fixed physical factor shares. For technical reasons, the Cobb-Douglas case is approximated with a substitution elasticity of 1.01 in the GAMS code. See, for instance, van der Mensbrugghe (2020) for the key analytical properties of the CES and constant-elasticity-of-transformation (CET) functions and the derivations of optimality problems using these functions.

The model differentiates between the pre-tax returns to factors and producers’ factor costs which comprise activity and factor specific ad-valorem factor taxes or subsidies. Similarly, for intermediates, producer cost can comprise activity and commodity specific ad-valorem taxes or subsidies.

² Capital vintages can be understood as *old* and *new* technologies of production. This is an “on-demand” feature of the model so that an analyst can choose to run the model with or without the vintage structure. The vintage structure allows for two impact channels. First, it is typically assumed that the *old* vintage has lower substitution elasticities for certain nests than the *new* vintage. This reflects that firms can choose for new investments between different technologies which, for instance, differ in their capital and energy intensity. Once the decision is made and the capital matches the chosen technology installed, the flexibility in these intensities is smaller. Higher saving rates will lead to a higher share of the *new* vintage and thus greater overall flexibility in adjusting production to the changes in input prices and productivity. The second channel is through the allocation of capital across sectors. *New* capital is assumed to be perfectly mobile across sectors whereas *old* capital is linked to the *old* vintage. In declining sectors, the return to capital will be less than the economy-wide average and some old capital will be released according to an upward-sloping supply curve, such that *old* capital in expanding sectors is immobile and sluggish in declining ones. This is explained in greater detail in the market equilibrium section.

Figure 2.1: Example of a typical flexible production nesting structure



In Figure 2.1, the boxes in green depict inputs – i.e., intermediate inputs (Armington demand for different goods and/or services) or factors of production (here capital, different types of labor and land). These bought inputs are combined, here over multiple layers of CES aggregates, to produce total output depicted in dark blue at the top. The light blue boxes depict these CES aggregates, composed of these inputs and/or of other CES aggregates.

A top nest combines the value-added-energy bundle and non-energy intermediate inputs into output via a CES function. The non-energy bundle is a CES aggregate of all non-energy intermediate commodities. The value-added-energy bundle is a CES aggregate of demand for an unskilled labor bundle, demand for land (where appropriate for a sector) and the capital/skilled labor/energy bundle. unskilled labor is a CES aggregate of male and female unskilled labor, where such differentiation is available in the data. The composite bundle capital/skilled labor/energy plays a crucial role in climate change abatement. It depicts substitution between the bundle of energy and the capital/skilled labor bundle. The latter comprises capital and skilled labor bundle of male and female skilled labor. Finally, the energy bundle depicts the substitution possibilities between different energy carriers, such as coal, oil, gas and petroleum products.

Without the flexible nesting, extending or modifying a nesting structure such as shown above requires (re)defining several (new) variables (quantities and prices), equations and parameters (shares, shifts, and elasticities of substitution) for each node, and modifying or adding GAMS code for the benchmarking. With the flexible nesting, an appropriate production structure for a specific country or project can now be easily defined, including exemptions for specific activities. Instead of being fixed in the GAMS code, the names of the nests, their structure and substitution elasticities are defined in an EXCEL workbook, called

the bridge-file. This file also comprises all other project specific sets, data and parameters, such as the SAM for the country.

Figure 2.2 shows how a flexible production nesting is specified in the bridge-file, using a quite evolved example. The top row comprises the parent nests. For example, *f-top*, *va* and *nd* denote total production, value-added and total intermediate, respectively. The second row defines the default elasticity of substitutions in the parent nests, activity and vintage specific exemptions can be inputted in another sheet in the bridge-file. The Leontief case with a zero-elasticity must be inputted by *eps*. A flag of 1 in the third row activates the additive variant of the CES function, if the cell is left empty, the default form of the CES function is used. The first column in the table comprises either the label “all” to indicate the default nesting structure for all activities or a label of an activity to define activity specific nesting structures replacing the defaults. The second column gives the name of the child nest. The numbers in each child row indicate the distribution of this child’s cost at the benchmark to one or several parent nests. These shares must add up to unity. In the example below, only 1:1 relations between children and parents nests are depicted.

With the exemption of the top-level nest *f-top*, all other parent nests shown in the first row must also occur as a child in the table. Additionally, all intermediate inputs and all factors of productions must be found in a row in the table to allocate them in shares to one or multiple parent nests.

Figure 2.2: Example of flexible nesting information for the production function in the bridge-file

		labSk	labus	cap_labSk	va	cap_ener	ener	ely_tnd	gas_gdt	ener_gasoil	ener_fossil	wtr_land	aez_land	crops	anim_meat	f-top	nd
	subsElas	0.5	0.5	0.2	0.7	0.25	0.5	eps	eps	0.7	0.5	0.5	7.5	2	2	eps	eps
	additive						1			1	1						
		labSk	labus	cap_labSk	va	cap_ener	ener	ely_tnd	gas_gdt	ener_gasoil	ener_fossil	wtr_land	aez_land	crops	anim_meat	f-top	nd
all	f-off_mgr_pros	1															
all	f-tech_aspros	1															
all	f-m_skl	1															
all	f-f_skl	1															
all	f-clerks		1														
all	f-service_shop		1														
all	f-ag_othlowsk		1														
all	f-m_nsk		1														
all	f-f_nsk		1														
all	labSk			1													
all	f-Capital			1													
all	f-NatlRes				1												
all	labus				1												
all	cap_ener				1												
all	wtr_land				1												
all	cap_labSk					1											
all	ener					1											
all	ener_fossil						1										
all	ely_tnd						1										
all	c-p_c						1										
all	c-TnD							1									
all	c-ely							1									
all	c-gdt								1								
all	c-gas								1								
all	c-oil									1							
all	gas_gdt									1							
all	ener_gasoil										1						
all	c-coa										1						
all	f-Land											1					
all	aez_land											1					
all	f-water											1					
all	f-AEZ1												1				
all	f-AEZ2												1				
all	c-pdr													1			
all	c-wht													1			
all	c-gro													1			
all	c-v_f													1			
all	c-osd													1			
all	c-c_b													1			
all	c-pfb													1			
all	c-ocr													1			
all	c-pcr													1			
all	c-ctl														1		
all	c-ctlRais														1		
all	c-oap														1		
all	c-rmk														1		
all	c-wol														1		
all	c-cmt														1		
all	c-omt														1		
all	va															1	
all	nd																1
all	crops																1
all	anim_meat																1
all	c-frs																1

Note: The colors are used to better visualize the structure but have no impact on the nesting itself.

The production structure under the flexible nesting is represented by the following five equations which encompass the differentiation between the standard CES and its additive variant. Equation (1) determines the optimal level of demand for child in sector a and vintage v at time t $xFac_{Chl,a,v,t}$ in case of the normal CES function. The term Chl states that the factor or commodity is a *child* of a parent factor or commodity

nest denoted by Par .³ Parents combine factors and/or commodities into production bundles as well as bundles into bundles. For instance, a composite commodity of skilled labor $labsk$ could represent the parent factor of male and female skilled labor (f_m_skl and f_f_skl) and others, while being a child of a parent cap_labSk along with capital $f_capital$ as in Figure 2.2.

$xFac_{Chl,a,v,t} = \sum_{Par} \left(\alpha_{Chl,Par,a,v}^{xFac} (1 + \chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}) (1 + twist_{Chl,Par,a,v,t}) \lambda_{Chl,a,v,t}^{xFac} \sigma_{Par,a,v}^{xFac-1} \left(\frac{pxFac_{Par,a,v,t}}{pxFac_{Chl,a,v,t}} \right)^{\sigma_{Par,a,v}^{xFac}} xFac_{Par,a,v,t} \right)$	(1)
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The above optimality condition states that the quantity demanded of a child (a single factor, a commodity or a CES composite) depends on the factor augmenting (Hicks neutral) productivity change $\lambda_{Chl,a,v,t}^{xFac}$, the prices including taxes $pxFac_{Par,a,v,t}$ and $pxFac_{Chl,a,v,t}$, the share parameter of the child in the parent $\alpha_{Chl,Par,a,v,t}^{xFac}$, the quantity of parent $xFac_{Par,a,v,t}$, and the elasticity of substitution between children within a parent $\sigma_{Par,a,v}^{xFac}$. $\chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}$ is non-zero only under the forward-looking extension. Similarly, $twist_{Chl,Par,a,v,t}$ remains zero if the twist option is not active. Finally, the demand for a child in a parent nest is summed over the parent nests to find the total demand for the child, to accommodate the case where a child is comprised in shares in multiple parents.

Equation (2) determines the price index of each parent nest. It is a non-linear aggregator of the prices (and/or price indexes) of the children comprised in this parent, taking the share parameters and productivity changes into account.

$pxFac_{Par,a,v,t} = \left(\sum_{Chl} \alpha_{Chl,Par,a,v}^{xFac} (1 + \chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}) (1 + twist_{Chl,Par,a,v,t}) \left(\frac{pxFac_{Chl,a,v,t}}{\lambda_{Chl,a,v,t}^{xFac}} \right)^{1-\sigma_{Par,a,v}^{xFac}} \right)^{\frac{1}{1-\sigma_{Par,a,v}^{xFac}}}$	(2)
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For additive CES functions, the expression in (1) is written as follows:

³ The terms Chl and Par are used here for convenience only. The GAMS codes have $fpChl$ and $fpPar$ for Chl and Par respectively to differentiate them from the other flexible nests in the model.

$ \begin{aligned} & xFac_{Chl,a,v,t} \\ & = \sum_{Par} \left(\alpha_{Chl,Par,a,v}^{xFac} (1 + \chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}) (1 \right. \\ & \quad \left. + twist_{Chl,Par,a,v,t}) \lambda_{Chl,a,v,t}^{xFac} \frac{-\sigma_{Par,a,v}^{xFac}}{\left(\frac{pxFacIND_{Par,a,v,t}}{pxFac_{Chl,a,v,t}} \right)^{\sigma_{Par,a,v}^{xFac}}} xFac_{Par,a,v,t} \right) \end{aligned} $	(3)
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The dual price index $pxFacIND_{Par,a,v,t}$ in equation (2) is determined in equation (4).

$ \begin{aligned} & pxFacIND_{Par,a,v,t} \\ & = \left(\sum_{Chl} \alpha_{Chl,Par,a,v}^{xFac} (1 + \chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}) (1 \right. \\ & \quad \left. + twist_{Chl,Par,a,v,t}) \lambda_{Chl,a,v,t}^{xFac} \frac{-\sigma_{Par,a,v}^{xFac}}{pxFac_{Chl,a,v,t}} \right)^{\frac{1}{-\sigma_{Par,a,v}^{xFac}}} \end{aligned} $	(4)
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In the additivity case, one needs to define the revenue exhaustion price $pxFac_{Par,a,v,t}$ as a function of $pxFacIND_{Par,a,v,t}$ as follows:

$ \begin{aligned} & pxFac_{Par,a,v,t} = \left(\sum_{Chl} \alpha_{Chl,Par,a,v}^{xFac} (1 + \chi_{Chl,Par,a,v,t}^{\alpha_{xFac}}) (1 \right. \\ & \quad \left. + twist_{Chl,Par,a,v,t}) \lambda_{Chl,a,v,t}^{xFac} \frac{-\sigma_{Par,a,v}^{xFac}}{pxFac_{Chl,a,v,t}} \right)^{\frac{1}{-\sigma_{Par,a,v}^{xFac}}} pxFacIND_{Par,a,v,t}^{\sigma_{Par,a,v}^{xFac}} \end{aligned} $	(5)
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Equations (6) and (7) link $xFac_{Chl,a,v,t}$ and $pxFac_{Chl,a,v,t}$ with the intermediate Armington demand $xa_{i,a,t}$ and the user-specific price $paf_{i,a,t}$ for commodity i . The intermediate demand for commodity i from activity a is $xa_{i,a,t}$ which is the sum over the vintages of the factor demands for this commodity, while the prices faced by the vintages are identical and equal to the tax inclusive use price.

$xa_{i,a,t} = \sum_v xFac_{i,a,v,t}$	(6)
$pxFac_{i,a,v,t} = \chi_i^{pa} paf_{i,a,t}$	(7)

A so-called twist is a cost-neutral shift in two share parameters in a CES nest. In MANAGE-WB, it is typically used to target a cost share or total cost of a specific production factor. One might be tempted to simply add respectively decrease these parameters by the same amount, but doing so neglects the impact of a changed price relation compared to the benchmark. The update formula is therefore somewhat more evolved and requires the updated cost share of one of the children, called here $shrxFac$ and defined in equation (9) below. For a derivation, see van der Mensbrugghe (2020). If the twist option is active for a combination of two children in a parent nest, the following equation defines $twist_{Chl,Par,a,v,t}$ where $\chi_{Par,a,t}^{twFa}$ and $\chi_{Par,t}^{twF}$ are endogenous or exogenous productivity shifters affecting the demand of the parent nest.

$twist_{Chl,Par,a,v,t} = \frac{1 + (\chi_{Par,a,t}^{twFa} + \chi_{Par,t}^{twF})}{1 + ((\chi_{Par,a,t}^{twFa} + \chi_{Par,t}^{twF})shrxFac_{Chl,Par,a,old,t-1})} (1 + twist_{Chl,Par,a,v,t-1}) tw_{Par,a,v,t}$	(8)
$shrxFac_{Chl,Par,a,old,t} = \frac{\sum_v xFac_{Chl,a,v,t} pxFac_{Chl,a,v,t}}{\sum_v xFac_{Par,a,v,t} pxFac_{Par,a,v,t}}$	(9)

Equation (10) determines the level of input augmenting productivity change:

$\lambda_{fpNest,a,v,t}^{xFac} = \chi_{fpNest,t}^{xFacf} \chi_{a,t}^{xFacA} \chi_{v,t}^{xFacV} \chi_{fpNest,a,t}^{xFacFA} (1 + \chi_t^{xFacL}) (1 + \chi_t^{xFacK}) (1 + \chi_t^{xFacKL}) (1 + \chi_t^{xFacFP}) \chi_{aagr,t}^{xFacAgr} (1 + \chi_t^{xFacNRG})$	(10)
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where the set $fpNest$ contains all Chl and Par sets, $\chi_{fpNest,t}^{xFacf}$ is the general productivity shifter for nest $fpNest$ for all activities and vintages, $\chi_{a,t}^{xFacA}$ is the sector-specific productivity shifter for all vintages and nests, $\chi_{v,t}^{xFacV}$ is the vintage-specific productivity shifter for activities and nests, $\chi_{fpNest,a,t}^{xFacFA}$ is the activity- and nest-specific productivity shifter for all vintages, χ_t^{xFacL} is the productivity shifter for labor in all sectors, χ_t^{xFacK} is the productivity shifter for capital in all sectors, χ_t^{xFacKL} is the productivity shifter capital and labor, χ_t^{xFacFP} is the productivity shifter for natural factors (land, labor, capital etc.), $\chi_{aagr,t}^{xFacAgr}$ is the productivity shifter for a group of activities and $\chi_t^{xFacNRG}$ is the productivity shifter for energy carriers.

Equation (11) determines the aggregate unit cost for each sector $px_{a,t}$ as the weighted average of the vintage-specific unit costs, using vintage-specific output levels as weights. In this equation, the subscript $fptop$ stands for the top node of the nested structure – i.e., output. Equation (12) determines the final market price for output $pp_{a,t}$ that is equal to the aggregate unit cost augmented by the ad-valorem output tax ($\tau_{a,t}^{Prd}$ and $\tau_{a,t}^{Phn}$), the ad-valorem subsidy $\tau_{a,t}^{Psb}$ and the emission tax $\tau_{em,a,t}^{Emi}$. The emission tax is a function of the emissions per unit of activity $\rho_{em,a}^{EmiXP}$, a parameter determining whether the sector's emissions are taxed $\phi_{em,a,t}^{EmiXP}$, and an emission- and sector-specific tax value shifter $\chi_{em,a,t}^{EmiXP}$. The latter allows to differentiate the emission tax rates by the type of emission and sector.

The equivalence of the tax-adjusted unit cost to the output price is an implication of two key assumptions: constant-returns-to-scale technologies where average costs are equal to marginal costs, and perfect competition such that no mark-ups over marginal after-tax costs occur. These assumptions also exclude fixed production costs.

$px_{a,t} xp_{a,t} = \sum_v xFac_{fptop,a,v,t} pxFac_{fptop,a,v,t}$	(11)
$pp_{a,t} = (1 + \tau_{a,t}^{Prd} + \tau_{a,t}^{Psb} + \tau_{a,t}^{Phn}) px_{a,t} + \sum_{em} \tau_{em,a,t}^{Emi} \phi_{em,a,t}^{EmiXP} \chi_{em,a,t}^{EmiXP} \rho_{em,a}^{EmiXP}$	(12)

Equation (13) allows to target the $yield_{fp,a,t}$ for factors (output per unit of factor use) of production in each sector. It is active if the yield is fixed.

$xp_{a,t} = m_{dmgAgg_{dmg, "fp,a"}} yield_{fp,a,t} \sum_v xFac_{fp,a,v,t}$	(13)
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MANAGE-WB allows for differentiated labor productivity for primary, secondary and tertiary sectors of production during baseline construction. The variable $labProd_t$ refers to the average labor productivity in service sectors and is implicitly defined by the targeted real GDP level. Labor productivity differences for non-service sectors are a quadratic function of real GDP growth as expressed in equation (14). For details, see Roson (2019) and Roson and Britz (2021).

$\chi_{fp,a,t}^{xFacFA} = \min \left(-labProd_t, -labProd_t \left(prodShift_{a',a} + prodShift_{b',a} \left(\frac{rGDPmp_t}{rGDPmp_{t-1}} - 1 \right) + prodShift_{c',a} \left(\frac{rGDPmp_t}{rGDPmp_{t-1}} - 1 \right)^2 \right) + 1 + labProd_{t-1} \right)$	(14)
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As in other CGE models, the supply of land and natural resources is not directly linked to population growth such as in case of labor or to capital accumulation and solely reacts with a typical low factor supply elasticity to changes in land rental rates. In a growing economy, this implies that land rental price may grow at an unreasonable rate. This can be addressed by assuming productivity improvement in the use of land and natural resources. In the model, equations (15) and (16) provide an option to link productivity shifts for land and natural resources with a multiplicative factor to changes in labor productivity – either with $labProd_t$ if differentiated labor productivity is selected or χ_t^{xFacL} if not.

$\chi_{lnd,t}^{xFacF} = 1 - \min(0, -(labProd_t + labProd_{t-1}) \text{ or } \chi_t^{xFacL})$	(15)
$\chi_{natres,t}^{xFacF} = 1 - \min(0, -(labProd_t + labProd_{t-1}) \text{ or } \chi_t^{xFacL})$	(16)

Multi-input and multi-output specifications

This section of the supply block describes the multi-input and multi-output specifications of the model. Multi-input allows for multiple activities producing the same commodity which can be quality differentiated across these activities based on a CES function or assumed to be homogenous implying the law-of-one-price such that the unit cost of production is identical across activities. A multi-input specification is useful, for instance, to depict electricity production differentiated by hydro, solar, oil or coal powered such that power mix adjust under carbon taxation. The multi-output part of the model allows for joint production of multiple outputs by the same activity, for example producing both ethanol and DDGS in the ethanol production activity.

To address the multi-output case, activity a produces quantities $x_{a,i,t}$ of a suite of commodities indexed by i . In the case of a diagonal make matrix, each activity will produce exactly one commodity. Distributing the total production capacity of activity a to its potentially multiple outputs is implemented on the basis of a Constant-Elasticity-of-Transformation (CET) structure, where ω_a^p depicts the elasticity of transformation. Using a transformation elasticity of zero implies fixed physical output shares. Equation (17) defines the supply of $x_{a,i,t}$, emanating from production output $xp_{a,t}$ where the law-of-one-price holds in the case of infinite transformation. Equation (18) represents the zero-profit condition or the revenue balance for the multi-output production function. Again, $\chi_{a,i,t}^p$ is a set of parameters used as

normalization factors so that the price variables are initialized at 1. Wherever the normalized prices are used in equations, they need to be scaled by the relevant $\chi_{a,i,t}^p$ to preserve the correct accounting.

$\left\{ \begin{array}{l} x_{a,i,t} = ap_{a,i,t} \left(\frac{\chi_{a,i,t}^p p_{a,i,t}}{pp_{a,t}} \right)^{\omega_a^p} xp_{a,t} \quad \text{if } \omega_a^p \neq \infty \\ \chi_{a,i,t}^p p_{a,i,t} = PP_{a,t} \quad \text{if } \omega_a^p = \infty \end{array} \right\}$	(17)
$\left\{ \begin{array}{l} pp_{a,t} = \left(\sum_i ap_{a,i,t} \chi_{a,i,t}^p p_{a,i,t}^{1+\omega_a^p} \right)^{\frac{1}{1+\omega_a^p}} \quad \text{if } \omega_a^p \neq \infty \\ xp_{a,t} = \sum_i \chi_{a,i,t}^p x_{i,a,t} \quad \text{if } \omega_a^p = \infty \end{array} \right\}$	(18)

In the next step, multiple streams of output, such as electricity produced from oil, gas, coal etc., can be combined into a single supplied commodity $xs_{i,t}$ based either on a CES aggregator or assuming homogeneity. In the latter case, the cost of each stream must be equal, considering the price normalization factors. Equation (19) determines the demand for produced commodity $x_{a,i,t}$. In the case of a finite elasticity of substitution σ_i^s , it is a CES formulation. With an infinite substitution elasticity, the law-of-one price must hold – i.e., the producer price of each component must be equalized in efficiency units. Equation (20) determines the equilibrium condition in the form of the cost function or (primal) volume equality if the law-of-one price holds. The formulas allow for shifts in preferences via the $\gamma_{a,i,t}^p$ parameter. One possible preference shift can emerge from a cost neutral shift in the preference for one component of the CES function.

$\left\{ \begin{array}{l} x_{a,i,t} = as_{a,i,t} (\gamma_{a,i,t}^p)^{\sigma_i^s - 1} \left(\frac{\chi_i^{ps} ps_{i,t}}{\chi_{a,i,t}^p p_{a,i,t}} \right)^{\sigma_i^s} xs_{i,t} \quad \text{if } \sigma_i^s \neq \infty \\ p_{a,i,t} \chi_{a,i,t}^p = \chi_i^{ps} ps_{i,t} \quad \text{if } \sigma_i^s = \infty \end{array} \right\}$	(19)
$\left\{ \begin{array}{l} ps_{i,t} \chi_i^{ps} = \sum_a \left(as_{a,i,t} \left(\frac{\chi_{a,i,t}^p p_{a,i,t}}{\gamma_{a,i,t}^p} \right)^{1-\sigma_i^s} \right)^{\frac{1}{1-\sigma_i^s}} \quad \text{if } \sigma_i^s \neq \infty \\ xs_{i,t} = \sum_a x_{a,i,t} \quad \text{if } \sigma_i^s = \infty \end{array} \right\}$	(20)

The optimality conditions and the dual price indexes in equations (21) and (22) are derived in the case of a normal CES structure. In the case of an additive CES structure with $\sigma_i^s \neq \infty$, they can be written as follows:

$x_{a,i,t} = as_{a,i,t} (\gamma_{a,i,t}^p)^{-\sigma_i^s} \left(\frac{\chi_i^{ps} pIND_{i,t}}{\chi_{a,i,t}^p p_{a,i,t}} \right)^{\sigma_i^s} xs_{i,t}$	(21)
$pIND_{i,t} \chi_i^{ps} = \sum_a \left(as_{a,i,t} (\gamma_{a,i,t}^p \chi_{a,i,t}^p p_{a,i,t})^{-\sigma_i^s} \right)^{\frac{1}{-\sigma_i^s}}$	(22)

Again, the revenue exhaustion price in the case of an additive CES function needs to be defined as follows:

$ps_{i,t} = \left(\sum_a as_{a,i,t} (\gamma_{a,i,t}^P)^{-\sigma_i^S} p_{a,i,t}^{1-\sigma_i^S} \right) pIND_{i,t}^{\sigma_i^S}$	(23)
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Equation (24) computes the share of electricity generated with each electricity source $elecMix_{aElec,iElec,t}$ where $aElec$ is a set of activities producing a set of electricity commodities $iElec$.

$elecMix_{aElec,iElec,t} = \frac{x_{aElec,iElec,t}}{\sum_{aElec} x_{aElec,iElec,t}}$	(24)
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Income block

Households

The first block of equations below describes income generation. Equation (25) defines gross factor income $factY_{fp,t}$ – i.e., the pre-tax level of total factor remuneration from each factor of production defined in the fp set in each activity a and vintage v covering wages, capital, natural resource and land rents. Pre-tax refers here to factor taxes paid by the activities. Equation (26) defines gross profits $kapY_t$ as the sum of all capital earnings. Equation (27) defines gross income $yH_{h,t}$ for household h based on three terms. The first one distributes the economy-wide payments of each factor of production across households based on their shares in the respective factor income governed by the matrix of coefficients $\chi_{h,fp,t}^{fac}$. It is used for the factors of production where one aggregate agent manages the economy-wide factor stocks. The second term considers the case in which factor stocks are allocated and managed by the households (for more, see the section on factor supplies).

The third term adds up $transfers_{h,inst,t}$ from other institutions to household h and the fourth term is the debt payment (principal and interest) received from the government $debtPay_{h,t}$. Household's disposable income $yHDsp_{h,t}$ is defined in equation (28) as the gross income, net of personal income taxes and deducting transfers $totTr_{h,t}$ from household h to the other institutions.

$factY_{fp,t} = \sum_{a,v} pxFac_preTax_{fp,a,v,t} xFac_{fp,a,v,t}$	(25)
$kapY_t = \sum_{cap} factY_{cap,t}$	(26)
$yH_{h,t} = \sum_{fp} \chi_{h,fp,t}^{fac} factY_{fp,t} + \sum_{fmp,vf} xfmSup_{fmp,fmp,h,vf,t} pfmSup_{fmp,fmp,h,vf,t} + \sum_{inst} transfers_{h,inst,t} + debtPay_{h,t}$	(27)
$yHDsp_{h,t} = yH_{h,t} (1 - t_{h,t}^{Pit}) - totTr_{h,t}$	(28)

The next block of equations shows the allocation of households' income. Equation (29) determines household savings $savH_{h,t}$ which is a share $asav_{h,t}$ of the disposable income and evolves with a shifter

χ_t^{ms} and the returns to savings $fundPrice_t$ relative to a composite price $pindH_{h,t}$. The GDP deflator $pGDPmp_t$ enters in these equations to maintain the stationarity of the ratio in the event of positive inflation. The price dependence of the marginal propensity to save vanishes if ω_h^{sav} is set to zero. According to equation (30), $pindH_{h,t}$ is a composite price index of the returns to savings (a weighted average of expected returns to new investment and the interest rate on new government debt, see section on domestic fund markets below) and the aggregate expenditure price for household consumption $pf_{h,t}$. Equation (31) defines the marginal propensity to save. If not fixed, it serves for information only. Equation (32) specifies the level of transfers from households to other institutions, $totTr_{h,t}$ as a share of total income. Equation (33) determines available household income for purchases of goods and services $yf_{h,t}$ as the difference between disposable income and savings.

$savH_{h,t} = \chi_t^{ms} asav_{h,t} yHDsp_{h,t} \left(\frac{fundPrice_t pGDPmp_t}{pindH_{h,t}} \right)^{\omega_h^{sav}}$	(29)
$pindH_{h,t} = \left(asav_{h,t} (pGDPmp_t fundPrice_t)^{\omega_h^{sav}} + (1 - asav_{h,t}) pf_{h,t}^{\omega_h^{sav}} \right)^{\frac{1}{\omega_h^{sav}}}$	(30)
$aps_{h,t} = \frac{savH_{h,t}}{yHDsp_{h,t}}$	(31)
$totTr_{h,t} = \chi_{h,t}^{totTr} yH_{h,t}$	(32)
$yf_{h,t} = yHDsp_{h,t} - savH_{h,t}$	(33)

Equation (34) approximates the Gini coefficient using income of households in different income groups $yH_{h,t}$ the relative position of each element in the set of households (usually this will be the position of a decile in income distribution) ord_h and the total number of household groups $card_h$. Equation (35) computes the dependency ratio as a ratio of the population of a given cohort to the working-age population.

$gini_t = 2 \frac{\sum_h ord_h yH_{h,t}}{\sum_h card_h yH_{h,t}} - \frac{card_h + 1}{card_h}$	(34)
$depRate_{cohort,t} = \frac{pop_{cohort,t}}{pop_{15-64,t}}$	(35)

Government

Government tax revenues stem from multiple tax streams $yG_{g,t}$ where the set g comprises the following tax elements: factor taxes in equation (36), subsidies⁴ on production in equation (37), production taxes in equation (38), phantom taxes in equation (39), domestic and import value-added taxes in equations (40) and (41), export and import taxes in equations (42) and (43), total agent-specific sales taxes and subsidies in equation (44), taxes on emissions (from both the use of energy commodities and process emissions in equations (45) and (46), respectively) and direct taxes in equation (47). Note that there is no set called dtx in the model code. It is introduced here for convenience only. Rather, the model code

⁴ Subsidies are considered as negative taxes in the model. In general, however, they are viewed as public expenditure in government statistics.

calculates household-specific direct tax revenue under $yG_{pit,h,t}$ and enterprise-specific direct tax revenue under $yG_{cit,entr,t}$.

Whereas production taxes and subsidies are directly observable in the data base, phantom taxes are not observable but can be used to recover exogenous sector shares, for instance, for different activities producing electricity. The sum of the phantom tax revenue is fixed to zero to render them revenue neutral. For instance, fossil-fuel based power sectors can be taxed and renewables subsidized. Keeping the sum of these taxes and subsidizes at zero will imply that the phantom taxation does not change (directly) the average electricity price.

$yG_{fac,t} = \sum_a \sum_v \tau_{a,fp,t}^{fac} px_{fac_preTax_{fp,a,v,t}} x_{Fac_{fp,a,v,t}}$	(36)
$yG_{psb,t} = \sum_a \tau_{a,t}^{psb} px_{a,t} xp_{a,t}$	(37)
$yG_{prd,t} = \sum_a \tau_{a,t}^{prd} px_{a,t} xp_{a,t}$	(38)
$yG_{phn,t} = \sum_a \tau_{a,t}^{phn} px_{a,t} xp_{a,t} = 0$	(39)
$yG_{vatDom,t} = \sum_i (\chi_{i,t}^{pd} pd_{i,t} + pMarg_{i,t} tmg_{i,t}^d) \tau_{i,t}^{vat,d} xd_{i,t}$	(40)
$yG_{vatImp,t} = \sum_i (eR_t(1 + \tau_{i,t}^{imp}) pwm_{i,t} + pMarg_{i,t} tmg_{i,t}^m) \tau_{i,t}^{vat,m} xm_{i,t}$	(41)
$yG_{exp,t} = \sum_i \tau_{i,t}^{exp} \chi_{i,t}^{pe} pe_{i,t} xe_{i,t}$	(42)
$yG_{imp,t} = \sum_i \tau_{i,t}^{imp} eR_t pwm_{i,t} (xm_{i,t} + mdelst_{i,t})$	(43)
$yG_{paf,t} = \sum_i \sum_{aa} \tau_{i,aa,t}^{paf} \chi_i^{pa} pa_{i,t} xa_{i,aa,t}$	(44)
$yG_{emi,t} = \sum_{aa} \sum_{emSrc} \sum_{em} \tau_{em,t}^{emi} emi_t^{cal} \chi_{em,i,aa,t}^{emi} \rho_{em,emSrc,aa}^{emi} \phi_{em,emSrc,aa}^{emi} XA_{emSrc,aa,t}$	(45)
$yG_{emiXp,t} = \sum_a \sum_{em} \tau_{em,a,t}^{emi} \rho_{em,a}^{emiXp} \phi_{em,a,t}^{emiXp} \chi_{em,a,t}^{emiXp} xp_{a,t}$	(46)
$Y_{Gdtx,t} = \sum_h \tau_{h,t}^{pit} yH_{h,t} + \sum_{entr} \tau_{h,t}^{cit} entrY_{entr,t} + gTax_t$	(47)

For information, the model code calculates factor tax revenue by activities for each production factor, agent-specific tax revenue for each commodity under $yG_{paf,i,t}$, and emissions tax revenue from energy consumption for Armington agents under $yG_{emi,aa,t}$ and from process emissions for activities under $yG_{emiXp,a,t}$.

The next block of equations determines closures for the government accounts. Equation (48) defines the total revenues for the government $yGov_t$, comprised of three components: aggregate tax revenues net of subsidies yG_{Tot}_t as defined in equation (49), transfers from the other institutions and shares on factors income. Equation (50) calculates government direct taxes on its revenue. Equation (51) shows that total public transfers are a share, $\chi_{gov,t}^{tottr}$, of gross domestic product (GDP at market prices), and are

affected by $fiscalRespDrv_t$ generated by the difference between actual and target debt-to-GDP ratios, when the fiscal responsibility option is activated. The term $relExpInc$ captures the strength of the debt reducing policy via expenditure cuts.

$yGov_t = yGTot_t + \sum_{inst} transfers_{gov,inst,t} + \sum_{fp} \chi_{gov,fp,t}^{Fac} factY_{fp,t}$ $+ \sum_{fmp,vf} xfmSup_{fmp,fmp,gov,vf,t} pfmSup_{fmp,fmp,gov,vf,t}$	(48)
$yGTot_t = \sum_g yG_{g,t}$	(19)
$gTax_t = dgovTax_t yGov_t$	(50)
$totTr_{gov,t} = \chi_{gov,t}^{tottr} (1 + relExpInc/100 fiscalRespDrv_t) GDPmp_t$	(51)

The primary balance $primaryBalance_t$ as defined in equation (52) is the difference between total government revenue $yGov_t$ minus taxes paid by government, public transfers, public consumption and government (productive and adaptive) investments. It does not consider any financial flows related to government debt. The fiscal balance $fiscalBalance_t$, as defined in equation (53), considers additionally interest payments, $DebtInterest_{instfd,t}$, to foreign and domestic debt holders $instfd$, while the government balance in equation (54) takes additionally the principal payments $DebtPrincipal_{inst,t}$ into account.

In case of a positive $govBalance_t$, the government will provide additional savings to finance private investments, otherwise, this balance will be equal to new government debt $debtNew_t$ (see section on fund markets).

$primaryBalance_t$ $= yGov_t - gTax_t - totTr_{gov,t} - yf_{gov,t} - \sum_{ginv} yf_{ginv,t}$ $- \sum_{adpinv \in GOV} yf_{adpinv,t}$	(52)
$fiscalBalance_t = primaryBalance_t - \sum_{instfd} debtInterest_{instfd,t}$	(53)
$govBalance_t = fiscalBalance_t - \sum_{instfd} debtPrincipal_{instfd,t}$	(54)

Enterprises

The next set of equations relates to the enterprise accounts as pass-through accounts with minimal behavior.⁵ Equation (55) defines enterprise incomes from three sources: from factors of production,

⁵ Enterprises represent financial and nonfinancial corporations in the System of National Accounts.

income from transfers, and from debt payments received. Equation (56) determines enterprise savings that add to the funds available for savings/investment and domestic new government debt, depending on the share parameter $asavEntr_t$ and a shifter variable χ_t^{ms} . The share reacts to changes in the returns to the average return to savings, $fundPrice$ if $\omega_{savEntr,t}$ is non-zero. Equation (57) determines aggregate enterprise transfers as a residual, after direct taxes paid by the enterprise and enterprise savings are subtracted from total enterprise income.

$entrY_{entr,t} = \sum_{fp} \chi_{entr,fp,t}^{fac} factY_{fp,t} + \sum_{fmp,vf} xfmSup_{fmp,fmp,entr,vf,t} pfmSup_{fmp,fmp,entr,vf,t} + \sum_{inst} transfers_{entr,inst,t} + debtPay_{entr,t}$	(55)
$savEntr_{entr,t} = \chi_t^{ms} asavEntr_t entrY_{entr,t} (1 - \tau_{entr,t}^{cit}) fundPrice_t^{\omega_{savEntr}}$	(56)
$totTr_{entr,t} = entrY_{entr,t} (1 - \tau_{entr,t}^{cit}) - savEntr_{entr,t}$	(57)

Transfers

The next set of equations in the income block allocates transfers across institutions, handling the domestic and foreign transfers separately. Equation (58) allocates the total transfers of domestic agents $totTr_{dinst,t}$, indexed by $dinst$, using a matrix of distribution coefficients $\chi_{inst,dinst,t}^{tr}$ to $inst$. The transfers from the rest of world $transfers_{inst,t}^{ROW}$ are exogenous and fixed in foreign currency terms and are converted to domestic currency units by the exchange rate eR_t in equation (59).

$transfers_{inst,dinst,t} = \chi_{inst,dinst,t}^{tr} totTr_{dinst,t}$	(58)
$transfers_{inst,row,t} = eR_t transfers_{inst,t}^{ROW}$	(59)

Domestic fund markets

MANAGE-WB defines a virtual domestic investor who collects households' saving $savH_{h,t}$ and enterprise savings $savEntr_t$ into a variable called $fundT_t$ as defined in equation (60):

$fundT_t = \sum_h savH_{h,t} + \sum_{entr} savEntr_{entr,t}$	(60)
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The investor distributes $fundT_t$ to finance private investments $fundP_t$ and the government new domestic debt $fundG_t$ in equation (61).

$fundT_t = fundP_t + fundG_t$	(61)
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The distribution is based on maximizing revenues under an additive CET function and equation (62) shows the composite price $fundPriceD_t$ as an average of relative changes in the interest rate $interest_t$ and in the expected returns to capital $rore_t$.

$fundPriceD_t = \left(aFundG_t \left(\frac{interest_t}{interest_{t0}} \right)^{\omega_{fund}} + (1 - aFundG_t) \left(\frac{rore_t}{rore_{t0}} \right)^{\omega_{fund}} \right)^{\frac{1}{\omega_{fund}}}$	(62)
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The normalization with the benchmark variables $interest_{t0}$ and $rore_{t0}$ implies that $fundPriceD_t$ is equal to unity at the benchmark. $aFundG_t$ and $aFundP_t$ are share parameters calibrated from the benchmark portfolio composition. If ω_{fund} is zero, these distribution shares are fixed.

The government's new domestic debt $fundG_t$ is defined in equation (63):

$fundG_t = aFundG_t \chi_t^{fundG} fundT_t \left(\frac{interest_t}{interest_{t0} FundPriceD_t} \right)^{\omega_{fund}}$	(63)
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The additive CET form implies that the dual price aggregator $fundPriceD_t$ is not equal to the exhaustion price $fundPrice_t$ which is therefore defined in equation (64):

$fundPrice_t fundT_t = \frac{interest_t}{interest_{t0}} fundG_t + \frac{rore_t}{rore_{t0}} fundP_t$	(64)
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The expected rate of return to capital $rore_t$ is defined as:

$rore_t = \left(\frac{kstocke_t}{kstock_t} \right)^{\varepsilon_t^{ror}} rorc_t$	(65)
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where the cost adjusted rate of return to capital $rorc_t$, often called Tobin's Q, is determined by the economy-wide return to capital $trent_t$ and the price index of investments $pf_{inv,t}$ and the depreciation rate $depr_t$. If the forward looking option is used, $rore_t$ is defined over a set of the current and future years $tNext$ and related weights $exptWgts_{tNext}$:

$rore_t = \frac{\sum_{tNext \geq t} exptWgts_{tNext} \left(\frac{kstocke_t}{kstock_{tNext}} \right)^{\varepsilon_t^{ror}} rorc_{tNext}}{\sum_{tNext \geq t} exptWgts_{tNext}}$	(66)
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The economy-wide depreciation rate $depr_t$ is a weighted average of the activity specific depreciation rates $depr_{a,t}$, which can be subject to climate change damages to capital stocks captured by the macro $m_dmgAgg_{dmgDepr,"a"}$. The capital demands $xFac_{cap,a,v,t}$ act as weights.

$rorc_t = \frac{trent_t}{pf_{inv,t}} - depr_t$	(67)
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$depr_t = \frac{\sum_{cap} \sum_a \sum_v xFac_{cap,a,v,t} depr_{a,t} m_dmgAgg_{dmgDepr,"a"}}{\sum_{cap} \sum_a \sum_v xFac_{cap,a,v,t}}$	(68)
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Another relevant variable is the capital utilization rate ror_t which is determined in the following equation.

$ror_t = \left(\frac{trent_t}{trent_{t0} pGDPmp_t} \right)^{\varepsilon^{ror}}$	(69)
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The capital utilization rate ror_t is a ratio between capital stock $kStock_t$ and capital use $kapS_t$ in equation (70). $kapS_t$ is equal to the economy-wide supply of capital and thus exhausts the sum of capital demands of the production activities. If ε^{ror} is zero, the capital utilization rate is constant. A non-zero ε^{ror} implies that an increase in the returns to capital leads to a higher utilization rate of capital stock and vice-versa. This, in turn, should have consequences on the capital depreciation rate, but this feedback is not yet captured in the model. Accordingly, it is recommended to keep ε^{ror} at zero.

$kapS_t = ror_t kStock_t$	(70)
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New foreign government debt

The contribution of the ROW to new government debt $fundF_t$ is captured by equation (71). The variable $fundF_t$ is defined as a share of domestic GDP at current market prices $GDPmp_t$, modified by changes in the interest rate on foreign debt of government $debtIntF_t$ according to elasticity σ^{fint} and a sovereign risk correction term driven by a sigmoid function depending on $debtGDP_t$, the debt to GDP ratio:

$fundF_t = \gamma_t^{fundF} (interest_t riskPrem_{instf,t})^{\sigma^{fint} \frac{GDPmp_t}{GDPmp_{t0}}} [1 - \min(0, (sigmoid([debtGDP_t - 0.5] * 4) - 0.5) * 2)]$	(71)
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The risk correction term $-\min(0, (sigmoid([debtGDP_t - 0.5] * 4) - 0.5) * 2)$ is equal to zero at a debt to GDP ratio below 50%, ~0.2 at 60%, ~0.38 at 70%, ~0.54 at 80%, ~0.66 at 90%, 0.76 at 100%, showing the typical saturation towards unity for a sigmoid function. The correction term implies that foreign investors will require higher interest rates for the same amount of new debt at given GDP if the debt to GDP ratio increases.

Government borrowing

The amount of government borrowing $debtNewT_t$ in equation (72) is the new gross debt and equal to the difference between government income $yGov_t$ and government spending (consumption, investments, transfers, debt principal payment and interest on debt) – i.e., negative $govBalance_t$.⁶

$debtNewT_t = \min(0, govBalance_t)$	(72)
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The total amount of new debt, as defined in equation (73), is equal to the sum of foreign and private funds allocated to government debt:

⁶ The minimum operator is not differentiable. To avoid related numerical problems, it is technically implemented in the code based on a differentiable smoothing function.

$debtNewT_t = fundG_t + fundF_t$	(73)
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Equations (74) and (75) correspond to the demand and supply of government total borrowing and defines the interest rate $interest_t$ paid on new government debt. In addition, the following conditions hold where the sets $instd$ and $instf$ have domestic and foreign institutions respectively.

$debtNew_{instd,t} = fundG_t$	(74)
$debtNew_{instf,t} = fundF_t$	(75)

Debt stocks and savings

The next set of equations governs the dynamics of government debt. Equation (76) determines the stock of government debt held by institutions $debtStk_{instfd,t}$, indexed with $instfd$ which contains both domestic and foreign institutions. This stock is equal to the stock in the previous period plus the new government debt $debtNew_{instfd,t}$ minus debt principal payments $debtPrincipal_{instfd,t}$. Equation (77) calculates the total debt stock $debtStkT_t$ by summing over the debt stocks of the different institutions. Equation (78) defines the debt-to-GDP ratio $debtGDP_t$. Equation (79) determines the total debt payments by government to different institutions $instfd$. It considers the interest payments $debtInterest_{instfd,t}$ as defined in equation (80) and principal payments $debtPrincipal_{instfd,t}$ as defined in equation (81). $rDebt_t^{StkT}$ and $rDebt_{instfd,t}^{Stk}$ are common and institute specific repayment rates respectively, while $avgDebtInt_{instfd,t}$ is the average interest rate, calculated in equation (82) as the weighted average of past interest rates. Equation (83) computes the total new debt.

$debtStk_{instfd,t} = debtStk_{instfd,t-1} + debtNew_{instfd,t} - debtPrincipal_{instfd,t}$	(76)
$debtStkT_{instfd,t} = \sum_{instfd} debtStk_{instfd,t}$	(77)
$debtGDP_t = \frac{debtStkT_t}{GDPmp_t}$	(78)
$debtPay_{instfd,t} = debtInterest_{instfd,t} + debtPrincipal_{instfd,t}$	(79)
$debtInterest_{instfd,t} = avgDebtInt_{instfd,t} (debtStk_{instfd,t} - debtNew_{instfd,t})$	(80)
$debtPrincipal_{instfd,t} = rDebt_t^{StkT} rDebt_{instfd,t}^{Stk} debtStk_{instfd,t-1}$	(81)
$avgDebtInt_{instfd,t} = \frac{\sum_{\tau=0}^{t-1} \max(0, debtNew_{instfd,t} (1 - rDebt_{instfd,t}^{Stk})) interest_{\tau} riskPrem_{instfd,t}}{\sum_{\tau=0}^{t-1} \max(0, debtNew_{instfd,t} (1 - rDebt_{instfd,t}^{Stk}))}$	(82)
$debtNewT_t = \sum_{instfd} debtNew_{instfd,t}$	(83)

Equation (84) is the ubiquitous investment equals savings equation. Nominal productive investment $yf_{inv,t}$ is equal to $fundP_t$, the amount of households and enterprise savings not used for new domestic government debt, government savings (in case of a positive government balance) and FDI (the difference between total foreign savings $savF_t$ multiplied by the exchange eR_t and new foreign government debt

$fundF_t$). In order to define productive investments, the value of stock building⁷ and non-productive private investment dedicated to climate adaptation are subtracted.

$yf_{inv,t} = fundP_t - \max(0, -govBalance_t) + eR_t savF_t - fundF_t - tdelst_t - \sum_{adpinv \in inv} yf_{adpinv,t}$	(84)
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Demographic equations

The following block of equations defines a set of demographic variables. Equations (85) and (86) define population by age $popAge_{age,t}$. $popAge_{age=1,t}$ is the birth-rate $brate_t$ times the last period working age population between 15 and 64 years old $pop_{15-64,t-1}$. $popAge_{age>1,t}$ is determined by the deathrate by age $drate_{age,t}$ and the population in previous age and climate damage $m_dmgAgg_{dmg,"age"}$. Equation (87) aggregates population by age into cohorts. Equation (88) computes the population of each household group, h where $t0$ implies the initial period and assumes that all households grow with the same factor.

$popAge_{age=1,t} = brate_t pop_{15-64,t-1}$	(85)
$popAge_{age,t} = popAge_{age-1,t-1} drate_{age,t} + m_dmgAgg_{dmg,"age"}$	(86)
$pop_{cohorts,t} = \sum_{age \in cohorts} popAge_{age,t}$	(87)
$pop_{h,t}^h = \frac{pop_{h,t0}^h}{pop_{total,t0}} pop_{total,t}$	(88)

Demand block

Households

The top-level nest of the households' demand system is depicted by a constant-difference-in-elasticity (CDE) utility function, the demand system found in the GTAP standard model. It defines the demand for single products or product bundles which are CES composite of commodities or other CES composites. This demand system is defined by a flexible nesting structure. Shares of a commodity can be allocated to multiple nests so that the total demand for it can be added up over multiple nests. For instance, part of energy demand might be allocated to a transport service and the remainder to a household (heating, cooking, lighting etc.,) bundle. The CDE is based on an implicitly additive utility function and can collapse to a CES utility function under certain conditions. As it comprises three vectors of product (nest) specific parameters, it provides a fair amount of flexibility and is typically calibrated to external estimates of both income and own-price elasticities.

To ease the implementation of the CDE demand system, equation (89) defines an auxiliary variable $cdeInt_{h,k,t}$ for each household h and a product bundle k . Adding up $cdeInt_{h,k,t}$ over k as in equation (90)

⁷The value of imports of stocks is the tariff-inclusive domestic price-excluding margin and other indirect tax wedges.

defines a scaling factor for each household h which is then used in the household's Marshallian demand $xak_{h,k,t}$ for each bundle in equation (91). Equation (92) determines household utility (on per capita basis) $uh_{h,t}$ and is an implicit function of the terms $cdeInt_{h,k,t}$ and the price related parameters $bh_{h,k,t}$. Equation (93) is an implicit function of money metric utility $mmuh_{h,t}$ which can be used to quantify the equivalent variation to estimate welfare effects in monetary terms. Equation (94) defines the budget share $xakShr_{oa,k,t}$ for all final demand agents (oa), including households h , and for bundle k . Equations (95)-(97) describe alternative ways to define the household specific consumer price index (CPI) $pf_{h,t}$. In the default settings, CPI is computed using a Paasche index as in equation (95). Alternatively, the user could choose to use a Laspeyres price index in equation (96), or to compute CPI using a weighted sum of consumer goods prices in equation (97).

$cdeInt_{h,k,t} = akf_{h,k,t}bh_{h,k,t}uh_{h,t}^{eh_{h,k,t}bh_{h,k,t}} \left(\frac{pxak_{h,k,t}}{yf_{h,t}/pop_{h,t}^h} \right)^{bh_{h,k,t}}$	(89)
$consScale_{h,t} = \sum_k cdeInt_{h,k,t}$	(90)
$xak_{h,k,t} = \frac{cdeInt_{h,k,t}}{consScale_{h,t}} \frac{yf_{h,t}}{pxak_{h,k,t}}$	(91)
$\sum_k \frac{cdeInt_{h,k,t}}{bh_{h,k,t}} \equiv 1$	(92)
$\sum_k akf_{h,k,t}uh_{h,t}^{eh_{h,k,t}bh_{h,k,t}} \left(\frac{1}{mmuh_{h,t}/pop_{h,t}^h} \right)^{bh_{h,k,t}} = 1$	(93)
$xakShr_{oa,k,t} = \frac{xak_{oa,k,t}pxak_{oa,k,t}}{yf_{oa,t}}$	(94)
$pf_{h,t} = \left(\sum_i xa_{i,h,t}paf_{i,h,t} \right) / \left(\sum_i xa_{i,h,t}paf_{i,h,t0} \right)$	(95)
$pf_{h,t} = \left(\sum_i xa_{i,h,t0}paf_{i,h,t} \right) / \left(\sum_i xa_{i,h,t0}paf_{i,t0} \right)$	(96)
$pf_{h,t} = \sum_k xakShr_{h,k,t}pkf_{h,k,t}$	(97)

Other final demand for bundles: government and investment

The demand decomposition of the other final demanders (i.e., government and investment) uses CES functions. Equation (98) determines the demands for a top-level bundle k for each other final demand agent f where $xfd_{f,t}$ is the aggregate volume of expenditure and is determined in specific closure rules. Equation (99) reflects the standard CES dual price aggregator determining $pf_{f,t}$.

$xak_{f,k,t} = ak_{f,k,t} xdf_{f,t} \left(\frac{pf_{f,t}}{pxak_{f,k,t}} \right)^{\sigma_f^f}$	(98)
$pf_{f,t} = \left(\sum_k af_{f,k,t} pxak_{f,k,t}^{1-\sigma_f^f} \right)^{\frac{1}{1-\sigma_f^f}}$	(99)

Final demand for sub-demand bundles and single products

The following set of equations refers to the nested CES structure for all final demand accounts oa (households, investment, governments). Demand decomposition of oa uses a flexible CES nesting structure in which $knest$ is a child and a parent and $knest1$ is its alias. Equation (100) determines the demand $xak_{oa,knest,t}$ for a child bundle $knest$ for each oa where $knest1$ are potentially multiple parent nests to which the child bundle is allocated in shares. Equation (101) shows the composite price $pxaf_{oa,knest,t}$ of a bundle which could include commodities captured by $pafi_{i,oa,t}$ and bundles of commodities captured by $pxak_{oa,knest1,t}$ where $knest1$ is a child nest. $\lambda_{oa,i,knest}^{oa}$ and λ_t^{nrg} (which is zero for non-energy commodities) are efficiency variables capturing technological progress. The macro $m_dmgAgg_{dmgDem,"oa,i,knest"}$ captures Climate Damages to final demand efficiency.

$xak_{oa,knest,t} = \sum_{knest1} af_{oa,knest,knest1,t} xak_{oa,knest1,t} \left(\frac{pxak_{oa,knest1,t}}{pxak_{oa,knest,t}} \right)^{\sigma_{oa,knest1}^c}$	(100)
$\begin{aligned} &pxaf_{oa,knest,t} \\ &= \left(\sum_i af_{oa,i,knest,t} \left(\frac{\chi_i^{pa} pafi_{i,oa,t}}{m_dmgAgg_{dmgDem,"oa,i,knest"} \lambda_{oa,i,knest,t}^{oa} (1 + \lambda_t^{nrg})} \right)^{1-\sigma_{oa,knest}^c} \right. \\ &\quad \left. + \sum_{knest1 \in knest} af_{oa,knest1,knest,t} pxak_{oa,knest1,t}^{1-\sigma_{oa,knest}^c} \right)^{\frac{1}{1-\sigma_{oa,knest}^c}} \end{aligned}$	(101)

Equation (102) defines the Armington demands $xa_{i,oa,t}$. The summation over $knest$ and indexing the share parameter $af_{oa,i,knest,t}$ both with i and $knest$ indicates that one product can be demanded through multiple bundles, for instance, fuel demand both by a transport bundle and a heating service bundle. The equation also comprises preference shifters, for instance, related to climate change.

$\begin{aligned} xa_{i,oa,t} = \sum_{i \in knest} af_{oa,i,knest,t} xak_{h,knest,t} &\left(m_dmgAgg_{dmgDem,"oa,i,knest"} \lambda_{oa,i,knest}^{oa} (1 \right. \\ &\left. + \lambda_t^{nrg}) \right)^{\sigma_{oa,knest}^c - 1} \left(\frac{pxak_{oa,knest,t}}{\chi_i^{pa} pafi_{i,oa,t}} \right)^{\sigma_{oa,knest}^c} \end{aligned}$	(102)
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The share parameters in investment demand can be dynamically updated on-demand depending on activity specific investment vectors. In this case, the nesting for investment demand comprises solely one level called “top” with a zero-substitution elasticity in equation (100). The shifters for the investment

demand $\lambda_{inv,i,top}^{inv}$ become endogenous and update the investment composition. The update first defines a volume index for total capital demand $newInv_t$ from the capital demands of the new vintages and a share of each old vintage $oldCapInvShare$ which by default is 0.25%, about half of a typical economy wide depreciation rate. Considering small shares of the *old* vintage in the volume index stabilizes the behavior in case where sectors shrink.

$newInv_t = \sum_a xFac_{a,cap,new,t} + oldCapInvShare \cdot xFac_{a,cap,old,t}$	(103)
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The update of investment shares is based on activity specific share parameters $\gamma_{i,a,t}^{inv}$, weighted according to their contribution to $newInv_t$. The normalization factor $\lambda_{inv,i,t0}^{inv}$ considers the differences between the volume index and total investment demand at the benchmark.

$\lambda_{inv,i,top}^{inv} / \lambda_{inv,i,t0}^{inv} = \sum_a \gamma_{i,a,t}^{inv} (xFac_{a,cap,new,t} + oldCapInvShare xFac_{a,cap,old,t}) / newInv_t$	(104)
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The activity specific share parameters are driven by a CES demand function. The related price index $invPrice_{a,t}$ is defined in equation (105) and used to update the benchmark investment shares of the individual activities in equation (106).

$invPrice_{a,t} = \left[\sum_i \gamma_{i,a,t}^{inv} (\chi_i^{inv} paf_{i,inv,t})^{1-\sigma_a^{inv}} \right]^{1/1-\sigma_a^{inv}}$	(105)
$\gamma_{i,a,t}^{inv} = \gamma_{i,a,t0}^{inv} \left(\frac{invPrice_{a,t}}{\chi_i^{inv} paf_{i,inv,t}} \right)^{\sigma_a^{inv}}$	(106)

This mechanism is especially relevant for stock changes as part of investments which ensure that stock levels present a certain share of domestic output.

Demand for trade and transport margins

The final set of equations in the demand block refers to the optional demand for goods and services for the trade and transport margins. The model has three different nodes for margins: 1) from farm or factory gate to domestic markets ('d'); 2) from farm or factor gate to the domestic border for exports ('x'); and 3) from the border to domestic markets for imports ('m'). Though the wedges can differ across nodes, the unit cost of the margin for each commodity i , is the same across all nodes. This implies that the 'production' function of the trade and transport margins is identical across all nodes. Equation (107) expresses the demand for margin services for good j across all three nodes $xmarg_{i,t}$. This demand generates demand for 'Armington' goods, where an additive CES function is assumed for the volumes as in equation (108).⁸ The price index for the additivity constraint $pmargInd_{j,t}$ is in (109) where it is corrected to be unity in the initial period while the average cost of the trade margins is reflected in the

⁸ Margin services are not associated with specific indirect taxes – though this could be changed in future revisions.

exhaustion price $pmarg_{j,t}$ which is determined in equation (110). The differential cost structure for margins across goods will reflect, for instance, the different energy intensities as reflected in the cost shares, and the indirect demand for energy through the input output table.

$xmarg_{j,t} = tmg_{j,"d",t}xd_{j,t} + tmg_{j,"x",t}xe_{j,t} + tmg_{j,"m",t}xm_{j,t}$	(107)
$xa_{i,j,t} = amarg_{i,j}xmarg_{j,t} \left(\frac{pmargInd_{j,t}}{\chi_i^{pa}pa_{i,t}} \right)^{\sigma_j^{mg}}$	(108)
$pmargInd_{j,t} = \frac{\left(\sum_i amarg_{i,j}(\chi_i^{pa}pa_{i,t})^{-\sigma_j^{mg}} \right)^{\frac{1}{-\sigma_j^{mg}}}}{\left(\sum_i amarg_{i,j}(\chi_i^{pa}pa_{i,t0})^{-\sigma_j^{mg}} \right)^{\frac{1}{-\sigma_j^{mg}}}}$	(109)
$pmarg_{j,t} = \sum_i amarg_{i,j}(\chi_i^{pa}pa_{i,t})^{1-\sigma_j^{mg}} pmargInd_{j,t}^{\sigma_j^{mg}}$	(110)

Equation (111) shows the price and volume split of total expenditure $yf_{oa,t}$ for the final demand agents. What this equation determines will be reflected by specific closure rules to be discussed below.

$yf_{oa,t} = pf_{oa,t}xf_{d_{oa,t}}$	(111)
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Stock change behavior

Endogenous stock changes are introduced in the current version of the model only if the forward-looking option is selected where the model is solved simultaneously for the current and a number of future years. The first equation related to stock change is the usual stock-flow relation for each product i :

$st_{i,t} = st_{i,t-1} + delst_{i,t}$	(112)
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where $st_{i,t}$ is domestic stock level for i and $delst_{i,t}$ is the change in its stock. The second related equation (113) depicts the share of last year's stock added or released in the current year based on (a) the expected price change $psExpRel_{i,t}$ as in equation (114) and (b) a desired average stock level $\alpha_{i,t}^{st}$ relative to expected output $xsExp_{i,t}$:

$delst_{i,t} = st_{i,t-1} \left[\{ \max(0, -psExpRel_{i,t}) \}^{\rho_i^{pdelst}} - \{ \min(0, psExpRel_{i,t}) \}^{\rho_i^{pdelst}} \right] + xsExp_{i,t} \left\{ \frac{2\alpha_{i,t}^{st}}{(st_{i,t} + st_{i,t-1})/xsExp_{i,t}} \right\}^{\rho_i^{xdelst}}$	(113)
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Using both current and last year stock levels in the expression in the second line dampens the impact of current year prices induced changes on targeted stock levels. The \min and \max expressions in equation (113) imply a symmetric behavior of stock changes for positive and negative expected relative price changes. As \min and \max are not differentiable, they are implemented instead based on an NCP Veelken-Ulbrich smooth minimum, sinus-based function $ncpVU_{sin}$ in GAMS. $\alpha_{i,t}^{st}$ defines the desired stock level

relative to domestic output, considering the elasticity ρ_i^{xdelst} , whereas ρ_i^{pdelst} defines how sensitive stock changes to react to the variable $psExpRel$ ⁹, defined as

$psExpRel_{i,t} = \frac{psExp_{i,t} - ps_{i,t}}{psExp_{i,t}}$	(114)
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This variable thus defines differences between expected average and current year realized prices, relative to expected average ones. Price expectations are formed over a number of future years $tNext$ which are simultaneously solved for in each model solve:

$psExp_{i,t} = \frac{\sum_{tNext < t+n} ps_{i,tNext} expWgts_{tNext}}{\sum_{tNext < t+n} expWgts_{tNext}}$	(115)
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where n is the maximal foresight period and $expWgts$ are the weights for the future time points. Note here that for the final years of the simulation horizon, not all n future points can be considered due to the finite simulation horizon. It might therefore be appropriate to extend to simulation period by n years beyond the last reported year. The same approach as for prices is used to define expected output levels:

$xsExp_{i,t} = \frac{\sum_{tNext < t+n} xs_{i,tNext} expWgts_{tNext}}{\sum_{tNext < t+n} expWgts_{tNext}}$	(116)
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An alternative to define desired stock levels relative to domestic output would relate them to total market appearances, i.e., production plus imports minus exports. This could be an appropriate option if, for instance, stochastic world market price shocks for fossil fuel importers are analyzed and it is assumed that the country analyzed maintains stocks of these fuels.

Trade block

The demand equations as discussed above determine the so-called Armington demand for goods $xa_{i,t}$ as the composite of import and domestic demand for different agents: activities (a), households (h), other final demand (f) and for margin delivery (j). The union of these sets defines all Armington agents aa which are assumed to share the same preferences for domestic and imported goods.¹⁰ Each Armington good i is thus assumed homogenous across agents and aggregated across them in volume terms, see equation (119).

⁹ Tests have shown that 0.75 is a reasonable level for ρ_i^{pdelst} . As $psExpRel$ measures relative deviations in prices, the related absolute changes are usually closer to zero than to unity. Using small value of ρ_i^{pdelst} , such as 0.1, implies that a difference between expected and realized prices of 1% will release 63% of previous stocks.

¹⁰ Some national SAMs and the GTAP database allow for the decomposition of Armington demand into its domestic and import components by agent. Even more detailed databases, such as a multi-regional input-output table (MRIO), allow for agent-specific preferences for imports by region of origin.

Equations (117) and (118) define the import and domestic prices $pma_{i,t}$ and $pmd_{i,t}$, respectively. In equation (117), defining $pma_{i,t}$, the CIF price is reflected in the variable $pWm_{i,t}$, multiplied by the exchange rate eR_t and adjusted by the import tariff $\tau_{i,t}^{Imp}$ and trade and transport margins from the border to the point of sale $pMarg_{i,t}tmg_{i,m',t}$. Finally, an import specific value added tax, $\tau_{i,t}^{vat,m}$, is added to the import price after tariffs to become the agent's price of imported goods $pma_{i,t}$. Similarly, according to equation (118), the agents' price of domestically produced goods $pda_{i,t}$ is equal to the domestic producer price $pd_{i,t}$ augmented by domestic trade and transport margins $pMarg_{i,t}tmg_{i,d',t}$ and the domestic value added tax $\tau_{i,t}^{vat,d}$. The terms χ_i^{pma} , χ_i^{pda} and χ_i^{pd} are the price normalization factors and $tmg_{i,m',t}$, and $tmg_{i,d',t}$ are the margin rates. Notice that these agents' prices are uniform across the Armington agents and do not include further agent specific sales and emission taxes.

$\chi_i^{pma} pma_{i,t} = (eR_t(1 + \tau_{i,t}^{Imp})pWm_{i,t} + pMarg_{i,t}tmg_{i,m',t})(1 + \tau_{i,t}^{vat,m})$	(117)
$\chi_i^{pda} pda_{i,t} = (\chi_i^{pd} pd_{i,t} + pMarg_{i,t}tmg_{i,d',t})(1 + \tau_{i,t}^{vat,d})$	(118)

Equation (119) aggregates Armington demand at the national level $xaT_{i,t}$ as the sum across all agents. Aggregate national demand for domestic goods $xd_{i,t}$ is a fraction of $xaT_{i,t}$ which is sensitive to the price of the domestically produced good $pda_{i,t}$ relative to the Armington composite good price $pa_{i,t}$ as shown in equation (120). The key parameter, known as the Armington substitution elasticity, is σ_i^m . Similarly, equation (121) determines the demand for imports $xm_{i,t}$. Note these equations (120) and (121) are derived from a normal CES function while equations (122) and (123) are from an additive CES form.

$xaT_{i,t} = \sum_{aa} xa_{i,aa,t}$	(119)
$xd_{i,t} = ad_{i,t}\gamma_{i,t}^{md(\sigma_i^m-1)} \left(\frac{pa_{i,t}}{pda_{i,t}} \right)^{\sigma_i^m} xaT_{i,t}$	(120)
$xm_{i,t} = am_{i,t}\gamma_{i,t}^{mm(\sigma_i^m-1)} \left(\frac{pa_{i,t}}{pma_{i,t}} \right)^{\sigma_i^m} xaT_{i,t}$	(121)
$xd_{i,t} = ad_{i,t}\gamma_{i,t}^{md(-\sigma_i^m)} \left(\frac{paIND_{i,t}}{pda_{i,t}} \right)^{\sigma_i^m} xaT_{i,t}$	(122)
$xm_{i,t} = am_{i,t}\gamma_{i,t}^{mm(-\sigma_i^m)} \left(\frac{paIND_i}{pma_{i,t}} \right)^{\sigma_i^m} xaT_{i,t}$	(123)

Equations (124) and (125) define the Armington composite price of commodity i in the cases of the normal and additive CES forms, respectively.

$pa_{i,t} = (ad_{i,t}(pda_{i,t}/\gamma_{i,t}^{md})^{1-\sigma_i^m} + am_{i,t}(pma_{i,t}/\gamma_{i,t}^{mm})^{1-\sigma_i^m})^{\frac{1}{1-\sigma_i^m}}$	(124)
$paIND_{i,t} = (ad_{i,t}(pda_{i,t}/\gamma_{i,t}^{md})^{-\sigma_i^m} + am_{i,t}(pma_{i,t}/\gamma_{i,t}^{mm})^{-\sigma_i^m})^{\frac{1}{-\sigma_i^m}}$	(125)

If the additive CES form is chosen for some commodities, the exhaustion price $pa_{i,t}$ needs to be defined as follows.

$pa_{i,t} = \left(ad_{i,t} (pda_{i,t} / \gamma_{i,t}^{md})^{1-\sigma_i^m} + am_{i,t} (pma_{i,t} / \gamma_{i,t}^{mm})^{1-\sigma_i^m} \right) paIND_{i,t}^{\sigma_i^m}$	(126)
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Tax rates and tax-inclusive Armington price

Equations (127)-(137) define various tax rates as a function of a base tax rate (indicated by a superscript 0), and a set of additive (superscript D) and multiplicative (superscript M) shifters. The τ_t^{Mall} is a multiplicate shifter that affects all the tax rates.

Factor specific taxes for each activity $\tau_{a,fp,t}^{Fac}$ are defined in equation (127). Departing from the base rate $\tau_{a,fp,t}^{Fac0}$, it considers general factor tax shifters as well as factor-specific (labor, capital and land) shifters.

$\tau_{a,fp,t}^{Fac} = \left(\tau_{a,fp,t}^{Fac0} + \tau_t^{FacD0} + \tau_{a,t}^{FacD1} + \tau_{fp,t}^{FacD2} \right) \left(\tau_t^{FacMLab} \tau_t^{FacMLabA} + \tau_t^{FacMCap} + \tau_t^{FacMlnd} \right) \tau_t^{FacM0} \tau_{a,t}^{FacM1} \tau_{fp,t}^{FacM2} \left(\text{sign}(\tau_{a,fp,t-1}^{Fac}) (\tau_t^{Mall} - 1) + 1 \right)$	(127)
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Equation (128) defines the activity specific production subsidies $\tau_{a,t}^{Psb}$. Shifters can apply to all activities ($\tau_t^{PsbD0}, \tau_t^{PsbM0}$) or can be activity specific ($\tau_{a,t}^{PsbD1}, \tau_{a,t}^{PsbM1}$):

$\tau_{a,t}^{Psb} = \left(\tau_{a,t}^{Psb0} + \tau_t^{PsbD0} + \tau_{a,t}^{PsbD1} \right) \tau_t^{PsbM0} \tau_{a,t}^{PsbM1} \tau_t^{Mall}$	(128)
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Equation (129) shows the activity-specific production tax rates. Phantom tax rates (activity-specific) are given in equation (130) where $aagr$ is a set of aggregated activities.

$\tau_{a,t}^{Prd} = \left(\tau_{a,t}^{Prd0} + \tau_t^{PrdD0} + \tau_{a,t}^{PrdD1} \right) \tau_t^{PrdM0} \tau_{a,t}^{PrdM1} \tau_t^{Mall}$	(129)
$\tau_{a,t}^{Phn} = \tau_{a,t}^{Phn0} + \tau_t^{PhnD0} + \tau_{a,t}^{PhnD1} + \sum_{a \in aagr} \tau_{aagr,t}^{PhnD2}$	(130)

Commodity-specific export and import tax rates are specified in equations (131) and (132). Symmetric to the activity tax rates above, shifters can apply to all commodities or can be commodity specific.

$\tau_{i,t}^{Exp} = \left(\tau_{i,t}^{Exp0} + \tau_t^{ExpD0} + \tau_{i,t}^{ExpD1} \right) \tau_t^{ExpM0} \tau_{i,t}^{ExpM1} \tau_t^{Mall}$	(131)
$\tau_{i,t}^{Imp} = \left(\tau_{i,t}^{Imp0} + \tau_t^{ImpD0} + \tau_{i,t}^{ImpD1} \right) \tau_t^{ImpM0} \tau_{i,t}^{ImpM1} \tau_t^{Mall}$	(132)

Equations (133) has the emission tax rates for emission type (em) and the Armington agents (aa). Here, shifters can apply to all emissions and agents (index 0), to specific emissions for all agents (index 1) or to specific agents for all emissions (index 2).

$\tau_{em,aa,t}^{Emi} = \left(\tau_{em,aa,t}^{Emi0} + \tau_t^{EmiD0} + \tau_{em,t}^{EmiD1} + \tau_{aa,t}^{EmiD2} \right) \tau_t^{EmiM0} \tau_{em,t}^{EmiM1} \tau_{aa,t}^{EmiM2}$	(133)
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Equations (134) and (135) specify the income tax rates for households (h) and enterprises ($entr$) respectively. They can relate to all households or enterprises (index 0) or be households or enterprise specific (index 1).

$\tau_{h,t}^{Pit} = \left(\tau_{h,t}^{Pit0} + \tau_t^{PitD0} + \tau_{h,t}^{PitD1} \right) \tau_t^{PitM0} \tau_{h,t}^{PitM1} \tau_t^{Mall}$	(134)
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$\tau_{entr,t}^{Cit} = (\tau_{entr,t}^{Cit0} + \tau_t^{CitD0} + \tau_{entr,t}^{CitD1}) \tau_t^{CitM0} \tau_{entr,t}^{CitM1} \tau_t^{Mall}$	(135)
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General value-added tax rates are given in equation (136) while the Armington agent-specific sales tax rates are in equation (137)). Similar to above, index 0 is a general shifter for all commodities and agents, index 1 refers to commodity specific shifters and 2 to agent specific ones.

$\tau_{i,n,t}^{Vat} = (\tau_{i,n,t}^{Vat0} + \tau_t^{VatD0} + \tau_{i,t}^{VatD1} + \tau_{n,t}^{VatD2}) \tau_t^{VatM0} \tau_{i,t}^{VatM1} \tau_{n,t}^{VatM2} \tau_t^{Mall}$	(136)
$\tau_{i,aa,t}^{Paf} = (\tau_{i,aa,t}^{Paf0} + \tau_t^{PafD0} + \tau_{i,t}^{PafD1} + \tau_{aa,t}^{PafD2}) \tau_t^{PafM0} \tau_{i,t}^{PafM1} \tau_{aa,t}^{PafM2} \tau_t^{Mall}$	(137)

Equation (138) defines the end-user price for each Armington agent $paf_{i,aa,t}$. In the absence of a carbon tax, the end-user price is equal to the basic Armington price adjusted by a user-specific sales tax, $\tau_{i,aa,t}^{Paf}$ as defined above. The carbon tax is composed of several parts. The parameter $\rho_{em,i,aa}^{emi}$ measures the quantity of emissions per unit of absorption at the benchmark. It can be adjusted over time by the factor $\chi_{em,i,aa,t}^{emi}$. The carbon tax is given by $\tau_{em,t}^{Emi}$ and is in local currency per unit of emission. The parameter $\phi_{em,i,aa}^{emi}$ allows for variable participation rates across end-users to the tax scheme. If it is set to 1, the participant is subject to the full tax, if set to 0, the agent is tax exempt. The carbon tax is scaled by the Emi_t^{cal} scale factor for accounting purposes.

$paf_{i,aa,t} = (1 + \tau_{i,aa,t}^{Paf}) pa_{i,t} + \frac{\sum_{em} \tau_{em,t}^{Emi} Emi_t^{cal} \chi_{em,i,aa,t}^{emi} \rho_{em,i,aa}^{emi} \phi_{em,i,aa}^{emi}}{\chi_i^{pa}}$	(138)
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Export supply

Analogous to the Armington specification described above, the model allows allocating domestic output to domestic and export markets based on a CET formulation. Alternatively, an infinite elasticity of transformation can be assumed which implies the law-of-one-price and physical aggregation of volumes of domestic sales and exports.

Equations (139) and (140) represent the revenue maximal supply to domestic $xd_{i,t}$ and export $xe_{i,t}$ markets, respectively, under the normal CET function. They drive implicitly the related prices $pd_{i,t}$ and $pe_{i,t}$. With finite transformation, these conditions are the standard CET first order conditions at given supply $xs_{i,t}$ (less stock building/variation $delst_{i,t}$). With perfect transformation, the implicitly driven prices are replaced with the law-of-one-price such that the domestic $pd_{i,t}$ and export $pe_{i,t}$ prices received by producers are set equal to the aggregate supply price $ps_{i,t}$ where γ_i^{xd} and γ_i^{xe} are price shifters.

$\left\{ \begin{array}{ll} \gamma_{i,t}^{xd} pd_{i,t} = ps_{i,t} & \text{if } \omega_i^x = \infty \\ xd_{i,t} = g_{i,t}^d \gamma_{i,t}^{XD(1-\omega_i^x)} \left(\frac{pd_{i,t}}{ps_{i,t}} \right)^{\omega_i^x} (xs_{i,t} - delst_{i,t}) & \text{if } \omega_i^x \neq \infty \end{array} \right\}$	(139)
$\left\{ \begin{array}{ll} \gamma_{i,t}^{xe} pe_{i,t} = ps_{i,t} & \text{if } \omega_i^x = \infty \\ xe_{i,t} = m_dmgAgg_{dmgTrd,"i"} g_{i,t}^e \gamma_{i,t}^{xe(1-\omega_i^x)} \left(\frac{pe_{i,t}}{ps_{i,t}} \right)^{\omega_i^x} (xs_{i,t} - delst_{i,t}) & \text{if } \omega_i^x \neq \infty \end{array} \right\}$	(140)

In the case of the additive CET form, the supply functions are written as in (141) and (142). In this case, the functional form guarantees that $xd_{i,t}$ and $xe_{i,t}$ add up to $(xs_{i,t} - delst_{i,t})$. :

$\left\{ \begin{array}{ll} \gamma_{i,t}^{XD} pd_{i,t} = ps_{i,t} & \text{if } \omega_i^x = \infty \\ xd_{i,t} = g_{i,t}^d \gamma_{i,t}^{xd\omega_i^x} \left(\frac{pd_{i,t}}{psIND_{i,t}} \right)^{\omega_i^x} (xs_{i,t} - delst_{i,t}) & \text{if } \omega_i^x \neq \infty \end{array} \right\}$	(141)
$\left\{ \begin{array}{ll} \gamma_i^{XE} pe_i = ps_i & \text{if } \omega_i^x = \infty \\ xe_{i,t} = m_dmgAgg_{dmgTrd,"i"} g_{i,t}^e \gamma_{i,t}^{xe\omega_i^x} \left(\frac{pe_{i,t}}{psIND_{i,t}} \right)^{\omega_i^x} (xs_{i,t} - delst_{i,t}) & \text{if } \omega_i^x \neq \infty \end{array} \right\}$	(142)

Equation (143) represents the market equilibrium. With perfect transformation (where all prices are uniform) domestic supply is equal to domestic sales and exports. The price aggregators $ps_{i,t}$ and $psIND_{i,t}$ correspond to the normal and additive CET forms.

$\left\{ \begin{array}{ll} xs_{i,t} = xd_{i,t} + xe_{i,t} + delst_{i,t} & \text{if } \omega_i^x = \infty \\ ps_{i,t} = \left(g_{i,t}^d (\gamma_{i,t}^{XD} pd_{i,t})^{1+\omega_i^x} + m_dmgAgg_{dmgTrd,"i"} g_{i,t}^e (\gamma_{i,t}^{xe} pe_{i,t})^{1+\omega_i^x} \right)^{\frac{1}{1+\omega_i^x}} & \text{if } \omega_i^x \neq \infty \\ psIND_{i,t} = \left(g_{i,t}^d (\gamma_{i,t}^{xd} pd_{i,t})^{\omega_i^x} + m_dmgAgg_{dmgTrd,"i"} g_{i,t}^e (\gamma_{i,t}^{xe} pe_{i,t})^{\omega_i^x} \right)^{\frac{1}{\omega_i^x}} & \text{if } \omega_i^x \neq \infty \end{array} \right\}$	(143)
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Under the additive CET form, the price aggregator differs from the revenue exhaustion price which is written as follows.

$ps_{i,t} = \left(g_{i,t}^d (\gamma_{i,t}^{xd})^{\omega_i^x} pd_{i,t}^{\omega_i^x+1} + g_{i,t}^e (\gamma_{i,t}^{xe})^{\omega_i^x} PE_{i,t}^{\omega_i^x+1} \right) psIND_{i,t}^{-\omega_i^x}$	(144)
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Equation (145) defines the exports price $pe_{i,t}$ from given world market price. The FOB price of exports $pWe_{i,t}$ is multiplied by the exchange rate eR_t to convert into local currency terms on the left-hand side. There are two wedges that affect the domestic producer price of exports: an ad-valorem export tax $\tau_{i,t}^{Exp}$ and the domestic trade and transport margin $pMarg_{i,t} tmg_{i',e',t}$ from factory gate to the border.

$\chi_i^{pe} pe_{i,t} (1 + \tau_{i,t}^{Exp}) + pMarg_{i,t} tmg_{i',e',t} = eR_t \chi_i^{pWe} pWe_{i,t}$	(145)
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The model allows for both a downward sloping export demand curve and an upward sloping import supply curve. Equation (146) reflects the export demand curve, with the possibility of an infinite demand elasticity in which case the FOB price of exports is fully exogenous. The variable $pWe_{i,t}^{ROW}$ represent the

exogenous world market price. If the export demand elasticity η_i^e is finite, export demand declines as the price of home country exports increases. Equation (147) provides an analogous treatment of import supply where Import supply increases with a rise in the price of imports into the home country (for example with a reduction in import tariffs). With an infinite supply elasticity ω_i^m , the price of imports is fixed at $pWm_{i,t}^{ROW}$.

$\begin{cases} pWe_{i,t} = \gamma_{i,t}^{PWE} pWe_{i,t}^{ROW} & \text{if } \eta_i^e = \infty \\ xe_{i,t} = \chi_{i,t}^e \left(\frac{\gamma_{i,t}^{pWe} pWe_{i,t}^{ROW}}{pWe_i} \right)^{\eta_i^e} & \text{if } \eta_i^e \neq \infty \end{cases}$	(146)
$\begin{cases} pWm_{i,t} = pWm_{i,t}^{ROW} & \text{if } \omega_i^m = \infty \\ xm_{i,t} + mdelst_{i,t} = \chi_{i,t}^m \left(\frac{pWm_{i,t}}{pWm_{i,t}^{ROW}} \right)^{\omega_i^m} & \text{if } \omega_i^m \neq \infty \end{cases}$	(147)

There are potentially three goods market equilibrium conditions as follows.

$$xd_{i,t}^s = xd_{i,t}^d$$

$$xe_{i,t}^s = xe_{i,t}^d$$

$$xm_{i,t}^s = xm_{i,t}^d$$

These equations are substituted away and the model implementation only carries a single variable for each of the equilibrium variables $xd_{i,t}$, $xe_{i,t}$ and $xm_{i,t}$.

Factor market closure

Flexible nesting

Similar to the nested structure in the final demand and in the production function, MANAGE-WB depicts factor markets based on a flexible nesting structure. Exempt from this structure is the supply of capital when capital vintages are active which is discussed below.

The factor allocation system considers the different factors fmp and supply by different institutions depicted by the set $instTot$. In the default case, one economy-wide aggregate agent $sumInst$ manages the allocation of economy-wide factor stocks to production activities. Factor prices received by the institutions (households, enterprises, government) are identical in this case and their income from each factor is a share of the economy-wide factor income, according to the distribution factors $\chi_{inst,fmp,t}^{Fac}$.

Alternatively, if information of the sourcing of each institution's income for a factor by activity is available and used, institutions manage their own factor stocks, such that, for instance, average wages by labor type can differ across households, while they are still identical at the level of the individual activity. This additional detail makes a sense for sluggish factor markets, only. Moreover, the factor stocks can be

broken into different age vintages captured by the set vf . This allows, for instance, to render the allocation of new entrants into the labor force more flexible compared to the existing labor stock.

The total supply of a mobile or sluggish, but not immobile primary factor $xfmSup_{fmp,fmp,instTot,vf,t}$ is defined by an iso-elastic supply schedule, where $atFmSup_{fmp,instTot,vf,t}$ is a shifter variable and $m_dmgAgg_{dmgSup,fmp}$ is an expression which adds-up potential shifters due to multiple Climate Change damage channels. The additional set fmp indicates the factor market, such as the labor category to which a labor sub-nest belongs. That eases the implementation as the same sub-nest names can be used for multiple markets.

$xfmSup_{fmp,fmp,instTot,vf,t} = m_dmgAgg_{dmgSup,fmp} atFmSup_{fmp,instTot,vf,t} \left(\frac{pfmSup_{fmp,fmp,instTot,vf,t}}{pGDPmp_t} \right)^{\varepsilon_{fmp}^{fmp}}$	(148)
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Supplies to sub-nests, including to individual activities $fmcHl$ under a sluggish parent nest $fmpPar$ are a share of the parent factor nest, updated with the price relation between the child and parent nest:

$xfmSup_{fmcHl,fmp,instTot,vf,t} = \gamma_{fmcHl,fmp,instTot,vf,t}^{fmp} xfmSup_{fmpPar,fmp,instTot,vf,t} \left(\frac{pfmSup_{fmcHl,fmp,instTot,vf,t}}{pfmSup_{fmpPar,fmp,instTot,vf,t}} \right)^{\omega_{fmpPar}^{fmp}}$	(149)
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The related exhaustion price indices for the sluggish case and non-additive CET cases are defined as:

$pfmSup_{fmpPar,fmp,instTot,vf,t} = \left(\sum_{fmcHl} \gamma_{fmcHl,fmp,instTot,vf,t}^{fmp} pfmSup_{fmcHl,fmp,instTot,vf,t}^{1+\omega_{fmpPar}^{fmp}} \right)^{\frac{1}{1+\omega_{fmpPar}^{fmp}}}$	(150)
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The additive case, which guarantees adding up, defines the dual price aggregator on a similar formula which differs in the exponent and is used instead of $pfmSup_{fmpPar,fmp,instTot,vf,t}$ in equation (149).

$pfmSupD_{fmpPar,fmp,instTot,vf,t} = \left(\sum_{fmcHl} \gamma_{fmcHl,fmp,instTot,vf,t}^{fmp} pfmSup_{fmcHl,fmp,instTot,vf,t}^{\omega_{fmpPar}^{fmp}} \right)^{\frac{1}{\omega_{fmpPar}^{fmp}}}$	(151)
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For the additive case, the exhaustion price $pfmSup_{fmpPar,fmp,instTot,vf,t}$ is defined as follows and is used where this price index enters the equation above for a child:

$pfmSup_{fmpPar,fmp,instTot,vf,t} = \left(\sum_{fChl} \gamma_{fChl,fmp,instTot,vf,t}^{fmp} pfmSup_{fChl,fmp,instTot,vf,t}^{1+\omega_{fmpPar}^{fmp}} \right) pfmSupD_{fmpPar,fmp,instTot,vf,t}^{-\omega_{fmpPar}^{fmp}}$	(152)
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For fully mobile or immobile (sub)-markets, the total supply is based on the adding-up condition:

$xfmSup_{fmpPar,fmp,instTot,vf,t} = \sum_{fChl \in fmpPar} xfmSup_{fChl,fmp,instTot,vf,t}$	(153)
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In this fully mobile case, component prices are equal to parent ones:

$pfmSup_{fChl,fmp,instTot,vf,t} = \sum_{fChl \in fmpPar} pfmSup_{fmpPar,fmp,instTot,vf,t}$	(154)
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The supply to individual activities in the set fma in case of immobile resources such as oil fields or fish stocks or other natural resources, is based on an iso-elastic supply schedule in case of increasing prices but exhibits strongly growing elasticities in case of dropping prices based on a sigmoid expression. The use rate of natural resource is hence flexibly adjusted if returns fall (lower extraction quantities of oil etc.).

$xfmSup_{fma,fmp,instTot,vf,t} = \min \left[\left(\frac{pfmSup_{fma,fmp,instTot,vf,t}}{pGDPmp_t} \right)^{\varepsilon_{fma,fmp}^{fma}}, \left(\text{sigmoid} \left(\frac{pfmSup_{fma,fmp,instTot,vf,t}}{pGDPmp_t} - 0.75 \right) * 25 \right) - 0.001 \right] * 10 \Big atFmSup_{fmp,t} \gamma_{fma,fmp,instTot,vf,t}^{fmp} m_dmgAgg_{dmg,fmp}$	(155)
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The economic-wide average price for immobile resources is defined by the exhaustion price:

$pfmSup_{fmpPar,fmp,instTot,vf,t} = \frac{\sum_{fChl} pfmSup_{fChl,fmp,instTot,vf,t} xfmSup_{fChl,fmp,instTot,vf,t}}{xfmSup_{fmpPar,fmp,instTot,vf,t}}$	(156)
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The link between factor supply and factor demand for each activity is defined by the following equation which adds-up factor supply over potentially multiple institutions and factor vintages on the left-hand-side and factor demand over the capital vintages on the right-hand-side.

$\sum_{instTot,vf} xfmSup_{fma,fmp,vf,t} = \sum_v xFac_{fmp,fma,v,t}$	(157)
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According to the physical adding up over vintages in the equation above, pretax factor prices for primary factors are identical across vintages and only assigned to the old vintage. Equally, according to adding up over supplying institutions and vintages, they are incidental across institutions and factor vintages.

$pxFac_preTax_{fmp,fma,old,t} = pfmSup_{fma,fmp,instTot,vf,t}$	(158)
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Technically, these equalities are introduced by a macro operator in GAMS which replaces $pfmSup_{fma,fmp,instTot,vf,t}$ by $pxFac_preTax_{fmp,fma,old,t}$ in the equations relating to the factor allocations.

Capital markets without vintage capital specification

If no vintage differentiation is present for the production activities, the equations above apply to capital markets. The link to capital stocks is based on the following equality which renders the shifter $atmSup_{cap,old,t}$ endogenous.

$kapS_t = atmSup_{cap,t}$	(159)
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The corresponding rental rate of capital $trent_t$ is determined as:

$trent_t = pfmSup_{cap,old,t}$	(160)
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Capital markets with the vintage capital specification

This section describes the capital allocation mechanism among the production activities under the assumption of two vintages for each activity. This specification is an on-demand option. Capital market equilibrium under the vintage capital framework assumes the following:

- *new* capital is perfectly mobile across production activities *a* which ensures an economy-wide uniform rate of return to capital $trent_t$. *new* capital is used in the *new* vintage only.
- *old* capital in expanding sectors – i.e., sectors which attract *new* capital – has the same productivity as *new* capital. Accordingly, the rate of return to *old* capital in expanding sectors is the same as the economy-wide rate of return to *new* capital.
- Declining sectors which do not attract *new* capital release *old* capital. The released *old* capital is added to the economy-wide the stock of *new* capital. The assumption here is that declining sectors will first release the most mobile types of capital which is, as being mobile, comparable to *new* capital, such as transportation equipment.
- The rate of return to capital in declining sectors reflects sector-specific supply and demand conditions and drives the share of released *old* capital.
- *Old* capital is used in the *old* vintage.

The result of these assumptions is that if there are no declining sectors, there is one single economy-wide rate of return to capital, identical to the pre-tax marginal return to capital in all sectors. Otherwise, there will be additional sector-specific rates of return to *old* capital in each declining sector.

The economy-wide rate of return on *new* capital $trent_t$ is derived from the market equilibrium condition in equation (161):

$kapS_t = \sum_{cap,a,v} xFac_{cap,a,v,t}$	(161)
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The stock of installed or *old* capital $oldCap_{a,t}$ at the beginning of the year for each sector is determined as the sum of capital use in the *old* and *new* vintages in the previous year corrected for depreciation:

$oldCap_{a,t} = (1 - depr_{a,t} m_dmgAgg_{dmgDepr,"a"}) (xFac_{cap,a,old,t-1} + xFac_{cap,a,new,t-1})$	(162)
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Equation (163) defines the quantity of *old* capital in use by the *old* vintage as a share $rrat_{a,t}$ of the total *old* capital stock of this activity. This share is upper bounded by unity as the *old* vintage cannot attract *new* capital.

$xFac_{cap,a,old,t} = rrat_{a,t} oldCap_{a,t}$	(163)
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Equation (164) shows that this share $rrat_{a,t}$ and depends on a activity specific supply schedule of *old* capital driven by the ratio between the returns to capital in the *old* vintage $pxFac_preTax_{cap,a,old,t}$ and the economy-wide return to capital $trent_t$. The lower the return to capital in a declining sector, the more of its capital is released and absorbed by expanding sectors, according to the elasticity $invElas_a$. Released *old* capital thus adds to *new* capital generated from net investment.

$rrat_{a,t} = (pxFac_preTax_{cap,a,old,t} / trent_t)^{invElas_a}$	(164)
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The condition for the *new* vintage to be active is determined as follows. As $rrat_{a,t}$ reaches unity, $pxFac_preTax_{cap,a,old,t}$ must be equal to $trent_t$ in the supply schedule. Equally, if the *new* vintage is not active for an activity, its production can stem solely from the *old* vintage and only *old* capital is used. In order to implement these two conditions, first an intermediate variable $rrat1_{a,t}$ is defined as the total capital demand by the activity over *old* capital:

$rrat1_{a,t} = \frac{\sum_{cap,v} xFac_{cap,a,v,t}}{oldCap_{a,t}}$	(165)
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The variable $rrat_{a,t}$ is then the minimum of 1 and $rrat1_{a,t}$:

$rrat_{a,t} = \min(1, rrat1_{a,t})$	(166)
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The condition above is either implemented with an upper bound on $rrat_{a,t}$ if the model is solved as a MCP or by a differentiable fudging function for the *min* operator when the model is solved as CNS or NLP.

The output of the new vintage is implicitly defined by adding up total activity output over the two vintages which assumes that these outputs are homogenous:

$xp_{a,t} = xFac_{ftop,a,old,t} + xFac_{ftop,a,new,t}$	(167)
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The vintage specification does not capture “stranded assets” in the sense of *old* capital no longer contributing to production. Capital no longer in use in declining sectors is released and contributes with the same marginal productivity to the overall productive capacity of the economy as new investments in the specification above. The economic-wide costs of shrinking sectors are captured by reduced returns to the part of their *old* capital which stays in their own production. The size of these costs depends first on the parameter $invEla_s_a$. The larger this elasticity, the faster *old* capital will be released if returns to capital drop in a sector and the smaller are the economy-wide cost. Second, the more flexibly sectors can substitute capital against other production factors, the lower the cost of adjustment as the necessary drop in returns to capital to reduce output becomes smaller. The usual assumption is that due to lock-in effects into specific technologies at the time of investments, capital substitution against other production factors is less flexible for the *old* vintage compared to the *new* vintage.

User cost of factors of production

Equation (168) converts the market price of factors of production $pxFac_Pretax_{fp,a,v,t}$ to the user-cost of factors of production $pxFac_{fp,a,v,t}$ by sectors by adding the activity specific ad-valorem factor tax or subsidy. Equation (169) adjusts the factor tax by the GDP price level for reporting purposes.

$pXac_{fp,a,v,t} = pxFac_preTax_{fp,a,v,t} (1 + \tau_{a,fp,t}^{Fac}) + \sum_{em} \tau_{em,a,t}^{Emi} Emi_t^{cal} \chi_{em,fp,a,t}^{emi} \rho_{em,fp,a}^{emi} \phi_{em,fp,a}^{emi}$	(168)
$\tau_{a,fp,t}^{rFac} = \frac{\tau_{a,fp,t}^{Fac}}{pGDPmp_t}$	(169)

Foreign savings and balance of payments

The balance of payments (BOP) in MANAGE-WB consists of four elements: (1) the trade balance (2) foreign savings contributing to investments, (3) foreign debt related flows $fundF_t$, see chapter “New foreign government debt” and (4) transfers between the ROW and domestic institutions (enterprises, households, government), see chapter “Transfers”.

For the sum of foreign contribution to savings and investments, different closures are available. The default case in equation (170) defines foreign savings are a share of nominal GDP at market prices $GDPmp_t$ in foreign currency by division with the exchange rate eR_t . The share is defined by the variable $savfGDP_t$ which is by default fixed to the benchmark ratio:

$savF_t = savfGDP_t \frac{GDPmp_t}{eR_t}$	(170)
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The share, fixed for this closure, is defined analogously, see equation (171). If other closure rules are used, the share is comprised for information in the model.

$savfGDP_t = \frac{savf_t eR_t}{GDPmp_t}$	(171)
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The second closure is one found in the GTAP model and equilibrates the normally exogenous global expected return to capital $rorg_t$ to the domestic one $rore_t$ such that the changes in the foreign contribution to domestic investment follow a given path of $rorg_t$:

$rore_t = rorg_t$	(172)
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The third rule defines foreign savings as a share γ_t^{sf2} of depreciated capital, the depreciation rate $depr_t$ times the capital stock $kstock_t$, driven by a term depending on the relation between $rore_t$ and $rorg_t$ dampened by a term which considers the debt-to-GDP ratio $debtGDP_t$, the exchange rate eR_t and the GDP price index $pGDPmp_t$:

$savF_t = \gamma_t^{sf2} depr_t kStock_t e^{\rho_t^{savf}(rore_t - rorg_t) - \omega_t^{savf} \frac{eR_t}{pGDPmp_t} debtGDP_t}$	(173)
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A fourth rule, inspired by the USAGE model, drives the growth rate of the capital stock grk_t . It considers the difference between domestic and global rate of returns to capital $rore_t$ and $rorg_t$ corrected for the country risk premium $rord_t$:

$devRor_t = rore_t - rord_t - rorg_t$	(174)
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This variable lets the growth rate of capital fluctuate in a corridor between grk_t^{min} and grk_t^{max} driven by a trend in capital growth grk_t^{trend} . The smaller χ_t^{grk} , the closer grk_t will follow the trend.

$grk_t = \frac{grk_t^{max} e^{\chi_t^{grk} devRor_t} + grk_t^{min} (grk_t^{max} - grk_t^{trend}) / (grk_t^{trend} - grk_t^{min})}{e^{\chi_t^{grk} devRor_t} + (grk_t^{max} - grk_t^{trend}) / (grk_t^{trend} - grk_t^{min})}$	(175)
---	-------

The last rule gives the user flexibility in how foreign savings react to the changes in capital rent $trent_t$ compared to a given global one $retROW_t$ and to the domestic per capita GDP growth $gGDPpc_t$ and the global one $gGDPpcROW_t$, by choosing appropriate parameters ($\gamma_t^{sf1}, \theta^{1a}, \theta^{2a}$):

$savF_t = \gamma_t^{sf1} \left(\frac{1 + trent_t}{1 + retROW_t} \right)^{\theta^{1a}} \left(\frac{1 + gGDPpc_t}{1 + gGDPpcROW_t} \right)^{\theta^{2a}}$	(176)
---	-------

Due to Walras' Law, one equation in the model can be dropped. The current implementation selects the balance of payments in equation (177) as the equation to remove. This equation does not restrict the model solution. Instead, it defines an additional variable $Walras_t$ which represents the equilibrium of the balance of payments. This variable should be zero (considering the numerical accuracy achieved by the solver) if the simulation has succeeded – i.e., Walras' law holds. This provides a model consistency check for the various exhaustion conditions. The code will throw an error during simulations if the absolute value of this test variable exceeds a certain share of GDP. Such an error typically points to logical errors in the equation structure. Note that the balance of payments equation is evaluated in foreign currency units and thus outbound transfers are divided by the exchange rate.

$ \begin{aligned} walras_t = & \sum_i \underbrace{\chi_i^{pWe} pwe_{i,t} x e_{i,t}}_{Exports} + \underbrace{savF_t eR_t}_{Net\ investment\ flows} \\ & + \sum_{inst} \underbrace{transfers_{inst,row,t}}_{Inbound\ transfers} - eR_t \sum_i \underbrace{pWm_{i,t} (xm_{i,t} + mdelst_{i,t})}_{Imports} \\ & - \sum_{inst} \underbrace{transfers_{row,inst,t}}_{Outbound\ transfers} - \sum_{instf} debtPay_{instf,t} \end{aligned} $	(177)
--	-------

Macro identities

The following block of equations provides the main macroeconomic identities. Equations (178)-(180) determine respectively nominal aggregate stock building, real aggregate stock building and the stock building aggregate price index.

$Tdelst_t = \sum_i (\chi_i^{ps} ps_{i,t} delst_{i,t} + eR_t pWm_{i,t} (1 + \tau_{i,t}^{Imp}) mdelst_{i,t})$	(178)
$rTdelst_t = \sum_i (\chi_i^{ps} ps_{i,t0} delst_{i,t} + eR_{t0} pWm_{i,t=0} (1 + \tau_{i,t}^{Imp}) mdelst_{i,t})$	(179)
$pdelst_t = \frac{Tdelst_t}{rTdelst_t}$	(180)

Equations (181) – (183) determine the nominal and real values for aggregate exports and the aggregate export price index. Aggregate exports are evaluated at FOB prices in domestic currency units.

$TotExp_t = eR_t \sum_i (\chi_i^{pWe} pWe_{i,t} x e_{i,t})$	(181)
$rTotExp_t = eR_{t0} \sum_i (\chi_i^{pWe} pWe_{i,t0} x e_{i,t})$	(182)
$pExp_t = \frac{TotExp_t}{rTotExp_t}$	(183)

Equations (184)-(186) determine respectively the nominal and real values for aggregate imports and the aggregate import price index. Aggregate imports are evaluated at CIF prices in domestic currency units.

$TotImp_t = eR_t \sum_i pWm_{i,t} (xm_{i,t} + mdelst_{i,t})$	(184)
$rTotImp_t = eR_{t0} \sum_i pWm_{i,t0} (xm_{i,t} + mdelst_{i,t})$	(185)
$pImp_t = \frac{TotImp_t}{rTotImp_t}$	(186)

Equation (187) defines nominal GDP at market prices, where the index fd in the sum covers the final demand accounts – household expenditures (h), government current expenditures (gov), private investment expenditures (inv) and government investment expenditures ($ginv$). Equations (188) and (189) define real GDP at market prices and the GDP deflator at market prices. Per capita real GDP is defined in equation (190) where $pop_{total,t}$ is total population. Equation (191) is relevant in dynamic simulations. In the baseline simulation, the growth in real GDP per capita $gGDPpc_t$ may be exogenous

(i.e., targeted) and another variable would be endogenous to achieve the growth target, typically some factor productivity parameter (discussed further below). In policy or alternative scenarios, the growth in real per capita GDP would be endogenous.

$GDPmp_t = \sum_{fd} yf_{fd,t} + Tdelst_t + TotExp_t - TotImp_t$	(187)
$rGDPmp_t = \sum_{fd} xfd_{fd,t} + rTdelst_t + rTotExp_t - rTotImp_t$	(188)
$pGDPmp_t = \frac{GDPmp_t}{rGDPmp_t}$	(189)
$rGDPpc = \frac{rGDPmp_t}{pop_{total,t}}$	(190)
$gGDPpc_t = \frac{rGDPpc_t}{rGDPpc_{t-1}} - 1$	(191)

Equation (192) defines nominal GDP only at factor cost as the sum of factor remunerations across all factors and activities of production – i.e., it does not include indirect taxes. Equation (193) defines real GDP at factor cost. It is a linearization of the true GDP function and is the weighted sum of the factors of production in efficiency units where the weights are given by the relevant base year prices. The GDP at factor cost deflator is defined in equation (194).

$GDPfc_t = \sum_a \sum_v \sum_{fp} pxFac_{fp,a,old,t} xFac_{fp,a,v,t}$	(192)
$rGDPfc_t = \sum_a \sum_v \sum_{fp} pxFac_{fp,a,old,t0} xFac_{fp,a,v,t}$	(193)
$pGDPfc_t = \frac{GDPfc_t}{rGDPfc_t}$	(194)

Additional macro identities provide more flexibility to define closure rules. Key macroeconomic shares, real and nominal government and investment expenditures as shares of GDP, are described in equations (195) and (196) respectively.

$rshrGDP_{oa,t} = \frac{xfdo_{a,t}}{rGDPmp_t}$	(195)
$nshrGDP_{oa,t} = \frac{yF_{oa,t}}{GDPmp_t}$	(196)

If the fiscal responsibility option is chosen, the required changes are reflected in $fiscalRespDrv_t$ as in equation (197) where $debtGDPTgt_t$ is a target debt-GDP ratio.

$fiscalRespDrv_t = \min \left(0, - \left\{ \text{sigmoid} \left(\frac{debtGDP_t - debtGDPTgt_t}{debtGDPTgt_t} \right)^{0.5} - 0.5 \right\}^2 \right)$	(197)
---	-------

If $fiscalRespDrv_t < 0$, normally exogenous $rshrGDP_{f,t}$ for $f = \{gov, ginv\}$ becomes endogenous and are driven by it as in equation (198). The term $relExpInc$ determines whether $fiscalRespDrv_t$ is solely on

government expenditures or both expenditures and revenue. If $relExpInc \neq 100$, it affects $tmall_t$ as in equation (199).

$\frac{xfd_{f,t}}{rGDPmp_t} = \frac{xfd_{f,t0}}{rGDPmp_{t0}} + \frac{relExpInc}{100} fiscalRespDrv_t$	(198)
$tmall_t = tmall_{t0} - fiscalRespDrv_t$	(199)

Default closure rules

The default closure rules of the model are as follows:

- Aggregate real government expenditures on consumption and investment are fixed as a share of real GDP.
- The government balance is endogenous, if negative, it is closed by new debt, if positive, it contributes to investment.
- Investment is savings driven. Household, enterprise and government savings were discussed above. Household and enterprise savings are usually parameterized to react to changes in returns to capital. Foreign savings, in the default closure, are fixed relative to real GDP. Thus, investments and capital accumulation are largely influenced through household and enterprise savings and new domestic government debt.¹¹
- The foreign saving is fixed as a share of GDP. Transfers from the ROW are fixed relative to GDP, transfers to ROW from domestic institutions are driven by changes in their income as detailed above. This implies that BOT is a fixed share of GDP corrected for changes in transfer to ROW relative to GDP. The latter correction is usually quite small.
- The contribution of foreign savings to investment depends on new government foreign debt minus principal and interest payments on foreign debt.
- The exchange rate is fixed to benchmark level and serves as the model numeraire.
- In the baseline simulation, the default closure fixes real GDP and endogenizes a uniform productivity shifter for all types of labor, or for labor in service sectors to which labor productivity changes in other sector are linked. The labor productivity changes can also be linked with a multiplicative factor to changes in land and natural resource productivity.
- In alternative (counterfactual or policy) simulations, the endogenous productivity shifter used in the baseline is fixed to the baseline level and real GDP becomes endogenous. Note that additional productivity shifters might be introduced both in the baseline and counterfactuals.

Emissions module

The model is set up to include any number of emissions, indexed by em , generated by final product use and input use (both intermediate and factor demand – e.g., herd-size or land), and by output (e.g., the case of cement, or methane emissions from landfills). Related emission factors are in most cases taken

¹¹ Alternative closures are conceivable, for example targeting investment (as a share of GDP) and allowing the household savings schedule adjusts to achieve the target.

from the GTAP data base, based on mapping products and activities in the country SAM to GTAP products and activities. GHG related emissions are scaled to match given emission inventories (energy, transport, etc.) at the benchmark which typically stems from CAIT. Other emissions, such as air emissions, are typically scaled with the same factor as the GHG emissions.

Equation (200) determines the level of emissions $EMI_{em,emSrc,aa,t}^{comm}$ by emission type em from the (Armington) consumption of emission source commodity $emSrc$ (a subset of i) by Armington agent aa (intermediate and final demand). The basic coefficient is $\rho_{em,emSrc,aa}^{emi}$ represents the benchmark level of emissions per unit of consumption, for example, tons of CO₂ per ton of oil equivalent. The $\chi_{emi,emSrc,aa,t}^{emi}$ parameters allow for (exogenous) changes in the emissions coefficients that could be brought about by autonomous improvements in emission efficiency. If needed, emissions can be scaled by the emi_t^{cal} scale factor for accounting purposes. Equation (201) determines the level of emissions associated with the use of each factor of production for each activity. In livestock this could pertain to the size of the herds, for example for methane emissions, and for crops, it could be linked to land use. Equation (202) determines the level of emissions generated by overall output – i.e., linked to the volume of production, $EMI_{em,a,t}^{xp}$ for each activity. This can be used to assess the level of emissions, so-called process emissions, from certain sectors such as emissions in cement, around 50 percent of which are not linked to the use of fuels, or methane emissions from landfills. Equation (203) determines emissions due to land use change for each activity.

$EMI_{em,emSrc,aa,t}^{comm} = emi_t^{cal} \chi_{emi,i,aa,t}^{emi} \rho_{emi,i,aa}^{emi} x a_{i,aa,t}$	(200)
$EMI_{em,fp,a,t}^{fact} = emi_t^{cal} \chi_{em,fp,a,t}^{emiFac} \rho_{em,fp,a}^{emiFac} \sum_v x Fac_{fp,a,v,t}$	(201)
$EMI_{em,a,t}^{xp} = emi_t^{cal} \chi_{em,a,t}^{emiXP} \rho_{em,a}^{emiXP} x p_{a,t}$	(202)
$EMI_{em,a,t}^{LUCF} = - \sum_{lnd} \rho_{em,lnd,a}^{emiLnd} (x Fac_{lnd,a,v,t} - x Fac_{lnd,a,v,t-1})$	(203)

Equation (204) computes emissions by the Armington agents which are aggregated to so-called emission inventories in equation (205). Equation (206) computes the real carbon tax by dividing nominal carbon tax by the GDP price deflator. Equation (207) determines the total level of emissions, for emission em , summing over all sources-intermediate and final consumption, and factor- and production-based emissions as well as emissions coming from the changes in land cover categories (defined in equation (208)) converted into CO₂ using a factor of 3.37. Land cover area $lCovArea_{lnd,lcov,t}$ is defined in equation (209) where $crops$, $anim$ and frs are sets of activities containing crops, livestock and forestry, respectively, $exolCovAreaChg_{lnd,lcov,t}$ is exogenous and the set $lCovDrv$ is land cover types that drive changes in unmanaged land according to the land transition matrix $lndTransMat_{lnd,lCovDrv,lCov}$. Equation (210) defines $lCovArea_{lnd,"builtupLand",t}$, one of $lCovDrv$, as a function of GDP per capita and population. It allows for an exogenous level of autonomous emissions – that could, for example, come from an external model. A subset of emissions is linked with greenhouse gases (GHG) such as methane,

nitrous oxides and the fluoridated gases. Equation (211) calculates the total of *GHG* emissions where the individual gases are weighted by the so-called global warming potentials $EmiConv_{em,polltype}$.¹²

$EMI_{em,aa,t}^{seca} = \sum_i EMI_{em,i,aa,t}^{comm} + \sum_{fp} EMI_{em,fp,aa,t}^{fact} + EMI_{em,aa,t}^{xp} + EMI_{em,aa,t}^{LUCF}$	(204)
$EMI_{ghgInvUsr,em,t}^{inv} = \sum_{a\$map_{aa_ghgInvUsr}} EMI_{em,aa,t}^{seca}$	(205)
$\tau_{em,aa,t}^{rEmi} = \frac{\tau_{em,aa,t}^{Emi}}{PGDPmp_t}$	(206)
$EMITot_{em,t} = \sum_{aa} EMI_{em,aa,t}^{seca} - 3.37 \sum_{lnd} \sum_{lcov} dCarbon_{lnd,lcov,t}$	(207)
$dCarbon_{lnd,lcov,t} = carbCnt_{lnd,lcov}(lCovArea_{lnd,lcov,t} - lCovArea_{lnd,lcov,t-1})$	(208)
$lCovArea_{lnd,lcov,t} = lCovArea_{lnd,lcov,t-1} + \sum_{crops} \sum_v (xFac_{lnd,crops,v,t} - xFac_{lnd,crops,v,t-1}) + \sum_{anim} \sum_v (xFac_{lnd,anim,v,t} - xFac_{lnd,anim,v,t-1}) + \sum_{frs} \sum_v (xFac_{lnd,frs,v,t} - xFac_{lnd,frs,v,t-1}) + exolCovAreaChg_{lnd,lcov,t} - \sum_{lCovDrv} (lCovArea_{lnd,lCovDrv,t} - lCovArea_{lnd,lCovDrv,t-1}) lndTransMat_{lnd,lCovDrv,lCov}$	(209)
$lCovArea_{lnd,"builtupLand",t} = \left(\frac{rGDPmp_t}{pop_{total,t}} / \frac{rGDPmp_{t0}}{pop_{total,t0}} \right)^{0.33503} \left(\frac{pop_{total,t}}{pop_{total,t0}} \right)^{1-0.17634}$	(210)
$EMI_{polltype,t}^{GHG} = \sum_{em} EmiConv_{em,polltype} EMITot_{em,t}$	(211)

Emission taxes can either set exogenously by fixing the emissions tax variables, $\tau_{em,aa,t}^{Emi}$, or can be endogenous under an emission cap, for example by fixing $EMITot_{em,t}$. Specific parameters (see the section on taxes) allow to define which emission sources are included in the tax scheme. Emission taxes increase government revenues and can interact with other taxes depending on the chosen closure for the government account.

Equation (212) shows the emission intensity of pollution types with respect to GDP.

$EMI_{polltype,t}^{Int} = \frac{EMI_{polltype,t}^{GHG}}{rGDPmp_t}$	(212)
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¹² See IPCC (1996), page 22 and also the Global Warming Potentials (IPCC Second Assessment Report) at http://unfccc.int/ghg_data/items/3825.php.

Climate change damage module

Climate change damages are implemented through shifters $m_{dmgAgg_{var,dims}}$ into a larger set of equations where the set var comprises the following variables and the set $dims$ the first index of each variable such as $fpNest$ and oa . Each variable carries the index $chnl$ depicting different channels through which climate change impacts the economy. The set of active channels can be defined by the analyst. The damage variables provide a flexible framework to integrate climate change related damages in a structured way into the model. For example, $m_{dmgAgg_{var,dims}} = \prod_{chnl}(1 + var_{dims,chnl,t})$. Technically, it is done by a macro operator in GAMS.

$dmgPrd_{fpNest,a,chnl,t}$	Acts as a productivity shifter
$dmgYld_{fpNest,a,chnl,t}$	Changes the yield (output over factor use)
$dmgSup_{fpNest,chnl,t}$	Changes factor supply, for instance, capital, labor or land supply
$dmgDem_{oa,i,knest,chnl,t}$	Preference shifter
$dmgTrd_{i,chnl,t}$	Export shifter
$dmgStk_{*,chnl,t}$	Changes capital stock or population size
$dmgDepr_{a,chnl,t}$	Depreciation rates

Defining damages as variables allows for an analysis where one can set up additional equations in a shock file which drive the damage variables as a function of others, for instance, the amount of adaptative investments. Without such additional equations, all damage variables need to be fixed and act as fixed parameters.

Model Dynamics

Model dynamics are driven by three factors, similar to most neo-classical growth models: changes in the labor stock, the capital stock and primary factor productivities. The labor force growth rate is typically equated to the growth rate of the working age population, assumed as the population aged between 15 and 64. Population growth and hence also labor stock changes are exogenous by default. However, in most applications, a labor supply function will render the active share of the labor force for each labor stock endogenous depending on real wages.

During the baseline construction, shifters introduced by the analyst might also let labor stocks' growth differ from changes in the working age population, for instance, to changes in composition by skill level.

The second factor is capital accumulation which let the capital stock grow. The aggregate capital stock in any given year for each type of capital $kStock_t$ is equated to the previous year's capital stock less depreciation at a rate of δ_t plus the previous period's volume of private investment $xf d_{inv,t-1}$ and

productivity adjusted public investment $\phi_{inv}^{pc} xfd_{ginv,t-1}$ where $\phi_{inv}^{pc} \leq 1$ and nonproductive investment $nonProdInv_{inv,t-1}$:

$kStock_t = m_dmgAgg_{dmgStk,"cap"}(1 - \delta_t) kStock_{t-1} + xfd_{inv,t-1} + \phi_{inv}^{pc} xfd_{ginv,t-1} - nonProdInv_{inv,t-1}$	(213)
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Government investments contribute with a factor at or below one. In the latter case, nonproductive investments (say in arts' collections or similar) can be captured. Equally, so-called adaptive investments to climate change which are defined as being not productive, are deducted.

$unemp = pop_{15-64,t} - \sum_l xfmSup_{l,t}$	(214)
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The last factor is changes in productivity. The default choice in baseline construction is one uniform labor productivity shifter which affects all labor types and activities to recover a given GDP growth path. Alternatively, the labor productivity of service sectors can be endogenized and linked to labor productivity in other sectors by a quadratic function of GDP growth.

The supply of other primary factors (land, water, natural resources, if present) is typically based on an iso-elastic supply scheme driven by a return to these factors. As reasonable choices for these elasticities suggest moderate elasticity values where large increases in rents can result if the economy grows strongly. The model, therefore, allows linking productivity changes in these sectors multiplicatively to changes in labor productivity.

Recursive dynamics versus limited forward-looking behavior

In the recursive-dynamic version, agents have rational expectations for product and factor prices and other variables relevant to their decisions taken in the current year. On-demand optimal decisions, for instance, use and allocation of resources, are based on realized prices for products and factors derived from market clearance. There are hence no surprises about market outcomes, planned is equal to realized production, including the consideration of stochastic shocks to production in the current (but not future) period. First order conditions hold at realized prices. But current year decision-taking is myopic as it does not consider expectations of future developments. This assumption can be challenged in cases where agents are informed about future changes in markets such as under announced upcoming tax reforms or known impacts of climate change.

The model, therefore, supports a modular extension where expectations for some key variables in MANAGE-WB are based on a simultaneous solution for a limited number of future years to overcome the limitation of the myopic solution. It establishes an intermediate option between a fully dynamic case where all future years are considered simultaneously and the recursive dynamic case which looks at one single year.

Considering solely a number of future years compared to perfect foresight over the full simulation horizon can be defended based on three arguments. First, with the increasing distance of a future time point to the current one, uncertainty on available technologies, preferences, policy instruments etc. increases.

Simulated future outcomes become hence less certain the larger the distance to the current year which renders their information value for current decision taking less valuable, compared to outcomes for the near future. Second, expectations are mostly important for decisions with a lock-in effect which impacts the future decision space of an agent such as in the case of investments. But lock-in effects are not infinite but soften over time, for instance, due to depreciation. Accordingly, the impact of current decisions on future flexibility also decreases over time which gives a further argument to consider only a limited number of future years when forming expectations. Lastly, time preference for money implies that impacts of current decision on future outcomes receive lower weights the farther away the future time point.

The solution is based on “rolling window” approach where a number of future years besides the current year are solved for simultaneously, and before the current year is moved forward by one period and the considered future period is prolong by one year as well. The module introduced stock changes, see chapter “Stock change behavior”. Additionally, the expected return to capital $rore_t$ uses the following equation:

$rore_t = \frac{\sum_{tNext < t+n} \left(rorc_t \left[\frac{kstock_{tNext} E_{tNext}}{kstock_{tNext}} \right]^{\varepsilon_t^{rore}} \right) expWgts_{tNext}}{\sum_{tNext < t+n} expWgts_{tNext}}$	(215)
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For explanation of $expWgts_{tNext}$ etc., see chapter “Stock change behavior”.

3. Model Implementation

The code for MANAGE-WB is structured into different files and folders (Figure) to support structured model development and maintenance as well as hosting the code on a version control system. The main folder can be named freely as all folder references in it are relative to the main folder.

To simplify the use of one model version with several country specific projects, country specific input files are stored in sub-folders under the **reg** sub-folder. They comprise the (potentially multiple) bridge files, implemented in EXCEL, which bundle all necessary information for the model: the SAM, additional data, parameters and sets. The file **load_base.inc**, when present, is read by the code to add non-default parameters and sets from the bridge file required in the shock file. In a sub-folder called **sim**, shock files are hosted. Shock files can also comprise declarations and definitions of additional variables and equations which are added to the core model. This allows project specific changes to the model structure without changes to the core model code under version control.

The subfolders in a MANAGE-WB installation host specific type of files while the main level is reserved for **.gms** files that are the start point of GAMS project (called **main files** from here on). All files that are included in such a main file have the extension **.inc** and are hosted in one of the sub-folders.

These sub-folders that organize the model files according to their type and use:

\dat folder hosts the country independent data files

\inc folder hosts the GAMS include files that are called by model code at various stages
 \res comprises sub-folders for the different countries which host the result **.gdx** files
 \sav file keeps all temporary files (e.g. debug **.gdx** files, saved model instances)
 \opt comprises solver option files
 \reg folder hosts sub-folders for the different countries
 \gui folders hosts files and sub-folders needed for the Graphical User Interface
 \jrexxx hosts a Java-Run-Time Engine needed for the Graphical User Interface

Model runs to construct a baseline or to run a counterfactual are started with **run.gms**. A counterfactual inherits the time series of variables and parameters from a counterfactual, as well as all set definitions etc. Starting a counterfactual with an empty shock file will hence replicate the baseline results, however, different from baseline mode with GDP turned endogenous and exogenous productivity shifters. Accordingly, shock files define changes in exogenous variables compared to the underlying baseline to render them transparent and should not repeat shocks already implemented in the baseline.

Figure 3.1: Folder Structure

Name	Date modified	Type ^	Size
.git	1/4/2022 11:57 AM	File folder	
dat	1/25/2022 11:15 AM	File folder	
inc	1/25/2022 11:24 AM	File folder	
res	1/26/2022 7:47 AM	File folder	
sav	1/4/2022 11:58 AM	File folder	
sim	1/4/2022 11:58 AM	File folder	
irencgedf.gpr	1/26/2022 9:31 PM	GAMS IDE project	125 KB
.gitignore	12/12/2021 4:28 PM	GITIGNORE File	1 KB
bau.gms	1/25/2022 8:59 AM	GMS File	3 KB
COMP.GMS	1/25/2022 8:54 AM	GMS File	4 KB
Maketab.gms	1/25/2022 10:11 AM	GMS File	2 KB
report.gms	12/15/2021 9:28 AM	GMS File	24 KB
result.gms	1/25/2022 9:06 AM	GMS File	10 KB
result0.gms	12/12/2021 4:28 PM	GMS File	10 KB
sim.gms	12/15/2021 10:58 AM	GMS File	3 KB
EqVaMatch_Reg.xlsm	12/12/2021 4:28 PM	Microsoft Excel M...	7,387 KB
EqVaMatch_v2_3.xlsm	12/12/2021 4:28 PM	Microsoft Excel M...	22,573 KB
path.op2	12/12/2021 4:28 PM	OP2 File	1 KB
path.op3	12/12/2021 4:28 PM	OP3 File	1 KB
path.op4	12/12/2021 4:28 PM	OP4 File	1 KB
path.opt	12/12/2021 4:28 PM	OPT File	1 KB
varList.txt	12/12/2021 4:28 PM	TXT File	7 KB
runall.cmd	12/15/2021 9:30 AM	Windows Comma...	1 KB
vscode.cmd	12/13/2021 4:33 AM	Windows Comma...	1 KB

Folder and File Structure

Table 3.1 lists the core files used for model implementation.

Table 3.1: Subfolders and files under main model folder

Name	Description
./	Main Folder
run.gms	Runs business as usual scenario and simulations
run_stoch.gms	Runs a set of stochastic counterfactuals
samest.gms	SAM updating, balancing and splitting
gtapSAMv6.gms	Generates as SAM and other data from the GTAP Data Base
result.gms	File to prepare results from scenarios quickly in gdx format.
fromGuiToRun.gms	Includes a file generated by the GUI, overwritten with each run
.gitignore	List files, folders and extension to be ignored by github (defines files not under version control)
.gitattributes	Internal files used by github
manage.bat	Batch file which opens the GUI
/dat	Subfolder for project independent data files
BridgeFile_GTAP_POW_template.xlsx	Template bridge file for GTAP derived SAMS (used by gtapSAMv6.gms)
cc_damages.gdx	CC damage estimates by Roson and Sartori (2016) converted into polynomial functions based on a regression analysis
debtData.gdx	Data on foreign debt stocks and related payments (used by gtapSAMv6.gms)
faoProd.gdx	Physical production output in 1000 tons for selected products from FAO production statistics (added to bridge-file by gtapSAMv6.gms)
gdld_2014_V10.gdx	Gender Differentiated labor data by sector and GTAP country, matching GTAP V10 (used by gtapSAMv6.gms)
ghg.gdx	GHG emission inventories from CAIT
gtapData11a.gdx	Basic data from GTAP database (user needs a GTAP license to obtain this file), used by gtapSAMv6.gms. run.gms uses the file to define physical land use and carbon stocks.
gtapGHG.gdx	Matching
gtapSets11a.gdx	Sets used in GTAP data
mfmmodGdp.gdx	Macro-economic projections from MFMOD, June 2023 (added to bridge-file by gtapSAMv6.gms)
nutrients_mng.gdx	Estimates calories, protein and fat contents per USD (added to bridge-file by gtapSAMv6.gms)
SAM_GTAP_POW_template.xlsx	Template SAM file for GTAP derived SAMS (used by gtapSAMv6.gms)
splitSharesWater.gdx	Shares to split up the land account in GTAP V11 crop sectors to land and irrigation water, based on CGEBox results (used by gtapSAMv6.gms)
unpop.gdx	Demographic projections by the UN
/inc	Subfolder for files included in main files
agsam5.inc	Code for update of SAM against macro totals, part of SAMest.gms

base.inc	Declares sets and parameters; reads data from excel file, part of run.gms and run_stoch.gms
checkInfes.inc	Prints lists of infeasible equations and variables hitting their bounds, part of run.gms and run_stoch.gms
fromFrEndExcel.in	Reads in shocks from EXCEL sheet, part of run.gms and run_stoch.gms
fromFrEndReadExcelSheet.inc	Reads in shocks from EXCEL sheet, part of run.gms and run_stoch.gms
inical.inc	Includes initial calibration of parameters and variables, part of run.gms and run_stoch.gms
inistoch.inc	Initializes stochastic variables for stochastic shocks, part of run_stoch.gms
iterloop.inc	Drives model dynamics and closures, part of run.gms and run_stoch.gms
kill_equ.inc	Removes equations symbols from past years to reduce memory load, part of run.gms and run_stoch.gms
macroSam3.inc	Aggregates SAM from Macro-SAM, part of samest.gms
mactotal_eg_agg.inc	Defines macro-totals, part of samest.gms
maketab.inc	Outputs CSV results for multiple scenarios, part of run.gms
model.inc	Model equations, part of run.gms and run_stoch.gms
opt.inc	Some core definitions such as folders and globals related to model extensions, part of run.gms and run_stoch.gms
postsim.inc	Processes raw model results to produce CSV files (on-demand), part of run.gms and run_stoch.gms
readIEC.gms	Example of a main reading CC damage information from CSV files provided by the IEC group
rescale.gms	Rescales quantities and values back to the level of the original SAM, part of run.gms and run_stoch.gms
report.inc	Writes raw model results to a data container to use in result.gms (on-demand), part of run.gms and run_stoch.gms
reportDecl.inc	Declares parameters and sets used in report.inc, part of run.gms and run_stoch.gms
resLastYear.inc	Resets all variables levels to the previous year values (used for the solution algorithm), part of run.gms and run_stoch.gms
resetNegatives.inv	In between repeated solves, resets variables back to plausible ranges, part of run.gms and run_stoch.gms
samCalc.inc	Calculates SAM from model results for each period, part of samest.gms
samStat.inc	Defines statistics on the SAM during balancing, part of samest.gms
result.inc	Generates a result matrix, optional part of run.gms and run_stoch.gms
RSCCDamages.inc	Processes CC damages by Roson and Sartori (2016), part of run.gms and run_stoch.gms
runStoch.gms	Helper code when running stochastic simulations, part of run_stoch.gms
title.inc	Helper code to update title bar in the GUI, part of run.gms and run_stoch.gms
title_def.inc	Definition for the title helper code, part of run.gms and run_stoch.gms
toGui.inc	Calculates the result parameter for the reporting backend, part of run.gms and run_stoch.gms
/opt	Subfolder with option files for the solvers (PATH, CONOPT4)

/res	Subfolder to write results, with regional specific folders (generated automatically)
/sav	Subfolder has temporary files that user does not need to access directly
/reg	Subfolder with country specific directories

Main Files

The **.gms** files in the main folder organizes the model code for different purposes by calling files from other folders. **run.gms** runs the baseline and other scenarios. **run_stoch.gms** runs a set of stochastic scenarios on a grid. **samest.gms** runs the code to update a SAM against macro data or a macro SAM, re-balance it and if, required, to split the SAM further. **gtapSAMv6g.gms** generates bridge-files from the GTAP data to be inputted into **samest.gms** for further processing, such as splitting, or to used directly with **run.gms** and **run_stock.gms**.

All other files are optional and not required for typical applications using the GUI. **result.gms** can be used to comparing scenario results with the baseline outside of the GUI. **maketab.gms** writes the model results to a **csv** file to be read by excel to prepare tables and figures.

Data input files

All other input data, sets and maps for MANAGE-WB are read directly from different sheets in EXCEL file found in **reg\xxx\BaseBridge_xxx_v??**.xlsx where **v??** denotes the version number and **xxx** denotes the country specific folder. Such an EXCEL file is called a bridge-file as it provides the bridge between the template, data driven structure of MANAGE-WB and the region and project specific model implementation. The bridge-file is hence the way to set-up an actual implementation of MANAGE-WB for a specific country (and project). It must follow certain conventions as detailed next.

The **layout** sheet (Figure 3.2) in the bridge-file keeps the information on where each set, map or parameter to feed into MANAGE-WB is found. The first column in this sheet defines the type of GAMS symbol to read (*set* for multi-dimensional maps, *dset* for one-dimensional sets and *par* for parameters), the second column defines the name of the GAMS symbol and the third column describes its location in the workbook. The three following columns define the number of row, column and total dimensions of the symbol, the last being optional. Based on this information, a GAMS utility converts the content of the entries in the layout sheet into a GDX container from which they are inputted in **base.inc**. Wrong ranges or other errors in settings up the **layout** sheet can lead to not or partial input of symbols or can let crash the conversion process. It is therefore recommended after changes to layout to carefully check the generated GDX container in the **sav** folder. In many cases, errors will lead to errors at compile time, but some others, such as reading only a part of a parameter table might not provoke errors at all but instead leads to faulty parameterization of the model.

The user can add data on sheets or new sheets to read additional data, for instance into shock files. Such additional loads can be performed by placing a file "**load_base.inc**" in the project specific folder under **reg**. This requires adding rows for such new symbols in the **layout** sheet.

Figure 3.2: Layout sheet

Comment	Data Type	Name	Location	row dimension	column dimension	Total dimension
				rdim	cdim	dim
Mapping from activities to GTAP	set	mapaGTAP	Maps!A6	2		
Mapping from products to GTAP	set	mapcGTAP	Maps!E6	2		
All accounts (plus nests)	dset	is	Sets!B6	1		
Energy products	dset	e	Sets!G6	1		
Debt account	dset	debt	Sets!T6	1		
Labor accounts	dset	l	Sets!H6	1		
Capital account	dset	cap	Sets!I6	1		
Land accounts	dset	lnd	Sets!J6	1		
Natural resources	dset	natres	Sets!K6	1		
All investment account	dset	inv	Sets!L6	1		
Government investment account	dset	ginv	Sets!M6	1		
Adaptive investment account	dset	adpinv	Sets!N6	1		
Stoch changes account	dset	stb	Sets!O6	1		
Rest-of-the-World	dset	row	Sets!P6	1		
Energy shifter nest	dset	nrgFac	Sets!U6	1		
Maps from tax types to tax accounts	set	maptax	Maps!K6	2		
Mapping from factor taxes to factors	set	mapftax	Maps!M6	2		
Aggregation from proto to final SAM	set	mapis	Maps!H6	2		
Demand bundle composition	par	mapkNestShr	mapk!B5	1		1
Substitutional elasticities in demand nests	par	sigmacNest0	mapk!C2			1
Map from aggregated to single sectors	set	mapaagr	Maps!R6	2		
Production nesting	par	pnest	pnest!B6	2		1
Substitution elasticities for production nests	par	subselas	pnest!C2	1		1
Factor market nestings	par	fmNest	fmNest!B7			
Parameters for factor market nests	par	fmNestPar	fmNest!B2			
For postSim.inc, not needed for GUI	set	mapRep	Maps!P6	2		
Products which emit	dset	emsrc	Sets!S6	1		
Link activities to aggregate emission accounts	set	map_is0_ghglnvL	Maps!Y6	2		
Link activities to aggregate emission accounts	set	map_is0_ghglnvL	Maps!U6	2		
Age cohorts	dset	cohorts	Sets!Q6	1		
Initial SAM	par	SAM0	SAM!A1	1		1
default parameter values	par	defPar	defPar!B2	1		
exceptions in activity parameters	par	actPar	actPar!A1	1		2
exceptions in commodity parameters	par	comPar	comPar!A1	1		1
exceptions in HH parameters	par	hohPar	hohPar!B9	1		2
exceptions in other parameters	par	othpar	othPar!A1	1		1
labor quantities for initial SAM, aggregated to model SAM dimensions in base.inc	par	labor0	labor!B1	1		1
GHG inventory positions	dset	ghglnvUstr	Sets!V6	1		
Official Emission inventory - combustion	par	emilInventory_OFI	emilnv!A2	1		1
Official Emission inventory - Process	par	emilInventoryXP_	emilnv!A12	1		1
Power mix	par	powerMix	powerMix!A1	2		1
GHG mitigation	par	GHGMitigation	GHGMitigation	2		1
Energy projections	par	energyProj	Energy!A1	3		1
Nutrient contents	par	nutCont	Nutrients!A1	1		1
Points on Marginal Abatement Cost Curves	par	macc	macc!A1	1		1
Regression parameter to render CDE income dependent	par	regparIncomeCDE	hohPar!A3	2		1
GDP composition (for targeting)	par	GDPComp	GDPComposi	2		1
Factor supply distribution across institutions	par	xfrmSupDistrInst	hohPar!B28	2		1

The **sets** sheet contains all the one-dimensional sets that are read into the model, while the **maps** sheet comprises two- and three-dimensional ones. Some sets are directly read from columns or row of maps and parameters and do not need to be defined a second time in the sheet. The file **bridgefile_GTAP_POW_template.xlsx** in the **dat** folder can be used as starting point for developing a new bridge-file as it comprises all necessary sets, maps and parameters for a default baseline construction.

The second row in the **sets** sheet uses a formula to get the set name from the fifth row, while the third row shows the column name automatically, again using a formula. These two rows are used in the **layout** sheet to automatically attach the location to a symbol name. A similar setup is used in the **maps** sheet. Sets are all one dimensional while maps can be two or more dimensional. It is good practice to add long-labels to the **is** set which defines the SAM accounts.

[illegible]

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Maps															
mapaGTAP	0	0	0	mapcGTAP	0		0	mapis	0	0	maptax	0	mapftax	0	mapRep
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
mapaGTAP				mapcGTAP			mapis			maptax		mapftax			mapRep
pdr	a-pdr_0			pdr	c-pdr_0		a-pdr_0	a-pdr		dtx	t-DIRTAX	f-Land	t-tf_E1Land		
wht	a-wht_0			wht	c-wht_0		a-wht_0	a-wht		ptx	t-PRODTAX	f-Capital	t-tf_E7Capital		
gro	a-gro_0			gro	c-gro_0		a-gro_0	a-gro		stx	t-tssd	f-NatlRes	t-tf_E8NatlRes		
v_f	a-v_f_0			v_f	c-v_f_0		a-v_f_0	a-v_f		mtx	t-tmm_world	f-tech_aspros	t-tf_E2tech_as		
osd	a-osd_0			osd	c-osd_0		a-osd_0	a-osd		ftx	t-tf_E1Land	f-clerks	t-tf_E3clerks		
c_b	a-c_b_0			c_b	c-c_b_0		a-c_b_0	a-c_b		ftx	t-tf_E2tech_as	f-service_shop	t-tf_E4service		
pfb	a-pfb_0			pfb	c-pfb_0		a-pfb_0	a-pfb		ftx	t-tf_E3clerks	f-off_mgr_prost	t-tf_E5off_mgr		
ocr	a-ocr_0			ocr	c-ocr_0		a-ocr_0	a-ocr		ftx	t-tf_E4service	f-ag_othlowsk	t-tf_E6ag_othl		
ctl	a-ctl_0			ctl	c-ctl_0		a-ctl_0	a-ctl		ftx	t-tf_E5off_mgr				
oap	a-oap_0			oap	c-oap_0		a-oap_0	a-oap		ftx	t-tf_E6ag_othl				
rmk	a-rmk_0			rmk	c-rmk_0		a-rmk_0	a-rmk		ftx	t-tf_E7Capital				
wol	a-wol_0			wol	c-wol_0		a-wol_0	a-wol		ftx	t-tf_E8NatlRes				
frs	a-frs_0			frs	c-frs_0		a-frs_0	a-frs		ftx	t-tf_F_skl				
fsh	a-fsh_0			fsh	c-fsh_0		a-fsh_0	a-fsh		ftx	t-tf_M_skl				
coa	a-coa_0			coa	c-coa_0		a-coa_0	a-coa		ftx	t-tf_F_nsk				
oil	a-oil_0			oil	c-oil_0		a-oil_0	a-oil		ftx	t-tf_M_nsk				
gas	a-gas_0			gas	c-gas_0		a-gas_0	a-gas		vat	na				
oxt	a-oxt_0			oxt	c-oxt_0		a-oxt_0	a-oxt		ctx	na				

The **defpar** sheet in Figure 3.4 comprises a longer list of core model parameters. Detail on these parameters as listed below can be found in the related section of the model equation documentation.

Figure 3.4: *defpar* sheet

Elasticity between demand bundles other final Demand	sigmaf0	1.01
Elasticity between savings and total consumption	omegaSav0	0.2
Elasticity between savings and transfers to HHs	omegaSavEntr0	0.05
Elasticity between government and private bonds	omegaFund0	4
Elasticity of capital supply to capital stock	epsRor0	eps
Elasticity of expected return on capital to capital stock	epsRorE0	0.9
Share of investment that is not added to capital stock	PhiPcInv0	0.75
Depreciation rate	depr0	0.05
Capital stock GDP ratio	capoutRate0	5
Initial interest rate (one plus interest rate)	interest0	1.1
Elasticity of labor productivity to air pollution	AirpolEff0	0.025
Elasticity among commodities when an activity produces multiple commodities	omegap0	0.20
Disinvestment Elasticity for declining sectors	invElas0	0.7
Elasticity among activities when a commodity is produced by multiple activities	sigmas0	inf
Elasticity between imports and domestic demand	sigmam0	0.054
Elasticity between exports and domestic supply	omegax0	3
Elasticity between margin commodities	sigmamg0	0.01
Import supply Elasticity	omegam0	inf
Export demand Elasticity	etae0	inf
debt to gdp default threshold	omegasavf0	0.5
positive scaling parameter	rhosavf0	10
Country risk premium	riskPrem0	1
expected return rate in the row	retrow0	0.05
expected per capita GDP growth rate in the row	ggdppcrow0	0.02
efficiency param. returns	eta10	7
efficiency param. GDP p.c. growth	eta20	7
efficiency param. returns	eta1a0	7
efficiency param. GDP p.c. growth	eta2a0	7
Elasticity of foreign new public debt to interest rate	sigmaFlnt0	1
Threshold after which gov consumption is dampended	debtGdpTgt0	0.5
Default income elasticity for aggregate commodity group (overwritten by hohpar)	incElas0	0.735
Default price elasticity for aggregate commodity group (overwritten by hohpar)	prcElas0	-0.54
Exchange rate Dom Currency per USD	USD	1
Foreign Debt Stock	DebtStkF0	0
Domestic Debt Stock	DebtStkD0	0
Interest on foreign debt	DebtIntF0	3.84
Interest on domestic debt	DebtIntD0	8.20E+00
SAM scale factor	inscale	1.00E-03
Population Scale factor	pscale	1.00E-03
Emissions scale factor	escale	1.00E+00
Elasticity of stock changes with regard to relative delta in expected price	rhoPDelSt0	0.5
Elasticity of stock changes to targeted stock over production	rhoXDelSt0	0.25
Target stock over production level	alphaSt0	eps

The *pnest* sheet defines the nested production structure and comprises two blocks. The first block defines the substitution elasticity for each parent nest. *f-top* is the fixed name for total output. The second block

defines the nesting shares. It comprises the children in the rows. The first column shows if the information along the row applies to all or certain activities. The proposed default nesting for a GTAP-derived SAM assumes a Leontief-relation between the value added *va* and intermediate *nd* bundles as child of *f-top*. This is indicated by eps as a zero would not be read. Equally, Leontief relations are assumed inside the intermediate bundle *nd*. Child nests of this bundle (*crops*, *anim_meat*) allow instead for substitution between individual intermediate inputs. Note also in the example below that the value-added bundle *va* comprises the child nests *cap_ener* and *wtr_land* which combine primary factors with intermediates or bundles thereof.

Figure 3.5: Example of a part of a *pnest* sheet

		labsk	labus	labsk_labus	cap_labsk	va	cap_labsk_ener	ener	ely_tnd	gas_gdt	ener_gasoil	ener_fossil	aez_land	crops	anim_meat	f-top	nd	wtr_land	nd_wtr_land	crops_land
	subElas	0.5	0.5	0.25	0.2	0.7	0.25	0.5	eps	eps	0.7	0.5	7.5	2	2	eps	eps	0.5	0.25	0.5
	additive																			
		labsk	labus	labsk_labus	cap_labsk	va	cap_labsk_ener	ener	ely_tnd	gas_gdt	ener_gasoil	ener_fossil	aez_land	crops	anim_meat	f-top	nd	wtr_land	nd_wtr_land	crops_land
all	f-off_mgr_pros	1																		
all	f-tech_aspros	1																		
all	f-m_skl	1																		
all	f-f_skl	1																		
all	f-clerks		1																	
all	f-service_shop		1																	
all	f-ag_othlowsk		1																	
all	f-m_rnsk		1																	
all	f-f_rnsk		1																	
all	labus			1																
all	labsk			0.5	0.5															
all	f-Capital				1															
all	f-NatIRes					1														
all	labsk_labus					1														
all	cap_labsk_ener						1													
all	f-land							1												
all	f-water								1											
all	aez_land									1								eps	eps	eps
a-pdr	nd_wtr_land																			
a-wht	nd_wtr_land																			
a-gro	nd_wtr_land																			
a-v_f	nd_wtr_land																			
a-osd	nd_wtr_land																			
a-c_b	nd_wtr_land																			
a-pfb	nd_wtr_land																			
a-ocr	nd_wtr_land																			
a-ctl	crops_land																			

The user enters non-default activity related input parameters in the *actpar* sheet. This allows to overwrite the defaults for three different parameters for each activity: *invElas0* (related to the release of old capital stock from the old vintage, in default *defpar*), *omegap0* (transformation of production to multiple outputs, default in *defpar*), and the substitution elasticities for the *old* and *new* vintages for the different nests (defaults in *pNest*). Empty cells do not imply a zero elasticity, but rather that the defaults will be used. Entering eps will overwrite the default with a zero.

Figure 3.6: *actpar* sheet

	A	B	C	D	E	F	G
1		omegap0	invElas0	va	va	cap_ener	cap_ener
2				Old	New	Old	New
3	a-pdr			0.195804	0.522144	0.1	0.80
4	a-wht			0.195804	0.522144	0.1	0.80
5	a-gro			0.195804	0.522144	0.1	0.80
6	a-v_f			0.195804	0.522144	0.1	0.80
7	a-osd			0.195804	0.522144	0.1	0.80
8	a-c_b			0.195804	0.522144	0.1	0.80
9	a-pfb			0.195804	0.522144	0.1	0.80
10	a-ocr			0.195804	0.522144	0.1	0.80
11	a-ctl			0.195804	0.522144	0.1	0.80

Table 3.2: User-specified production elasticities

Name	Description
xxx	Substitution elasticity for nests in production function, by vintage (old, new)
omegap0	Supply transformation elasticity of make matrix
invElas0	Dis-investment elasticity for sectors in decline

The factor supply nesting and related parameters are defined in the **fmNest** sheet. It comprises two blocks where the first defines the transformation elasticities *omegaCET* and indicates if the additive or standard CET functional form should be used. For primary factors, factor supply elasticities are inputted in the *factSupElas* row.

The second block defines the nesting. A special case occurs in case of immobile primary factors, such as for *f-natlRes* (natural resources) in the example below. In this case, factor supply to the activities is not governed by a CET mechanism, but by an isoelastic supply scheme. The values inputted there define the price elasticities, identical to *factSupElas* for primary factors in mobile or sluggish markets.

Figure 3.7: Example of part of a fmNest sheet

	cropland	grasland	agrland	fland	agrlab	f-natlRes	manulab	restlab	f-capital	f-water	f-tech_aspro	f-service_shc	f-off_mgr_pri	f-ag_othlows	f-clerks	f-m_skl	f-f_skl	f-m_nsk	f-f_nsk	AEZ1_cropla	AEZ2_cro
additive	1	1	1	1	1	1	10	10	inf	0.75	5	5	5	5	5	5	5	5	5	1	1
omegaCET	0.75	0.75	0.5	0.25	0.1	0.1	10	10	inf	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.75	0.75
factSupElas																					
a-pdr	1																				
a-vnt	1																				
a-gro	1																				
a-v_f	1																				
a-osa	1																				
a-c_b	1																				
a-pfb	1																				
a-scr	1																				
a-cti		1																			
a-oap		1																			
a-rmk		1																			
a-ctiRais		1																			
a-wol		1																			
cropland				1																	
grasland				1																	
a-fs				1			0.25														
a-fsh				1			0.25														
a-coa				1			0.25														
a-oil				1			0.25														
a-gas				1			0.25														
a-ost				1			0.25														
a-cmt				1			0.25														
a-cmt				1			0.25														

The **compar** sheet overwrites defaults found in the **defpar** sheet for a range of product related parameters: *sigmas0* (the substitution elasticity of outputs from multiple activities into one commodity), *sigmam0* (Armington substitution elasticities between imports and domestic sales), *omegax0* (Transformation elasticities of domestic output to domestic sales and exports), *sigmamg0* (substitution elasticities between different products in margin demands), *omegam* (import supply elasticity), *etae0* (export supply elasticity), *alphaST0* (desired stock level relative to domestic output), *rhoPDeISt* (changes in stock depending on changes in relative price differences between realized and expected prices), *rhoXDeISt0* (changes in stock depending on difference between actual and desired stock level).

Figure 3.8: compar sheet structure

	sigmas0	sigmam0	omegax0	sigmamg0	omegam0	etae0	alphaSt0	rhoPdelSt0	rhoXdelSt0
c-pdr		5.05					0.4		
c-wht		4.45					0.4		
c-gro		1.3					0.4		
c-v_f		1.85					0.4		
c-osd		2.45					0.4		
c-c_b		2.7					0.4		
c-pfb		2.5					0.4		
c-ocr		3.25					0.4		
c-ctl		2							
c-oap		1.3							
c-rmk		3.65							
c-wol		6.45							
c-frs		2.5							
c-fsh		1.25							
c-coa		3.05	0.05						
c-oil		5.2	0.05						
c-gas		17.2	0.25						
c-oxt		0.9							

Table 3.3: User-specified commodity elasticities

Name	Description
sigmas0	The CES aggregation elasticity for the make matrix
sigmam0	CES (Armington) elasticity between domestic and imported goods
sigmax0	CET transformation elasticity between domestic and export markets
sigmamg0	The substitution elasticity for the 'production' of trade and transport services
etae0(i)	Export demand elasticity
omegam0(i)	Import supply elasticity
alphaSt0(i)	Target stock to output ratio
rhoXdelSt0(i)	Elasticity of stock changes to deviation from target stock level
rhoPdelSt0(i)	Elasticity of stock changes to price relative deviations between realized and expected prices

The **hohpar** sheet comprises income (*incElas0*) and own-price elasticities (*prcElas0*) for the households which are used to calibrate the CDE parameters, as well the elasticity which update the saving rate (*omegaSav0*). Note that the regularity conditions such as adding up can imply differences between calibrated and targeted elasticities as entered in the sheet. The sheet can also comprise elasticities to update the expansion terms of the income elasticities depending on income changes.

Figure 3.9: hohpar sheet structure

	incElas0	incElas0	incElas0	incElas0	incElas0	prcElas0	prcElas0	prcElas0	prcElas0	prcElas0	omegaSav0
	food	Manu	Ener	Serv	Tran	food	Manu	Ener	Serv	Tran	
h-hhold	0.885	1.161556	0.85	1.1	0.695	-0.5	-1.1	-0.3	-1.25	-1.25	

Table 3.4: User-specified household demand related elasticities

Name	Description
incelas0	Household and commodity bundle specific targeted income elasticity
incelas0	Household and commodity bundle specific targeted own-price elasticity
omegaSav0	CET transformation elasticity between savings and consumption

Similarly, the **mapk** sheet defines the shares of products and sub-demand nests for final demands to their parents and the related substitution elasticities. Parent nests are shown in the columns, child nests in the rows. Parents which are not also child nests will become the bundles present in the top-level CDE demand nest of the households for which parameters must be inputted in the **hohpar** sheet. For other final demand, a CES top level demand system is used and the share parameters for these bundles are calculated automatically in the benchmark code. Note that child nest, including single products, needs to be assigned a least to one parent nest and that the sum of the shares must be one.

Figure 3.10: Example of part of a mapk sheet

	Crops	MeatDairy	FoodOther	Clothing	Manu	Hot_Trad	Serv	Tran	tranEner	TranRest	enerGas	enerFossil	enerEle	HousEner
	0.5	0.25	0.25	0.25	0.25	0.2	0.25	eps	eps	0.25	0.1	0.25	0.1	0.25
Crops								Tran	tranEner	TranRest	enerGas	enerFossil	EnerEle	HousEner
c-pdr	1													
c-wht	1													
c-gro	1													
c-v_f	1													
c-osd	1													
c-c_b	1													
c-ocr	1													
c-ctl		1												
c-ctlRais		1												
c-oap		1												
c-rmk		1												
c-fsh		1												
c-cmt		1												
c-omt		1												
c-mil		1												
c-trd		0.05	0.1	0.1	0.05	0.6				0.1				
c-vol			1											
c-pcr			1											
c-sgr			1											
c-ofd			1											
c-b_t			1											
c-pfb				1										
c-wol				1										
c-tex				1										
c-lea				1										
c-wap				1										
c-frs					0.5									0.5
c-oxt					1									
c-lum					1									

A set of additional parameters is defined in the **othPar** sheet. The user can add columns to define non-default substitution parameters for demand nests.

Table 3.5: User-specified other parameters

Name	Description
sigmaf0(f)	Substitution elasticity between commodity bundles in other final demand (government, investments)

The **GDPComposition** sheet keeps inter alia the GDP growth projections for the baseline and can host projections of the VA composition by broad sectors (here Manu, Serv, agri,mine, cons, hema) as well as of GDP composition (Government consumption govCons, net exports netExp, foreign savings savf).

Figure 3.11: Example of part of a GDPComposition sheet

		2017	2018	2019	2020	2021	2022	2023	2024
bas	Manu	0.13022814	0.13022814	0.13022814	0.13022814	0.13022814	0.13022814	0.13022814	0.1
bas	Serv	0.40855384	0.40855384	0.40855384	0.40855384	0.40855384	0.40855384	0.40855384	0.4
bas	GovCons	0.0879181	0.0879181	0.0879181	0.0879181	0.0879181	0.0879181	0.0879181	0.
bas	netExp	-0.07015892	-0.07015892	-0.07015892	-0.07015892	-0.07015892	-0.07015892	-0.07015892	-0.0
bas	savf	0.05698296	0.05698296	0.05698296	0.05698296	0.05698296	0.05698296	0.05698296	0.0
bas	agri	0.26585163	0.26585163	0.26585163	0.26585163	0.26585163	0.26585163	0.26585163	0.2
bas	mine	0.01807101	0.01807101	0.01807101	0.01807101	0.01807101	0.01807101	0.01807101	0.0
bas	cons	0.14393702	0.14393702	0.14393702	0.14393702	0.14393702	0.14393702	0.14393702	0.1
bas	hema	0.03335834	0.03335834	0.03335834	0.03335834	0.03335834	0.03335834	0.03335834	0.0
bas	GDP	6.79	5.44	5.80	2.00	4.28	5.34	5.70	

The **SAM** sheet keeps the SAM. The SAM is the key element in the data driven concept underlying MANAGE-WB. However, the user can aggregate this SAM based on the *mapis* map (in the maps sheet) to the products, sectors, factors, tax instruments etc., used in the project.

Figure 3.12: Example of part of a SAM sheet

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
SAM 2019, million LC																			
		a-agr0	a-frs0	a-coa0	a-oil0	a-ox0	a-fod0	a-b_r0	a-tex0	a-wod0	a-p_c0	a-chm0	a-bph0	a-omf0	a-l_s0	a-fmp0	a-ele0	a-mvh0	a-met0
a-oil0																			
a-ins0																			
a-oxg0																			
a-edu0																			
a-hh0																			
c-crp0	73534565	305318	26.1817	19.347726	2.48167	18986623	1781639	1.4E+07	18512	12.876553	24487.96	48289.2	17625.464				5.69536	2710.26	118804
c-frs0	260664.02	1644040	10958.6	4.6467703	8813.897	7634.345	36.5284		739075	160.15249	297420.7	7.355	40713.219						41085.6
c-coa0	4242.4125	1770.22	159.954	4.7318754	278.4959	167.2382		3451.95			12181.96				10004.57	13.7626			3.63326
c-oil0	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.488.6	*****	*****	*****	*****	*****	6.723.5	9.243.2
c-ox0	38252.686	2389.54	9481.77	623906.64	727793.7				43.9676	14127.651	400847.1	16.716	868102.29	6503588	9397.77				394.55
c-fod0	3512450.8	37478.3				6351279	568678	12663.6			21550291	24367.32	21.2234	889.67974	462.49126	232.605	10220.1	21.1054	6.83548
c-b_r0	477010.35	819392		36.327568	8.219236	216262.8	824506	665.443			3.7565044	130.8222	10.0295	260.08276	51456589	12.4709	61.1564		98.64
c-tex0	77599.608	6598.68	2370.63	669.41703	14095.75	40372.94	10710	1.4E+07	718.206	4862.5801	44769.18	485.852	89287.84	102392.7	4980.74	14912.2	873.23		26682
c-wod0	339823.8	7712.37	38771	1096.083	45941.16	518370.1	368080	222433	1198587	352.52476	61250.86	28332.5	965160.02	26327.78	8175.28	96569	7016.91		76363.6
c-p_c0	210046.3	114191	62006.7	174759.4	216328.8	93371.6	37021.7	80317.7	20660.8	3015261.9	484573.6	1845.35	210902.43	583821.84	5544.51	75145.7	22933.3		87535.9
c-chm0	2982907.6	57129.8	15165.5	33197.983	133258.7	222637.1	328402	1328959	184331	52464.447	6508093	64602.9	3023575.3	47142.79	68520.5	117318	96004.1		113674
c-bph0	274383.48	4143.03	47.0177	15.239804	5.742587	180851.7	4830.96	547.73	40.0145	436.21653	54983.33	747512	52421.963	719.1067	79.7734	49.3258	8.60294		64.9137
c-omf0	1290221.8	26307.9	61250.8	17845.199	424387.5	388874.1	715894	804035	15397.7	20267.188	181508.1	15349	3728909.5	1704171.4	268561	176373	98899		121365
c-l_s0	253088.02	4174.43	11233	107880.52	444695	17773.39	1886.62	120359	852.467	5627.574	25194.92	1069.91	904887.82	12633769	2197545	1735965	1657481		1626517
c-fmp0	138674.1	28430.3	7988.38	166727.01	262986.8	145151.4	397442	146597	64628.4	3490.5971	72577.16	687.425	491574.88	527180.02	825246	204677	261721		570795
c-ele0	509752.21	17009.6	14974.6	9630.2062	476643.4	123506	67630.5	79842.6	141802	14639.23	60040.45	1502.04	232352.55	474848.58	145614	3266080	116944		259367
c-mvh0	908922.37	20013.4	17965.2	247523.46	233052.1	111261	153775	323634	74398.5	16678.294	462631.3	960.004	153505.87	660336.51	36022.3	93827.8	628265		259350
c-met0	224721.27	5289.49	2245.74	11301.939	63176.36	46824.74	12339.5	33448.5	9523.98	2074.785	20328.14		48647.781	397344.9	1664.33	14503.3	9116.71		2.2E+07
c-TrnD	69467.115	2980.06	7942.12	358464.45	28626.44	73486.16	16647	108416	23752	63661.006	179826.9	7977.67	156443.53	424287.04	24338.5	68056.4	33639.9		324812
c-ely0	202034.66	8787.74	23128.5	1050340	84175.32	215045.7	49221.3	316613	69950.6	205142.49	520872.9	23435.4	455705.51	1223551.8	71343.1	198662	98698		95178.1
c-ent0	16896.742	55318	594.895	263.59159	9850.643	32723.35	5065.51	50706.6	45340	694.32297	31361.8	2038.95	69072.823	462399.68	49130.6	21029.4	24176.1		7484.83
c-omr0	150493.84	21719.1	3425.62	2470.432	32592.56	24102.2	23592.4	13463.1	181219	67722.641	36007.65	197.72	364755.63	43593.019	7287.93	8560.35	2365.85		16437.6
c-trd0	96168.059	39850.9	35771	38491.194	9401.698	115360	41823.3	16132.8	5544.11	1622.9745	17881.72	539.715	32080.334	82379.32	1359.36	27632.5	21431.6		44346.2
c-otp0	35374.324	12876.6	9.38644	5376.452	73426.37	235708	48805.4	39451.7	11282.8	8364.0409	176117.3	1054.54	706050.73	51139.65	10620.8	261627	4988.81		19416.9
c-atp0	512.85475	78.9406	423.831	1358.246	24.8168	259.3764	2649.65	490.897		324.62677	10492.14	264.516	4720.1647	1096.4015	117.025	444.362			3038.76
c-afu0	63758.442	1239.65	0.85562	4466.1195	1224.881	6242.49	10093.7	4969.23	58.4142	5393.4191	30603.7	225.775	7343.2777	6150.1008	309.01	2761.15	963.364		7412.83
c-ost0	1000694.8	67286.9	28481.3	245741.32	117631.7	318852.3	94491.3	186580	288888	8879.3144	489069.6	15081	298586.17	126263.79	63342.7	260633	71942.9		1020364
c-cmn0	6673.1553	1678.87	85.5221	15600.861	12504.33	14624.91	4909.24	11253.1	2487.98	1698.7006	44035.86	720.672	23394.308	27343.291	12211.9	4073.66	10757.5		15427.2
c-oil0	969654.38	22496.9	23957.1	1324694.6	348549.3	489810.1	71122.2	504425	97932.7	19980.752	362322.1	43080.8	550047.28	867105.76	112123	153138	103373		408812
c-ins0	107779.84	2064.47	1902.18	147023.53	63629.63	60255.05	8322.41	29304.8	2462.01	2030.4693	32259.69	1711.81	56137.163	87467.618	3724.25	14122.5	8570.21		32804.9
c-oxg0	1829.1777	91.1041	505.294	532.1703	62.18138	4291.398	740.237	2954.63	324.666	700.20195	2232.756	1022.41	4531.6305	10991.496	3024.48	2387.5	4943.66		6880.36
c-edu0	3663.8129	594.857	809.4	5375.7786	48.31329	738.2904	1806.44	505.564	145.652	448.39236	3797.962	107.13	3621.4146	12791.976	334.132	286.964	29.2372		2964.18
c-hh0	4864.9531	96.0765	308.238	407.39353	198.512	2413.894	1436.42	423.997	16.3916	384.2828	4093.965	21.0011	1970.0661	5194.2105	250.885	451.729	19.8862		892.577
f-fsk0	325708	9035	27504	208450	111524	82620	71089	419516	70200	74765	215435	41519	352617	539606	72228	146610	35141		244261
f-fsk0	3495768	110086	135421	1110162	594149	988033	418644	2245548	686626	399948	145594	146193	1664786	3226873	374699	520867	162344		1271478
f-fsk0	487784.76	104815	3860.54	30287.806	15520.21	104671.6	1374.2	61182.9	9695.39	10426.587	31377.81	1124.79	50205.651	97826.245	101314	4021.11	4876.12		35624.1
f-fsk0	37632.985	988.325	1752.22	13381.551	7011.463	9136.535	4635.5	26968.4	4394.68	4694.0253	13849.6	1533.54	22410.834	40766.272	4564.86	5470.43	2208.38		15752.6
f-fsk0	5532166.1	128042	47577.2	386896.95	197037.8	1270322	176792	782401	123220	133454.95	400551	10680.7	639223.99	1249221.8	128181	38107.0	61899.5		451777
f-fsk0	187663.24	3694.72	2586.14	21198.948	10451.26	37711.63	7933.32	42596.4	6488.46	7059.6253	21982.1	186.518	34517.777	63987.334	6898.69	666.603	3270.17		24873.9
f-capo0	11588423	4223369	136329	31149375	3396045	8110101	2396449	9107351	1218033	2149618.4	5829036	743227	7311865.6	28255582	1794808	1688391	898193		3677925
f-fnd0	63957155																		
f-fnd0	34084447																		
f-fsa0	541407.61	24856.7	4338.93	206936.33	48388.82	138185.7	47621.7	205337	29254.8	61800.399	113220.2	11627.7	134078.19	345935.04	33620	55798.8	26381.7		201780

The **emiinv** sheet is intended to keep emission inventory data and used to scale GTAP emission factors to these inventories.

Include files

The **inc** folder comprises all not-project specific includes files that are called by the main files. Inc files thus host the generic parts of the model code which are not in the mains. Currently these files are organized to separate declaration of sets and parameters used in a run, code for model benchmarking and loading the baseline information for a counterfactual, variables and model equations, recursive dynamics and reporting.

Project specific changes to these files should be avoided to ensure a smooth use of the versioning system. Modifying the code of files under version control will easily provoke so-called conflicts when model developers commit code to extend the model or correct bugs. Changes to the equation structure of the model can be introduced via the shock file. Additional data can be loaded in shock files or by providing the **load_base.inc** file in the project folder.

Main settings

opt.inc file sets the main settings for the model such as activating optional modules (optional modules, simulation abbreviations, time periods etc.). Global variables that manage the folder structure is also declared in **opt.inc** file.

Files for benchmarking

Benchmarking is the process of defining certain model parameters such that the observed *ex post* data, mainly found in the SAM, are replicated by the model under a no-shock experiment in the base year. This model calibration starts with declaring the necessary sets, maps and parameters in the **base.inc** file. Afterwards, the data in bridge-file as an Excel workbook is translated to a *gdx* file that is stored in the **sav** folder under the same name. Errors in the **layout** sheet, such as wrong cell references, will trigger error in the conversion such that the *gdx* file is not generated. This typically provokes many follow-up errors as necessary symbols cannot be inputted. This *gdx* file derived from the bridge-file can be inspected, for instance, with GAMS Studio to check if the translation has worked properly. The code of **base.inc** defines then the sets and reads afterwards the parameters using so-called domain checking - i.e., it makes sure that the labels used to define elements in sets, maps and parameters are found stem in the sets used in the symbol declaration. This excludes, for instance, that typos in the bridge-file go unnoticed.

After loading the project specific data tables, each parameter is assigned to the corresponding initialization parameter in the model, data from generic global data bases such as UN population projections are added. The user can add data to **base.inc** by providing a file called **load_base.inc** in the project specific *reg* folder.

After reading in the data, calibration of the parameters and assigning initial values to variables from the SAM and other satellite accounts is done in the **inical.inc** file. Like most CGE models, model parameters are calibrated such that the model implementation is able to re-produce the base dataset. The input to

the calibration procedure is the base year SAM, potentially some satellite accounts such as energy balances, population, etc., and a set of key parameters (mostly elasticities). More formally the model can be written as:

$$F(y, x, \theta, \Omega) = 0$$

where y is a multi-dimensional set of endogenous variables, x is a multi-dimensional set of exogenous variables such as tax rates, and the parameter set is divided into two: θ contains calibrated parameters and Ω contains key (or user-specified) parameters. A typical simulation is then solving F for y with x , θ and Ω fixed-where x deviates from its base value, for instance, tax rates might have changed. In the calibration phase, y and x are taken from the SAM and other data, the parameters Ω are inputted via the bridge-file and function F is used to solve for θ :

$$F(y_0, x_0, \theta, \Omega) = 0$$

In practice, calibration can be done recursively for block of equations without the need to invert the full equation system F formally.

The variables that are exogenous by default in the model, such as tax rates, emission and damage coefficients or productivities, are already fixed in the **inical.inc** file to keep the closure part of the **iterloop.inc** file simpler. If these variables need to become endogenous due to closure swaps specific to a counterfactual, the lower and upper bounds need to be set to minus and plus infinite in the shock file by the user.

Recursive Dynamic files

Most recursive dynamics in the model code are hosted in the **iterloop.inc** file. **iterloop.inc** is triggered only in the second and subsequent years of simulations. It first provides start values for the next solve by using some 'rule of the thumb' such as adding the GDP growth rate to previous year's result by calling the **resLastYear.inc** file. Although start values should not change the simulation results, they can affect model convergence and speed. For counterfactuals, the user can alternatively use the given benchmark results for last period as the starting point. This might speed up solution when the differences between the baseline run and the counterfactuals are limited over the full simulation horizon. **resLastYear.inc** also initializes the vintage capital structure for the first year by deriving the old capital from the benchmark capital use in each sector. Initial values, reflecting the SAM and satellite account for the base year, and lagged variables used in certain equations are also fixed in **iterloop.inc**. Capital accumulation and population dynamics together with some rules to update variables between periods exogenously are also done in the **iterloop.inc** file. Then different default closure options are implemented based on whether the model run is for the baseline or a counterfactual, depending on the **ifCal** parameter). **resLastYear.inc** is also used to reset the variable values in case of a model convergence failure, before attempting to solve the model with a different solver option file. **iterloop.inc** also updates on-demand the share parameters of the old vintages to represent a weighted average of the previous year's old and new vintage, similarly updates share parameters of the old labor force stock and can shift parameters in the CDE demand system on-demand.

iterloop.inc on-demand recursively sets start values for many variables from last year's results to reduce the number of infeasible equations and the sum of infeasibilities. This can especially help in case of large or disruptive shocks where differences between last and current solutions become large.

If the shock file in use implements certain non-standard closure swaps, providing a file **release.inc** in the project specific folder might be needed to problems resulting from applying default closure rules in **iterloop.inc**. Such a file is automatically called if present in the project specific folder and should add the necessary statements to re-define bounds, such as freeing some variable instances.

Model Files

The **model.inc** file contains variables and equation declarations, equation and model definitions. The listing of the model equations follows the same order as in this document and should allow for a relatively transparent comparison of the model write-up and its implementation in GAMS. The file is structured by blocks, such as production, income, government, consumption. Each equation is conditioned on the current solve year (i.e., if t is an element of ts) and other controls that drop equations in case where corresponding variables are not active. The **model.inc** file finally declares two version of the core model. One is paired – i.e., equations are matched to the corresponding variables. It can be used with a MCP solver to introduce complementarity conditions such that hitting a bound on a variable turns the related equation into an inequality. Using the MCP solver with the paired model version is also helpful in case where programming errors lead to a so-called non-square model – i.e., the number of non-fixed variables differs from the number of equations. The pairing information is neglected when a non-MCP solver is used. The non-paired version can be useful when the MCP solver is used with shock files using closure swaps. **model.inc** also declares two sub-models used to calibrate the CDE demand system.

Files used to solve the model

After a start point for the model is defined for the current year to solve and variables are fixed or freed according to the active closure swap, **run.gms** will attempt to solve the model. As a default, the first solve will use CONOPT4 in CNS mode (Constrained Non-Linear System of Equations). This will immediately fail if the model is non-square – i.e., if the number of non-fixed (or endogenous) variables is not equal to the number of equations. If the model is square, the maximal wall-clock time in seconds and number of iterations as inputted via the GUI can be used by the solver to find a solution. If this fails, the code will try repeatedly to solve the model. If the sum of infeasibilities after the first try is small, the second try will start from this solution. Otherwise, the start point will be reset based on a call to **iterloop.inc**. Subsequent solves will switch to DNLP mode in CONOPT4 which is usually slower than CNS but numerically more stable, they will also use solver option files were the feasibility tolerance is slightly higher. These repeated solves will also use different sets of bounds on some model variables to keep the solver away from regions where zero divisions and other math trap errors could occur. This is supported by calling **resetNegatives.inc** between solves which moves implausible values from the last solve into a range where numerical issues are unlikely. If the model is solved as a MCP, different option files are tried in repeated attempts to solve the model.

If no solution is found after the last solve – i.e., there are remaining infeasibilities or the Walras test fails, **run.gms** throws an error and aborts further execution. The intermediate results are stored in a **gdx** file comprises the string “debug”. It can be inspected to find suspicious developments in endogenous

variables and implausible levels for fixed ones. To help the analysis in case of infeasibilities, **checkinfes.gms** will report the first 100 infeasible equations and the first 100 cases where non-fixed variables hit their bounds. The latter should not happen as bounds should solely stabilize the solution behavior.

When the model is successfully solved, the code of **run.gms** will clean up the memory every third year by setting marginals and levels of equations back to defaults in **kill_equs.gms**. This file will also remove bounds which had been introduced by **run.gms** to stabilize the solution behavior. The next solve of the model will not use an in-memory call to the solver to trigger a re-organization of the internal memory of the GAMS base engine. This keeps the overall memory load of MANAGE-WB lower.

Reporting Files

Model results are now mostly explored based on the reporting backend of the GUI. The necessary information for the GUI reporting is assembled in **toGui.inc** which populates the parameters *p_toGUI*. It will also generate an XML file comprising the list of sectors, products, emissions etc., which is copied under the project name to the folder **gui/xml**.

For specific applications and to maintain backward compatibility, some other reporting possibilities are maintained. These alternatives report the model results in two ways: through the **results.gdx** file and through the **results.csv** file. Both options are run via different files: the **gdx** file is produced by running **result.gms** while the **csv** file is produced by running the **maketab.gms** file.

result.gms file reports levels, percentage change, level difference, year on year growth and cumulative growth for most variables and some parameters in the model and only for the variables and parameters in the model. It calls the simulation result files (**simname.gdx** files in the **res** folder) and reads the **rescon** parameter from each. It then writes the contents of the **rescon** parameter to a new container called **result** which has an extra dimension for the result type (e.g., level, percentage change from base etc.). Then it calculates the specific result types and writes them to variable/parameter specific result parameters which follow the naming convention of **variablename_r** using a macro that declares the new parameter and assign relevant values to that parameter from the **result** container. It then writes all **variablename_r** parameters to the **result.gdx** file in the **res** folder. The aim of the **result.gdx** file is to give the user a quick way to check the results while designing the scenarios.

Once the design of the scenarios is complete and the user has the set of simulations to report, running **maketab.inc** file would create a **csv** file with most relevant model results included. **maketab.gms** also calls the simulation **gdx** files and reads in all variables (not only **rescon** parameter). Then it calls the **postsim.inc** file to calculate specific indicators by using scenario results. The **csv** files are intended to be read into excel using data import facilities and be the underlying database for pivot tables.

The **samCalc.inc** file builds a SAM from the model results using variable values for each year in a simulation. It is activated by setting **ifSAM** parameter in the **opt.inc** file to 1. It uses model equations and solutions to fill in a post-simulation SAM which is very useful in spotting possible errors, especially those that causes a non-zero Walras.

Simulation Files

Simulations with the model use the same core GAMS file **run.gms** used for benchmarking. Simulation (or shock) files are generally project specific and therefore stored in the country specific folders in the **sim** folder. Such a shock file comprises of two parts, the declaration section which defines any additional symbols used in the shock file, and a transformation part which is called by **run.gms**. The following empty no-shock file shows this concept and gives an example on how a shock file can change the equation structure, here related to the pairing.

```
$iftheni.decl "%1"=="decl"

$setglobal modelVariant "rgdmp"

Model cge_%modelVariant%
/
  cge_
*
* --- implement BaU closure by changing pairing
*
  - rgdpmpeq
    rgdpmpeq.chixFacL
/

$else.decl

$endif.decl
```

Shock files can be used to cover a set of variants of a certain type of shock. For instance, a parameter read from a GDX container and inputted in the table in the declaration part could provide different sets of productivity changes, for instance, climate change damages for different future climate. The user can then via a global inputted on the user interface (see next) determine which future climate to use for the specific counterfactual.

Shock files are also regularly used during baseline generation to address cases which cannot be handled based on the features available from the GUI.

4. Graphical User Interface

Introduction

The Graphical User Interface (GUI) of MANAGE-WB serves mainly two purposes. First, it allows users to set run specific options such as files to use via the GUI rather than editing text files. Besides a touch and feel closer to modern software, this also avoids frequent changes to files under a version control. Second, the GUI generates reports to view and analyze model results, potentially comparing three or more scenarios which can represent baselines or counterfactuals.

The code underlying the GUI is the package GGIG (GAMS Graphical Interface Generator in Britz (2014)) is realized in Java of which license free versions portable across some operation systems are available. A

cost-free version of a Java run-time engine under the GNU license is shipped with the MANAGE-WB installation, no installation is necessary. GGIG as a package is in use for more than a decade with different economic modeling system, including at the OECD and the European Commission.

If the MANAGE-WB GAMS code uses consequently forward slashes as folder separators, this allows to run MANAGE-WB also natively on a MAC computer. Changes to the GUI layout require no knowledge of Java. Instead, the tasks and GUI controls are defined in **gui/manage.xml** and the reports in **gui/manageTables.xml**.

Baseline generation

Overview

The overview on the baseline process is depicted below. The user operates the controls on the GUI and once all settings are made, starts GAMS. Before the GUI executes GAMS, it generates the file **fromGuiToRun.gms** which maps the current control settings into GAMS statements. This file is included by **run.gms**, the main file of MANAGE-WB. The user can also start **run.gms** without the GUI, for instance from GAMS Studio and can manually change settings in **fromGuiToRun.gms**.

The code in **run.gms** defines some core information (for instance, directories to use, simulation horizon) in **opt.inc** and reads in the necessary data for benchmarking. This is based on two steps. First, the information from the selected base-bridge file in the country directory (**reg**) is converted into GAMS symbols in a GDX container.

Second, the sets and parameters in this GDX container are inputted into the GAMS process. These sets and parameters are mostly declared in **base.inc**. **base.inc** also comprises a longer list of data transformation statement which read in GTAP data on economic transactions and related emissions and maps them into emissions factors matching the structure of MANAGE-WB and the specific lists of products and activities in use. Country specific data input and transformation after reading and processing the default information from the base-bridge file can be done in a country specific file called **load_base.inc** which is automatically read if exists. Users should refrain from changes to **base.inc** or **inical.inc** to the extent possible to keep the master files clean. Instead, move country specific code into **load_base.inc**.

When working on **load_base.inc** or shock files, users should avoid fixing all potential instances of a variable to zero (such as *someVariable.fx(a,j,v,t) = 0;*). Instead, the code should use available flags sets such as *flagXFac* to restrict such statements to the variable instances actually entering the model equations. Otherwise, GAMS will need to allocate memory for all potential records and all fields of a variable (.lo,.up,.l,.scale,.m). The previous version comprised statements which fixed over a million records of which only a few ten thousand could actually enter some model equations. The more memory in use, the slower GAMS will get. Allocating a lot of memory might also mean that MANAGE-WB might fail to run on machines in training. It will also cost disk space when the results are saved.

The data read are then used to benchmark the model in **inical.inc**. The declarations of the variables are found in **model.inc** along with declarations of parameters used in the model. The benchmarking step defines starting values for all model variables and time points. Model parameters are either copied from

symbols found in the base-bridge file (mainly substitution and transformation elasticities) or derived from them and the starting values of the variables, such as share parameters in CES and CET functions.

Before the solving loop over the years starts which generates the baseline, declarations in the shock file for the baseline (the BAU for Business-as-usual scenario) are included. Declarations inside a loop are not allowed. Any shock file must be structured as seen below, even if the declaration part is empty. Otherwise, the shock will be applied before the simulation loop with unpredictable results:

```
$ifthen.decl "%1"=="decl"
```

```
* --- here all declarations which are specific to the shock file:
```

```
$else.decl
```

```
* Here come the statements which are executed in each simulation year, declarations are not allowed
```

```
$endif.decl
```

For each year, the code in **resLastYear.inc** will first copy last year's variable levels to the current year to solve. Quantity variables are additionally multiplied with next year's exogenous GDP growth. Some variables are then updated based on others (such as demography, debt stocks) and the closures applied - i.e., some variable levels are fixed, such as real GDP per capita, and other ones which are exogenous in simulation runs are turned endogenous.

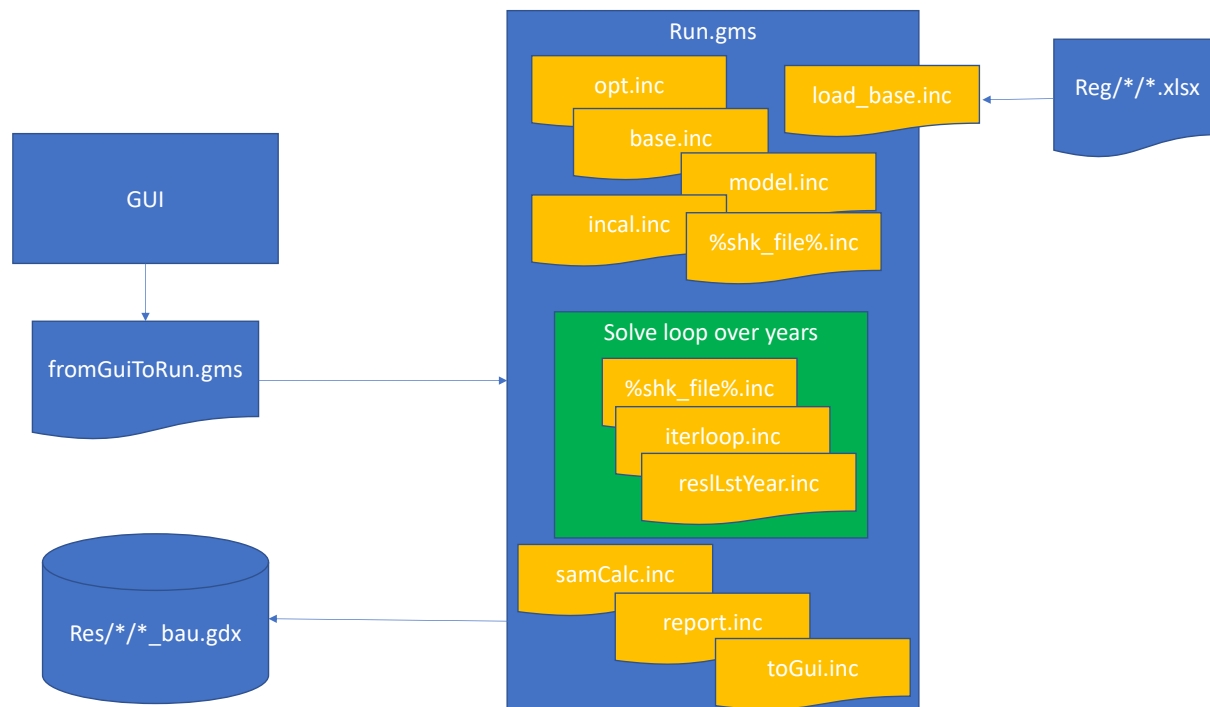
After setting the start values, the shock file is then included which introduces BAU specific changes, for instance, with regard to taxes or productivity shifters. The shock file can also define additional closure swaps or change the ones found in **iterloop.inc**. Afterwards, **iterloop.inc** is included again and fixes some tax rates and productivity shifters which do not depend on endogenous variables such that the related equations are not needed. This reduces the time for model generation and saves memory during model solving.

Should the solver not be able to solve the model in one go (too many iterations, too much time needed, declared infeasible), additional solving attempts are made. In MCP mode, different option files will be tested in repeated solve attempts. In CNS mode, subsequent solves will formally solve the model as a DNLP, using an empty objective function. This is useful as CONOPT4 applies a different solution strategy to square systems of (non)linear equations (CNS) and optimization problems (DNLP). The CNS mode is generally faster but might fail on certain model instances which can be solved as (pseudo) non-linear optimization problem. The tests in DNLP mode will introduce some bounds to avoid that CONOPT4 spends a lot of time in finding defined equations and to stabilize the solution process.

Once all years are solved, the post-model reporting codes are executed according to the options chosen by the user. In order to use the reporting back-end of the GUI, **toGui.inc** must be called. It maps variables and parameters and transaction derived thereof into one multi-dimensional parameters. Additionally, the user can generate a SAM for each year and produce output for the EXCEL based front-end.

In baseline mode, all symbols (sets, parameters, variables) are stored in a GDX container under the **res** folder in a country specific sub-folder. The name of the GDX container is by default countryName + “_” + “BAU”. In order to support multiple baselines, the user can add a simulation name *postfix* on the GUI as shown in Figure 4.2. This container is then loaded as a start point for counterfactuals.

Figure 4.1: Flowchart of baseline generation



GUI controls in baseline mode

General steering

The GUI controls in “General settings” are shown above. The user will select the sub-directory for the country (“Region dir”, found in **reg** folder) and based on this, one of the XSLX bridge-files found there (“SAM input (xlsx)”. Equally, the first and last year of the simulation period are chosen. The analyst can shorten the simulation horizon, if needed, by moving the “last year of simulation” before the last year. Otherwise, leaving “2100” is recommended. Choosing a different “last year of simulation” is useful if parameters are defined until the end of the simulation horizon such that domain errors might occur if a shorter horizon is defined based on “Last year” rather than by setting a “Last year of simulation”.

Figure 4.2: Screen shot “General settings” tab of the GUI in baseline mode

The second row of controls under the heading “Shock” allows to set a simulation name and to select the shock file used for the BAU run. Shock files are searched for under `/regname/sim/*.inc`. If multiple BAU runs are to be generated, the analyst can add a post-fix under “Postfix” next to “Simulation name”.

The content of the “Scen driver” fields will be written as entered by the user as additional lines in the generated include file. This content must hence comprise correct GAMS syntax. Most often, such additional lines provided by the user are used to introduce additional information, such as numerical settings or the choice of a specific scenario into a shock file. For instance, the size of a productivity shock could be stored on a global via one of these fields:

Scen driver 1 `$setglobal prodShockServ 1.01`

And then used in the code of a shock file, for instance:

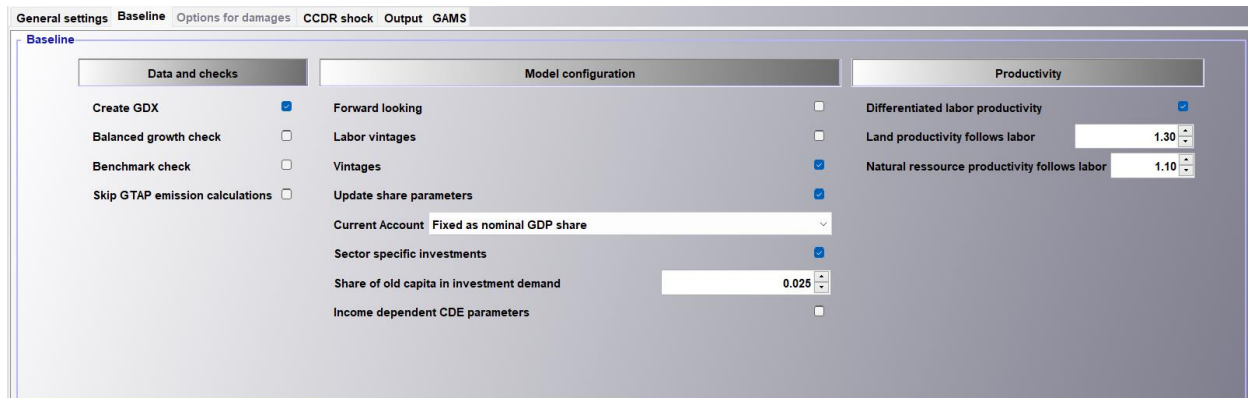
```
chixFacAgr.fx("serv","f-top",tt) = %prodShockServ%;
```

An appropriate use of such globals can help to keep the number of shocks files in a project small, especially to avoid that the same code block is found in many shock files. The user should however note that result files are by default named after the shock file such that either a post-fix must be set manually for results based on the same shock file, but using different settings, or the code in the shock file must define the postfix global based on the globals inputted by the user via the GUI.

The second last row of controls sets options such as “Air pollution damages” which refer to specific mechanism in MANAGE-WB and are detailed in the previous chapter. The last row comprises the option to choose the GDP composition and growth scenario from the “GDPComposition” sheet in the bridge-file and to target certain shares.

Options specific to the baseline mode are found under the “Baseline” tab:

Figure 4.3: Screen shot “Baseline” tab of the GUI



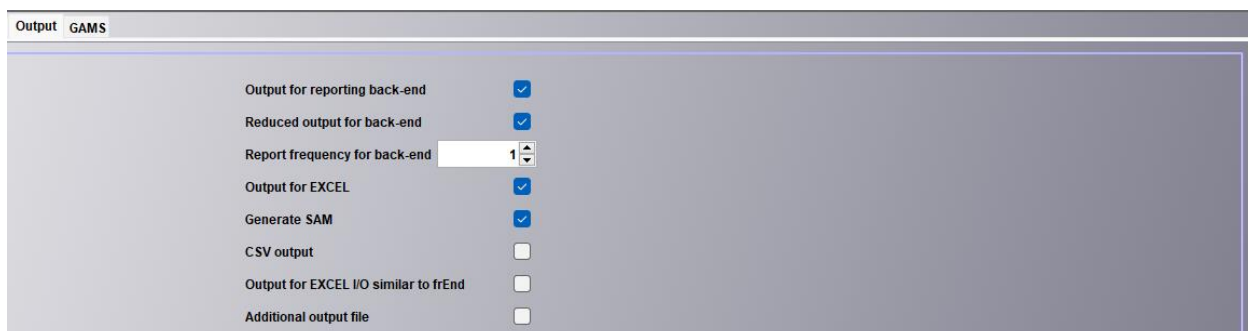
Note first the options under “Data and checks” are hidden for the user level “runner”. The different options available either are specific to the baseline, such as productivity changes implemented to recover a given growth path of real GDP found under “Productivity” tab or refer to model extensions which must be identically implemented in both the baseline and subsequent counterfactuals. The meaning of the different options to configure of the model is detailed in the methodological part of the documentation.

The option to switch on/off certain model mechanisms such as the *old* and *new* vintages can also help in cases when the model produces implausible results or gets infeasible to check if problems root in these optional model features.

Output controls

The second tab on the interface allows to (de)select the reporting options. Especially for large-scale sensitivity analysis, reducing the output generated by the reporting back-end (selected below) and increasing the report frequency such that results not for every year are generated can save considerable disk-space and speed up processing. The additional output file can be provided by the user to define, for instance, additional indicators to be stored to disk.

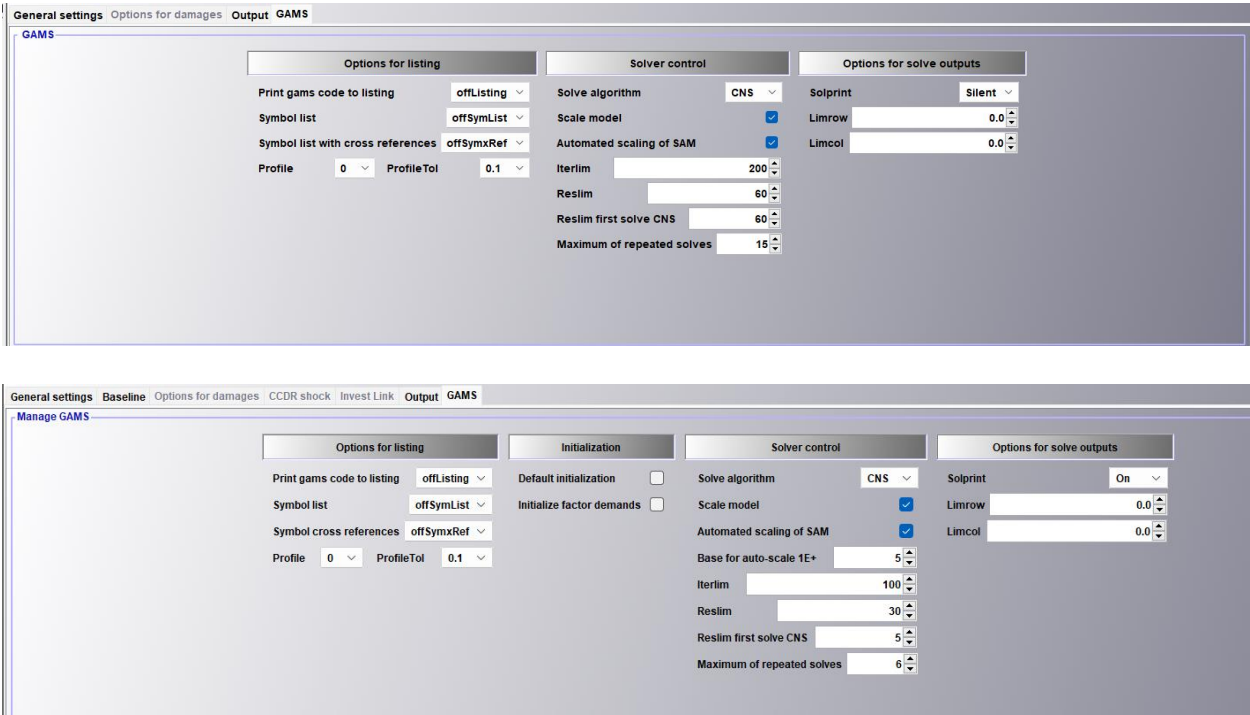
Figure 4.4: Reporting options



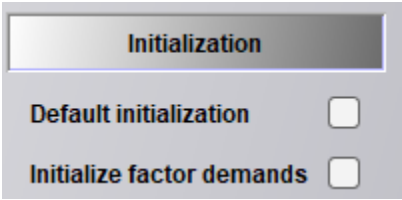
Solver control and GAMS specific settings

The last tab comprises options for listing and profiling, the solver control and for solve outputs. Adding the symbol list and the profiling controls are hidden for the user level “runner”.

Figure 4.5: Options which affect output and the solution process



During baseline generation, the solve for the next year will without the initialization options use the level of the endogenous variables from the last solve as the starting point, with volumes and values being updated accorded to the targeted GDP growth. For simulation, alternatively, the results from the baseline can be used. The using “Default initialization” will try to improve on this by, for instance, by using the demand system to capture income effects on household demands. It can be combined with initialization also the factor demands. In most cases, the default procedure works fine and the initialization options can be skipped.



The available solve algorithms are shown below.

Solve algorithm	CNS
Paired	MCP
Iterlim	CNS
	DNLP

MCP (Mixed Complementary Programming) allows the solver to change equations defined as equality constraints into inequalities if the variable paired with the equation hits its lower or upper bound. For instance, if the emission tax under an emission ceiling becomes zero, the ceiling can become non-binding. Such options are currently hardly used in MANAGE-WB. MCP requires specialized solver such as PATH which support the “redefinitions” of equality to inequalities. As not all users of MANAGE-WB might have a PATH license, the GUI allows to use the NLPEC solver instead for MCP which is comprised for free in any GAMS license. It will reformulate the MCP as a CNS and delegate the solution of the CNS to CONOPT4. This is about as fast as PATH itself. Solving the model as a MCP is usually slower. Numerical issues are also more frequent under MCP, but there are also cases where the model can be solved in MCP but CNS mode.

Solve algorithm	MCP
MCP Solver	PATH
Paired	PATH
	NLPEC

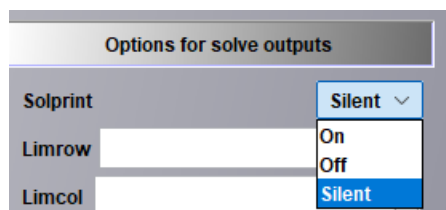
Under MCP, GAMS is able to check if the pairing between variables and equations is set-up correctly and will report unmatched variables. This is helpful for debugging as adding new equations to the model structure or closure swaps in shock files can introduce errors which lead to non-square models. In such cases, the number of equations is not equal to the number of non-fixed variables. A fully paired version of the model is available which eases finding variables not linked to equations or equations not linked to variables. This option is available under MCP, only, as the pairing information is neglected by GAMS if the model is not solved as an MCP. However, the default pairing defined in **model.inc** might not work if shock files introduce closure swaps. Even a model with the same number of equations and non-fixed variables can provoke errors and follow-up program aborts if the new closure rules violate pairing rules.

The most common case of pairing errors when using shock files is that GAMS will automatically remove equations which are paired to fixed variables. This behavior is consistent with the MCP rules: an equation is slack if the variable is at its lower or upper bound. If the variable is fixed, its lower and upper bound are equal and thus both active such that the equation is slack by definition and cannot provide information to the solution space. If no redefinitions are needed, pairing can be switched off and the MCP solver used to simply neglect such problems.

Solver control	
Solve algorithm	MCP
MCP Solver	PATH
Paired	<input checked="" type="checkbox"/>

The MANAGE-WB model is now defined such that it can be solved as a CNS (= square system of non-linear equations) as the default. This allows to use CONOPT4 which is usually somewhat faster than PATH.

The “*Iterlim*” and “*reslim*” options define the maximum iterations and seconds allowed by the solver for one single model instance. If these limits are exceeded, the model will be declared infeasible. As discussed above, MANAGE-WB can make repeated attempts to solve the model in this case. The number of repeated attempts is defined by the “Maximum of repeated solves”. In MCP mode with PATH, repeated attempts will use different option files.



Options for solve outputs	
Solprint	Silent
Limrow	On
Limcol	Silent

The last block of controls under “Options for solve outputs” steers the amount of information which GAMS puts in the listing for each model solve. If “*Silent*” is chosen for “*Solprint*”, no output from model solves is produced and the listing has its minimum size. This also saves times.

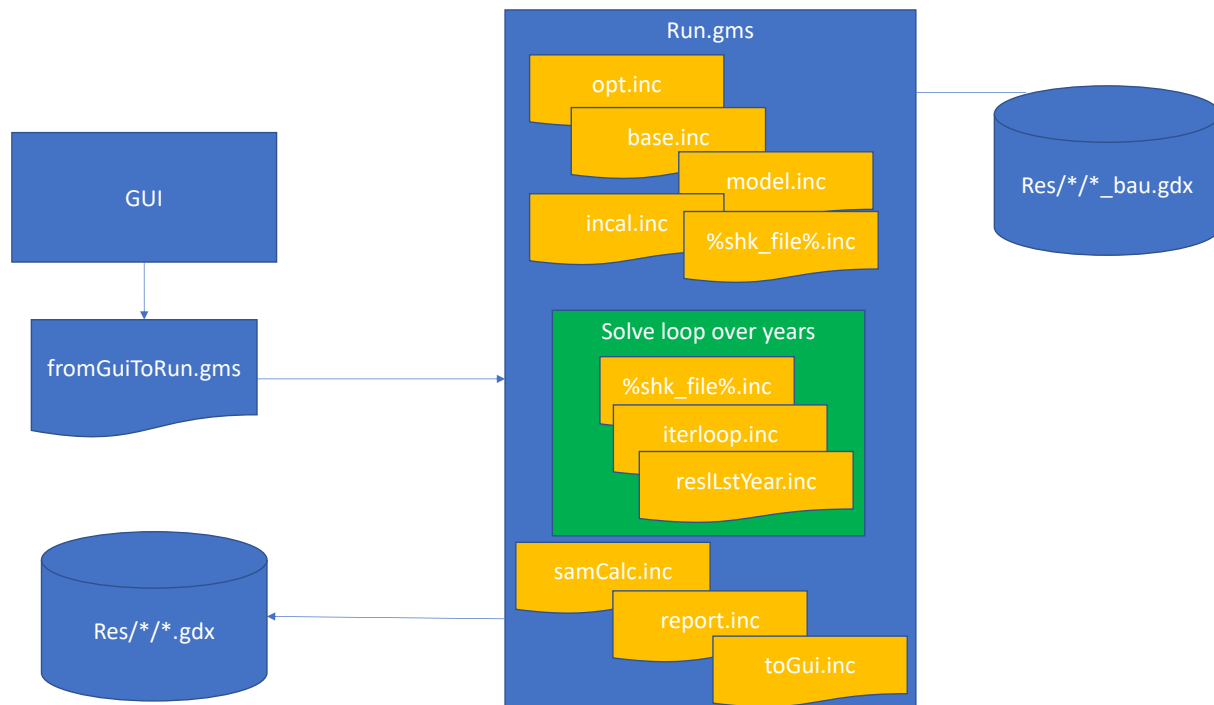
“*Off*” will report a summary for each solved model instance (number of equations and variables, time to solve etc.), only whereas “*On*” reports all equations and variables. If the model cannot be solved after repeated trials in a year, the last trial will always switch the generation of the full model listing on such that infeasible equations can be found. Generally, it is often more convenient to use GAMS Studio to inspect the variables from a model solution instead of using the equation listing. Setting “*Solprint*” to “*Silent*” or “*Off*” is therefore the recommended option.

Both with “*solprint*” set to “*off*” or “*on*”, “*limrow*” and “*limcol*” can be used to produce for the first *n* instances, as inputted in the text field, of each equation (*limrow*) or variable (*limcol*) a report with the record fields (level, lower and upper bound, marginal) and the Jacobian entries. Note that in case of non-linear expressions, the Jacobian entries are not constants and calculated at the given start values of all variables in an instance of an equation. They will hence change during the solution process. Switching “*limcol*” on by setting a non-zero number can be useful in case of unmatched variables to find out in which equations they occur. As the size of model listing increases substantially with “*limrow*” or “*limcol*” switched on, the two fields are by default set to zero.

Simulations

Overview on simulation

The steps in a simulation are largely identical to what was described above for the baseline. The main difference is that the data import from the base-bridge XSLX file and the subsequent benchmarking steps are skipped. Instead, **base.inc** reads the necessary sets from the GDX container produced by a previous baseline run and **inical.inc** reads all parameters and variable start levels from this GDX container as well. Instead of constructing a starting from last year’s solution, **iterloop.inc** can use in simulation mode on-demand the solution from the baseline for the year to solve to define starting values which can be faster.



GUI controls for simulation

The GUI options for the simulation are mostly identical to the baseline. The first and last year are loaded internally from the BAU GDX chosen as they cannot be changed. If the simulation name is not left empty, it defines the name of the GDX with the simulation results. Otherwise, the simulation output is named after the shock file. Entering a simulation name can also be used to pick a specific column from pre-defined shock parameters in the shock file.

The screenshot shows the **General settings** tab of the GUI. It is divided into four main sections: **Country and time**, **Shock**, **Options**, and **GDP Composition dynamics**.

- Country and time:**
 - Region dir: tza
 - BAU file (gdx): tza_BaU
 - Base year: 2017
 - Used as BaU: ☐
 - Last year of simulation: 2050
- Shock:**
 - Simulation name: [empty]
 - Postfix: [empty]
 - Shock file type: GAMS
 - Shock File (inc): ccd
 - Scen driver 1: [empty]
 - Scen driver 2: [empty]
 - Scen driver 3: [empty]
 - Scen driver 4: [empty]
 - Scen driver 5: [empty]
 - Scen driver 6: [empty]
- Options:**
 - Climate change damages: ☐
 - Air Pollution damages: ☐
 - Fiscal Responsibility: ☐
- GDP Composition dynamics:**
 - GDP (composition) scenario: bas
 - Aggregate sector shares on VA: ☐

Figure 4.5: GUI controls for simulation

The screenshot shows a software interface with a top navigation bar containing tabs: General settings, Options for damages, CCDR shock, Invest Link, Output, and GAMS. Below this, a 'Manage General settings' window is active, divided into four panels:

- Country and time:** Includes dropdowns for 'Region dir' (set to IND), 'BAU file (gdx)', 'Base year' (set to 2018), and 'Last year of simulation' (set to 2100).
- Shock:** Includes input fields for 'Simulation name' and 'Postfix', and dropdowns for 'Shock file type' (set to GAMS) and 'Shock File (inc)' (set to empty). Below these are six input fields labeled 'Scen driver 1' through 'Scen driver 6'.
- Options:** Includes checkboxes for 'Climate change damages' (checked), 'Air Pollution damages', 'Fiscal Responsibility', and 'Short run shocks'.
- GDP Composition dynamics:** Includes a dropdown for 'GDP (composition) scenario' (set to bas) and a checkbox for 'Aggregate sector shares on VA'.

As seen from the screen shot, solely the damage modules and the fiscal responsibility mechanism can be added into the simulation. Short run shocks do not change the equation set-up but allow the analyst to solve the model twice in each year and require a specific set-up of the shock file. During the first solve, the normal model set-up is used and solely *expected* exogenous changes by the agent are introduced, such as changes in average temperature and precipitation. Production decisions will be based on these expected changes, and using the normal model logic, resulting adjustments in price etc. are perfectly foresighted by all agents. The second solve will add unexpected changes, such as anormal high or low realizations of climate variables. Producers will have limited options to react to those and the second solve can therefore fix part of the factor allocation.

All other modular extensions imply changes in the model structure impacting the benchmarking step and hence require the construction of a new baseline before a counterfactual can be run where these extensions are active. Note that switching settings these options differently from the baseline will change the results compared to the baseline even if the shock file is empty.

Reporting back-end

Concept

The reporting back-end provides a longer list of pre-defined reports as tables or graphs. Its concept is summarized by the following points¹³:

- Each report combines a block of information helping to analyze specific aspects of the model solution.
- This often implies looking simultaneously at different model variables, say quantities, pre- and post-tax prices or values which is cumbersome with the model listing or GAMS Studio.

¹³ See Britz et al., (2015).

- The back-end therefore removes the technical dis-aggregation of the model's solution space into GAMS variables, parameters and equations.
- Each report comprises at least the following information:
 - Which dimension (for instance, SAM row or column) from *p_toGUI* is shown in columns and rows, (where applicable column or row groups) of the user's viewport, and which are moved to drop down lists. This information is called the pivot.
 - The selection of labels from each dimension in *p_toGUI*, for if sectors or products are shown. If no filter is given for a dimension, all found labels are included in the report. The selection can be based on a REGEX statement or selection lists defined by the GAMS code.
 - The view type (table, graph).

Further information can be added, such as the number of digits, if percentages changes should be additionally shown, if empty rows and columns should be hidden etc. The user can change the pivot, view type etc., but cannot add labels which are excluded by the filters which define the report. It is however possible to filter out additional labels.

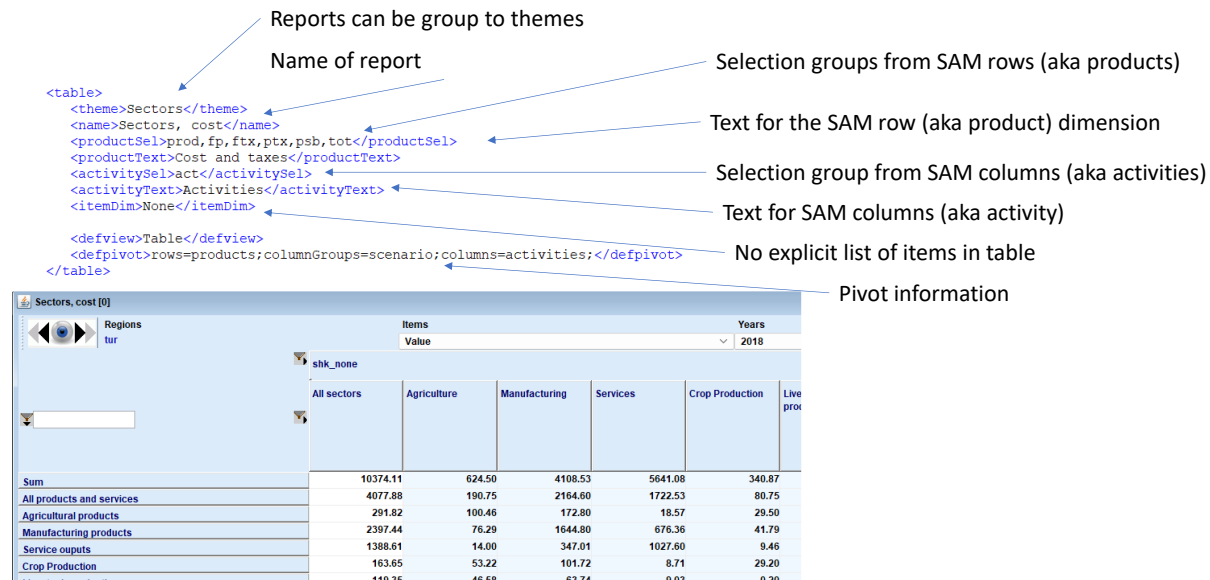
To enable this concept, variables, parameters and slacks in equations are copied by **toGui.inc** into a n-dimensional parameters (region, item, SAM row, SAM column, time):

- The item dimension covers Q=quantity, P=Price, V=Value, T=tax rate, G=tax income, plus emissions (set em).
- The SAM row dimension also comprises additional variables not found in the SAM, such as macro totals, cohorts, debt related variables, emissions, etc.
- The SAM column also comprises additional detail on such variables, such as emission sources.

If new variables are added to the model and should be reported, they need to be added in the **toGui.inc** file.

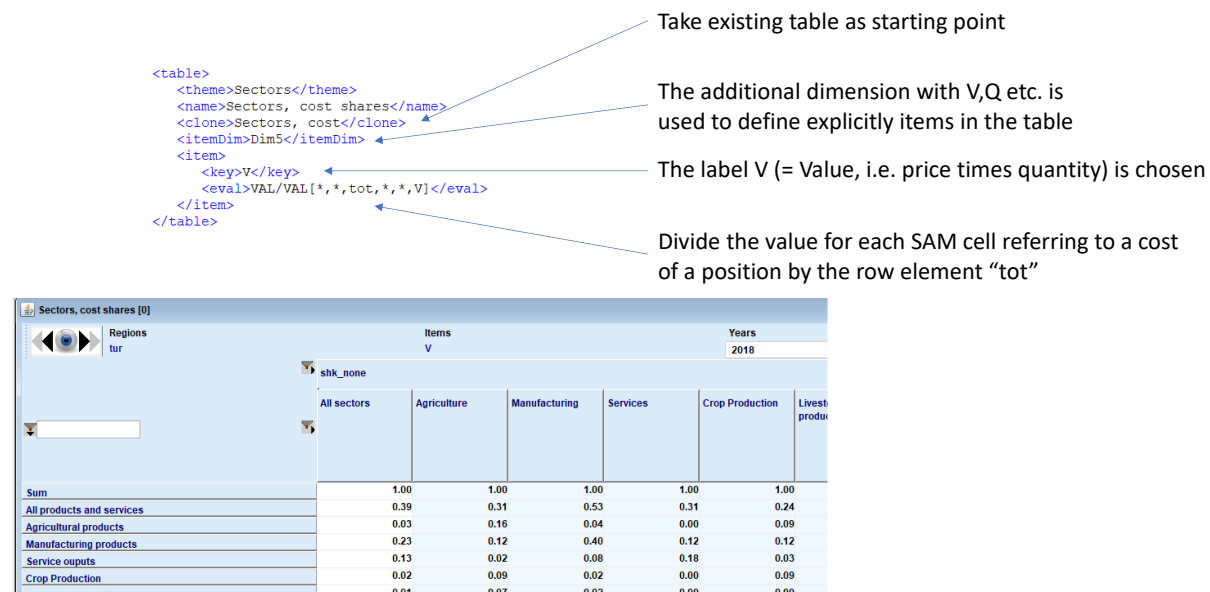
An example how reports are defined in **gui/manageTables.xml** is shown below.

Figure 4.6: Example of a report definition in XML and how it is shown in the reporting back-end



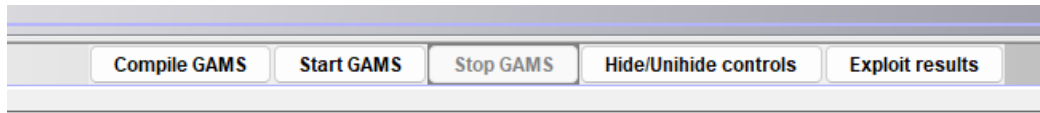
Existing reports can be “cloned” to define variants such as in the following example. It also shows how the expression evaluator built in the reporting tools can be used to define additional items in a table. Note that the evaluator requires to define a list of items for the table manually (not available with `<itemDim>none</itemDim>`).

Figure 4.7: Example of a report definition in XML which clones another report



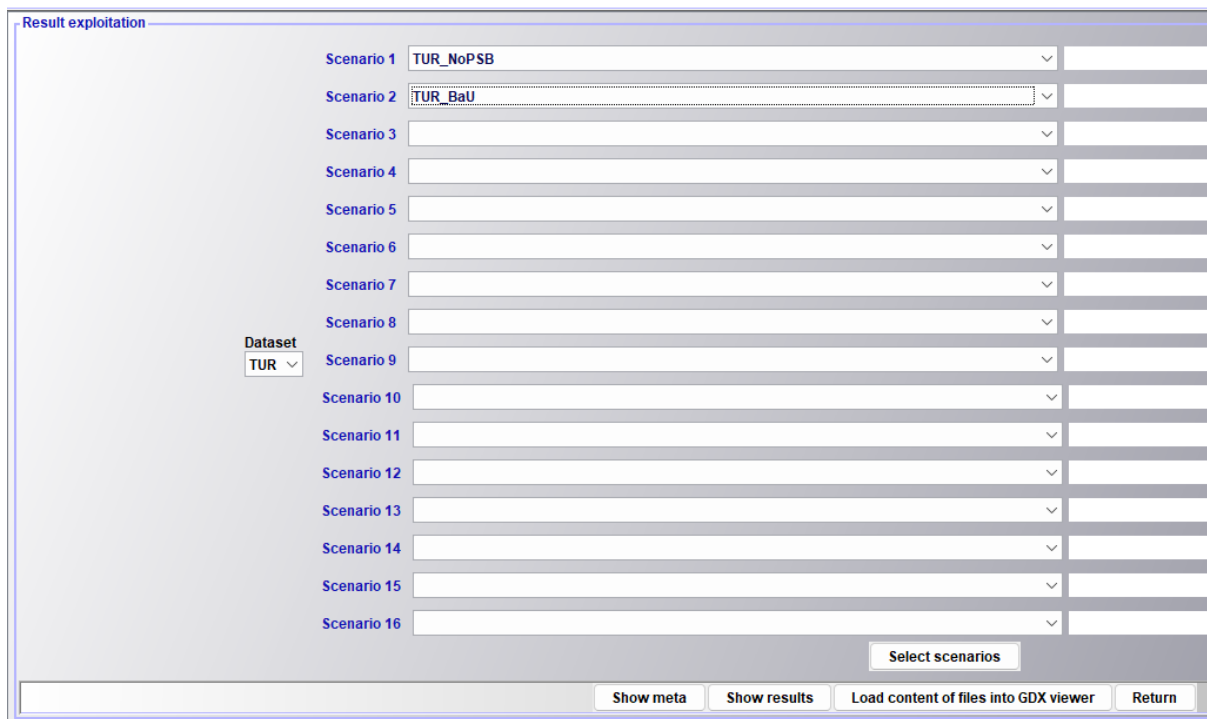
Using the reporting back-end and available reports

The back-end is opened by pressing “Exploit results” from the button bar:



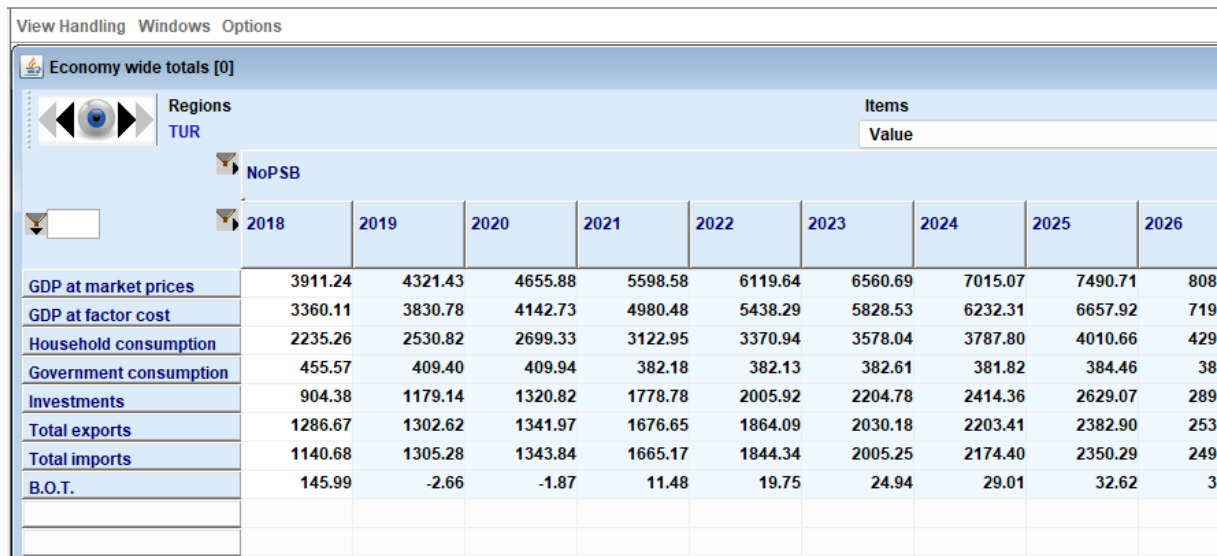
This opens a new content pane which allows to select the data set to use and shows the GDX containers with results available for this data set. To remove a scenario, leave the box empty. It is recommended to have the BAU scenario last.

Figure 4.8: Scenario selection panel of the reporting back-end



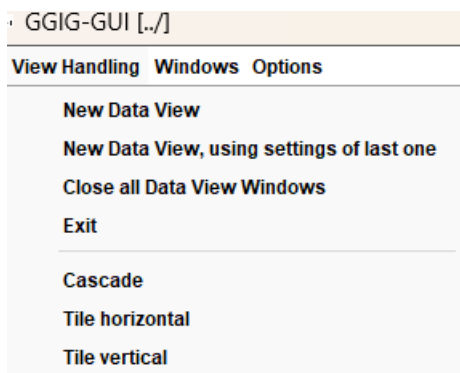
In order to return to the model steering controls, press return. When the “Show results” button is pressed, the GUI will load the GDX containers and show the views (if possible) opened during the last session. On a first-time load, the first report titled “Economy wide totals” will be shown:

Figure 4.9: First report in reporting back-end

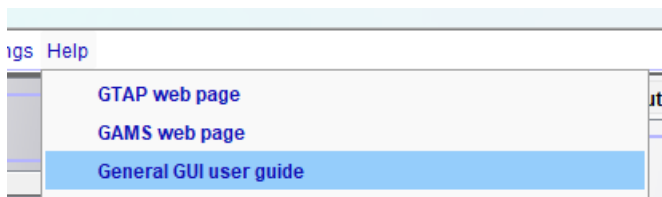


Economy wide totals [0]									
Regions	Items								
TUR	Value								
NoPSB	2018	2019	2020	2021	2022	2023	2024	2025	2026
GDP at market prices	3911.24	4321.43	4655.88	5598.58	6119.64	6560.69	7015.07	7490.71	808
GDP at factor cost	3360.11	3830.78	4142.73	4980.48	5438.29	5828.53	6232.31	6657.92	719
Household consumption	2235.26	2530.82	2699.33	3122.95	3370.94	3578.04	3787.80	4010.66	429
Government consumption	455.57	409.40	409.94	382.18	382.13	382.61	381.82	384.46	38
Investments	904.38	1179.14	1320.82	1778.78	2005.92	2204.78	2414.36	2629.07	289
Total exports	1286.67	1302.62	1341.97	1676.65	1864.09	2030.18	2203.41	2382.90	253
Total imports	1140.68	1305.28	1343.84	1665.17	1844.34	2005.25	2174.40	2350.29	249
B.O.T.	145.99	-2.66	-1.87	11.48	19.75	24.94	29.01	32.62	3

In order to return to the control settings, use “View Handling”, “Exit”:



Note that multiple reports can be kept open. Further detail on using the reporting tool can be found in the user guide for the reporting back-end available under the help menu. It details the various options to work with tables and graphs such as changing the pivot, filtering, sorting, hiding empty rows and columns, defining percentage differences etc.



The reports currently derived from the model solutions are:

- National accounts:
 - Macro totals (GDP, BOT etc.)

- Accounts for government, household, savings, enterprise (income and expenditure flows), some key household indicators
- Household indicators (population, CPI, real income and expenditures, money metric)
- Tax income composition
- Capital stock and depreciation
- Savings composition
- Debt
- Demography
- Sectors: Costs (intermediates, factors, factor taxes), cost shares, tech nests, tech nests as cost shares, make matrix
- Households: detailed expenditures, product demands, budget shares, detailed income composition
- Government: consumption, income flows
- Taxes: product, factor, sector related
- Markets: Factor markets, output/exports/domestic, sales/imports/domestic market appearances, demand at agent prices, Armington decomposition w/wo margins
- Air emission, emission inventories, GHG emissions
- Model diagnostics

It can be expected that with increasing use, more tables and variants of tables will be added.

5. Social Accounting Matrix: Estimation and Update

5.1 Constructing SAMs for MANAGE-WB from the GTAP data base

Motivation and data processing

The GTAP center produces the de-facto standard data set for global multi-regional CGE modeling. The latest version 11 being publicly released in spring 2023 covers 160 single countries with a benchmark year of 2017. While in some cases, the IO-Tables underlying single countries are rather old and have not much detail, the data base is still a good starting point for a quick start for single country work.

Figure 5.1: Options to construct a SAM from GTAP data base

The screenshot displays the 'MANAGE WB v6' software interface. On the left, a sidebar contains 'worksteps' with 'Build data base' selected, and 'tasks' with 'Prepare GTAP SAM' selected. The main window is titled 'General settings' and is divided into two panels: 'Input' and 'Debt'.

Input Panel:

- GTAP data base input:
- Calculate global cost shares: ☐
- Region(s):
- Minimum cost share E-:
- Load AEZ data: ☒
- Build gender differentiated SAM: ☒
- New SAM account cattle raising: ☒
- New SAM account electrified other transport: ☒
- Water-Land split: ☒

Debt Panel:

- Domestic debt stock relative to gov consumption:
- Interest rate on domestic debt:
- Principal rate on debt:
- New debt stock relative to gov consumption:

The program **gtapSAMV6.gms**, developed conducts the following steps:

1. Generates sets from the GTAP sets (activities, products, factors etc.).
2. Maps the different matrices of the global GTAP to a single country SAM in the MANAGE-WB format.
3. On-demand, adds data on land use and land cover at the level of Agro-Ecological Zones and integrates them into the SAM.
4. On-demand, uses the gender differentiated labor data base matching GTAP Version 10 to define SAM accounts relating to labor differentiated by skill and gender. Countries newly available in Version 11 use the shares of the regional aggregate they were comprised in Version 10. The more detailed skill categories in the Version 11 data base are linked to the skill levels in the gender differentiated labor base via mappings.
5. On-demand, splits the land account into irrigation water rents and land rents, based on FAO data on area and output by crop and production system (rainfed or irrigated).
6. On-demand, introduces two new sectors: cattle raising (derived from own use of ruminant meat and raw milk in the respective activities) and electrified other transport. For the latter, the production value is split according to the case share of electric use in the activity; its energy demand is set to the electric energy demand of the other transport activities and all other cost shares scaled to exhaust the constructed output. The remaining (fossil based) other transport activity is treated analogously.
7. Can filter out small shares. A setting of 1.E-10 will skip this step.
8. Balances this SAM in order to remove any remaining tiny imbalances.
9. Produces a simple macro-SAM from this micro-SAM.
10. Generates a new directory under **reg** for the region if not yet existing.
11. Copies a default template for the excel based input for SAMEST described above into the region directories and adds the generates proto-SAM and macro-SAM to the EXCEL sheet.
12. Copies a default template of the bridge file into the region directory and populates it with the SAM.
13. Reads in data on external debts and stores them into the **defpar** sheet.

14. Copies the CDE demand parameters, aggregated according to the mapping information in the **mapk** sheet, into the **hohpar** sheets and adds regression coefficients to updated them based on per capita income changes.
15. Reads in MFMod GDP projections and stores them into dynamics, along with time series which show the VA and GDP composition at the benchmark and some other indicators.

Standard aggregation

The template assumes a 1:1 aggregation from the initial SAM to the proto-SAM with the exemption of electricity which is aggregated over the different power generate products.

Missing bits and pieces

1. Currently, the user has to define the domestic debt stock relative to government consumption on the GUI and provides related interest and re-payment rates. Equally, the new debt (= government budget deficit) has to be defined relative to government consumption.
2. The program assumes that 50% of capital returns plus depreciation constitute the enterprise income. They are used to finance depreciation, the remainder flows as a transfer to households.
3. If the gender differentiated labor force is not used, not data on physical employment are available. In this case, **inc/base.inc** will use the SAM entries as distribution keys to generate a labor force equal to 2/3 of the population of the 15-64 cohort, at a wage rate of 10.000.

Once such as data base is constructed, it can be fed in the process detailed next, for instance for further splitting, or can be directly used to construct a baseline.

5.2. Updating SAMs to macro-totals, balancing and splitting

Updating an existing SAM based on Macro totals or Macro SAM and SAM balancing are an integral part of the MANAGE-WB system. Competing approaches to do so are discussed in the literature (e.g., Britz, 2021). Both from a computational and methodological viewpoint, so-called Highest Posterior Density (HPD) estimators are inviting. They can be understood as a Bayesian framework where the given data act as a-priori information and the balancing constraints as the data information. If normal distributed errors are assumed, the log likelihood of the maximization of the posterior density leads to minimization of squared (relative) errors which is computationally efficient.

The updated SAMEST program presented in this chapter was developed out of code used so far by the MTI-CGE group drawing on a Cross-Entropy approach with requires for each estimate a set of support points with attached a-priori probabilities. The support points capture typically moments of a distribution to approximate (such as mean and variance). The entropy criterion then minimizes the differences between the a-priori and posteriori distributions based on its specific loss function. While (cross-)entropy approaches have come in vogue a while ago, often accompanied by not-well funded criticism about alternative methods, cross-entropy requires more assumptions compared to a HPD framework and results are often harder to understand as the support space implicitly also restricts the estimates. The previous code of SAMEST comprised a range of quite evolved options with regard to Cross-Entropy such as different numbers of supports, cell specific choices for assumed standard errors, and with regard to data handling more generally such as automatic scaling etc. Very little of this flexibility was actual in use, leading to a large code base not easy to understand. The refactored version of SAMEST discussed in here removes most of this flexibility to arrive at a streamlined version, offers core options as GUI controls and is based on a HPD estimator.

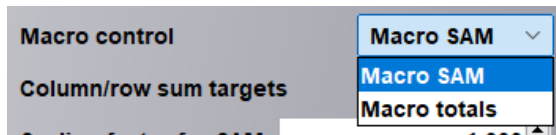
5.2.1 GUI choices

The user is supposed to put the EXCEL file which comprises the proto and the macro-SAM or macro-totals and related sets into the regional specific directory where later the bridge-file resides which is updated by SAMEST. Available GTAP data sets are searched for in the data directory by the GUI and shown in the GTAP data selection control. They can be used for splitting.

Figure 5.2: Core GUI controls for SAMEST

The screenshot displays the 'General settings' window of the SAMEST software. The window is divided into three main sections: 'Input', 'Algorithm control', and 'Split control for activities'. The 'Input' section includes dropdown menus for 'Region input dir' (set to 'BEN'), 'SAM input (xlsx)' (set to 'SAM_BEN_for-update_v1b'), and 'GTAP data' (set to 'gtapData11p'). It also features a checkbox for 'Write new SAM to bridge file' (checked) and a dropdown for 'Choose the Bridge file to write' (set to 'BaseBridge_BEN'). The 'Algorithm control' section includes a dropdown for 'Macro control' (set to 'Macro SAM'), a checkbox for 'Column/row sum targets', a text input for 'Weights for coefficients' (set to '0,5'), a text input for 'Scaling factor for SAM' (set to '1'), and a text input for 'Cutoff value for small SAM cells 1.E' (set to '-3'). The 'Split control for activities' section contains several sliders for lower bounds: 'Lower bound for make matrix entries' (0.30 to 100.00), 'Lower bound for factor demand' (0.30 to 100.00), 'Lower bound for intermediate demand' (0.30 to 100.00), 'Lower bound for indirect firm taxes' (0.30 to 100.00), 'Lower bound for final demand' (0.30 to 100.00), 'Lower bound for trade' (0.30 to 100.00), and 'Lower bound for indirect product taxes' (0.30 to 100.00).

Macro information can be inputted in the updating steps either based on a Macro SAM or Macro totals. The latter is an option currently explored, in order to use the MFMod data base in case where a Macro SAM is not available.



5.2.3 Main processing steps

The main steps in the application of SAMEST are:

1. Reading in the proto-SAM and macro-SAM and related sets and maps from the user supplied EXCEL file
2. Potentially aggregating the proto-SAM
3. Updating the (aggregated) proto-SAM based on the macro-SAM or macro-totals
4. Balancing of the updated SAM in a HPD framework, a macro-SAM is treated as fixed and given and must be balanced.
5. Defining split shares and application of a second HPD framework to balance the split accounts (on-demand)
6. Output of the resulting SAM into the bridge file

Data input

As before, the code reads the data, related sets and maps from the EXCEL sheet as selected on the GUI.

```

$$CALL "GDXXRW i=reg\%regGtapName%\%samXLS%.xlsx o=reg\%regGtapName%\%samXLS%.gdx index=Layout!A4 MaxDupeErrors = 100"
$$GDXIN "reg\%regGtapName%\%samXLS%.gdx"
*
  --- Micro sam Sets
  $$LOADdc ac acIni<samIni.dim1 acIniMac
  $$LOADdc c mrgn a f l k h g ginv e i w emrgn etax
  $$LOADdc gt gtind gtdir
  $$LOADdc acm
  $$LOADdc macro macro2<macroMat.dim1
  $$LOADdc macsam2
  $$LOADdc macromat
  $$LOADdc macscale
  $$loaddc macmap3 samIni map_acIni_ac map_acIni_acini
  $$GDXIN

```

If macro totals are not used, the table can be left empty. Similarly, empty sets and an empty table can be used if no macro-SAM will be used. GTAP data are only read and processed if related split sets are not empty (not shown here).

Initial aggregation

The code allows to first aggregate the original proto-SAM based on the *map_acIni_acIni* set.

```

sam(acIni,acInip,"read") = samIni(acini,acinip);
sam(acIni,acInip,"read")
= sum((map_acIni_acIni(acInipp,acIni),acInipp) $ map_acIni_acIni(acInipp,acInip),samIni(acInipp,acInipp));

```

The set must provide a full mapping (including the 1:1 cases), as otherwise, certain SAM accounts will be empty.

Macro SAM updating

In case of a macro-SAM being used, some checks are run first to ensure that each proto-SAM account is mapped exactly to one macro-SAM account.

```

*
* --- check for correct mapping of macro accounts
*
Parameter macMapCount(ac) "Count of ac entries in macmap3";
macMapCount(acnt) = sum(macmap3(acm,acnt), 1);

abort $ sum(acnt $ (macMapCount(acnt) ne 1),1)
      "All macMapCount entries should be 1", macMapCount;

abort $ (sum(acnt, macMapCount(acnt)) ne card(acnt))
      "Not all macro accounts are mapped to a account in the proto SAM",macMapCount;

macMapIni(acm,acIni) $ sum(map_acIni_ac(acIni,ac),macmap3(acm,ac)) = YES;

```

In order to update the proto-SAM based on a given macro-SAM, first a macro-SAM from the given proto-SAM is defined, based on the mapping information provided in the EXCEL sheet. This allows to define shares from the proto-SAM into its own macro-SAM.

```

* --- define macro SAM shares from SAM as read in

parameter macTotIni(acm,acmp)
macShares
;
macTotIni(acm,acmp) = sum((macmapIni(acm,acInipp),acInipp) $ macmapIni(acmp,acInipp),
      sam(acInipp,acInipp,"read"));
macShares(acInipp,acInipp)
= sum((macmapIni(acm,acInipp),acmp) $ (macmapIni(acmp,acInipp) $ macTotIni(acm,acmp)),
      sam(acInipp,acInipp,"read") / macTotIni(acm,acmp));

```

The special set *mapMapIni* can be used for 1:1 assignments from the given macro-SAM to the proto-SAM update. Related shares for these cases are hence unity.

```

alias (acIniMac,acIniMacp);
macShares(acIniMac,acIniMacp) $ sum((macMapIni(acm,acIniMac),acmp) $ macMapIni(acmp,acIniMacp),1) = 1;

```

Given the shares, either derived from the original proto-SAM or as 1:1 mappings to the given macro-SAM, we update the original proto-SAM.

```

* --- create the initial sam by applying shares dervied from read-in SAM to Macro SAM
sam(acIni,acInip,"ini") = sum((macMapIni(acm,acIni),acmp) $ macmapIni(acmp,acInip), macSam(acm,acmp,"in") * macShares(acIni,acInip));

```

This implies that blocks of SAM cells are updated with the same multiplicative factor. This will usually imply that the update results in an unbalanced SAM which motivates the follow up SAM balancing step. It is also useful if the original proto-SAM is not updated, but not yet fully balanced.

Balancing framework for the non-split proto-SAM

In opposite to the previous version of SAMEST, the preparation of split shares and balancing of the split cells is now handled in a second separate processing step as discussed below. This increases transparency and makes it easier to organize the workflow.

During balancing of the updated proto-SAM, all activities related costs as well as final demands are targeted as coefficients on the related column totals (total cost or expenditures). These coefficients are by far the largest part of the SAM such that they tend to dominate the outcome of the estimator. To match the coefficients, the estimator can also be tempted to put larger changes on account totals. To avoid this, both column/row sum targets can be used and the weights for relative deviations from the coefficient targets can be reduced. The user can also fix specific SAM cells as discussed below.

Based on the HPD approach, the balancing framework thus minimizes relative quadratic deviations for coefficients for all costs positions of activities and for final demand of households, government and for investment. All other cells are targeted as absolute values. Currently, all errors are defined as additive and sign changes are not allowed. The code, as before, will keep zero cells at zero. Column sum constraints are added on-demand only, and only if the column or related row comprise more than two non-zero entries. Row sum constraints are introduced if the related column total is empty. If both a column and row sum is given, the average of the two is targeted.

The first two constraints in the estimation framework target in absolute terms non-zero SAM cells *irow,icol*. The selection of these pairs is based on the *iValue* matrix. The errors can be defined as additive or multiplicative (currently not in use).

```
e_samFlow1(irow,icol) $ (iValue(irow,icol) and aCell(irow,icol)) ..
v_tSam(irow,icol)      =E= sam(irow,icol,"target") + v_errCell(irow,icol);

e_samFlow2(irow,icol) $ (iValue(irow,icol) and lCell(irow,icol)) ..
v_tSam(irow,icol)      =E= sam(irow,icol,"target")*EXP(v_errCell(irow,icol));
```

Similarly, the matrix *iCoeff* defines the cells where coefficients on related column totals are targeted, motivating the following two equations:

```
e_samCoeffA(irow,icol) $ (iCoeff(irow,icol) and aCell(irow,icol)) ..
v_coeff(irow,icol)      =E= (coeff0(irow,icol) + v_errCell(irow,icol));

e_samCoeffL(irow,icol) $ (iCoeff(irow,icol) and lCell(irow,icol)) ..
v_coeff(irow,icol)      =E= coeff0(irow,icol)*EXP(v_errCell(irow,icol));
```

Column and row sum errors can be defined on-demand, triggered by their presence on the sets *colTarget* or *rowTarget*.

```

* --- Column and row sum constraints

e_colSum(icol2) $ colTarget(iCol2) ..
v_colSum(icol2)  =E= colTarget(icol2) + v_errCRSum(icol2,"col1");

e_rowSum(irow2) $ rowTarget(iRow2) ..
v_rowSum(irow2)  =E= rowtarget(irow2) + v_errCRSum(irow2,"row1");

```

Column and row constraints together with additional constraints which equilibrate column and row sums ensure that the SAM is balanced.

```

* --- Define column and row sums and coefficients

e_colSumDef(icol) ..
v_colSum(icol)      =E= sum(irow, v_tSam(irow,icol));

e_rowSumDef(irow) ..
v_rowSum(irow)      =E= sum(icol, v_tSam(irow,icol));

* --- Row and column sum equality constraint
e_sam(acbal) ..
v_colSum(acbal)     =E= v_rowSum(acbal);

```

The targeted coefficients are defined from the column sum and the cell estimates.

```

e_coeffDef(irow,icol ) $ ((not izero(irow,icol)) and icolnz(icol) and iCcoeff(irow,icol)) ..
v_coeff(irow,icol)*v_colSum(icol) =E= v_tSam(irow,icol);

```

Two sets of constraints (from which only one is active) match the cell estimates against the macro-SAM or macro totals.

```

* -- Macro-sam constraints

e_macroSam(acmnt,acmntp) $ macmNz(acmnt,acmntp) ..

macSam(acmnt,acmntp,"used") =E=
sum(acnt,acntp) $ (macmap3(acmnt,acnt) and macmap3(acmntp,acntp)),
v_tSam(acnt,acntp);

* -- Macro constraints (different from v_macSam)

e_macro(macro2) $ macTotal(macro2,"scaled") ..
sum(irow,icol, macAgg(irow,icol,macro2)*v_tSam(irow,icol)) =E= macTotal(macro2,"scaled");

```

In opposite to previous version, no separate “aggregated SAM” is present. Equally, errors on the macro-SAM or the macro totals are no longer supported.

The objective function minimizes squared relative errors. If all weights are unity, this implicitly assumes that all errors have the same coefficient of variation.

```

e_hpd ..
v_hpd * [ sum((iCol2,rwcl) $ (colTarget(iCol2) $ sameas(rwcl,"coll")),1)
+ sum((iRow2,rwcl) $ (rowTarget(iRow2) $ sameas(rwcl,"rowl")),1)
+ sum((iRow,iCol) $ ( (sam(iRow,iCol,"target") $ iValue(irow,icol)) or (coeffTarget(iRow,iCol) $ iCoeff(irow,icol)), p_Wgt(iRow,iCol)
] /100

=E= sum((iCol2,rwcl) $ (colTarget(iCol2) $ sameas(rwcl,"coll")),
sqr(v_errCRSum(iCol2,rwcl)/colTarget(iCol2)))

+ sum((iRow2,rwcl) $ (rowTarget(iRow2) $ sameas(rwcl,"rowl")),
sqr(v_errCRSum(iRow2,rwcl)/rowTarget(iRow2)))

+ sum((iRow,iCol) $ ( (sam(iRow,iCol,"target") $ iValue(irow,icol)) or (coeffTarget(iRow,iCol) $ iCoeff(irow,icol)),
sqr(v_errCell(iRow,iCol)/( sam(iRow,iCol,"target") $ iValue(irow,icol)
+ coeffTarget(iRow,iCol) $ iCoeff(irow,icol)) * p_wgt(iRow,iCol));

```

Fixing of estimates

A regular headache for an analyst is the fact that such a balancing framework will never exactly recover a target for a single cell which is not equal to a macro-SAM cell or a macro total. The old approach allowed to provide standard errors for each individual cell. Setting the standard errors to zero for specific estimates implicitly removed the related error from the estimation framework. The estimate was then forced to the target. This option is now supported by the sheet “fixes” in the EXCEL workbook.

	A	B	C	D	E	F
1	debt-0	g-govt-0	9,077,280			
2						
3						
4						
5						
6						
7						
8						

The example above fixes new government debt. This option must be used with caution as too many fixed entries are likely to (1) provoke infeasibilities and (2) shift the balancing errors into other cells. Fixes can be applied in the step where the proto-SAM is balanced or in a follow-up split step.

Splitting

The next step consists of splitting certain accounts (on-demand). This will be regularly done to with multiple households in MANAGE-WB. Multiple households are typically not part of the proto-SAM. In other cases, also specific activities or commodities are split, such as for power generation.

In a first step, split factors as provided by the user in the EXCEL sheet are applied. Split factors are scaled to unity such that, for instance, missing split factors for households will imply that each household gets the same share on the total assigned as the targeted value. Such cases should be avoided, if possible.

```

--- missing user provided split shares are set to unity, 1:1 relation to SAM updated from Macro SAM

sumSplit(acIni,acIniP) = sum((map_acIni_ac(acIni,ac),acp) $ (map_acIni_ac(acIniP,acp) $ split(ac,acp)) , splitShr(acIni,ac,acIniP,acp));

splitShr(acIni,ac,acIniP,acp) $ ( (map_acIni_ac(acIni,ac) and map_acIni_ac(acIniP,acp))
$ (not sumSplit(acIni,acIniP)) $ split(ac,acp)) = 1;

sumSplit(acIni,acIniP) $ ((not sumSplit(acIni,acIniP)))
= sum((map_acIni_ac(acIni,ac),acp) $ map_acIni_ac(acIniP,acp),splitShr(acIni,ac,acIniP,acp));
splitShr(acIni,ac,acIniP,acp) $ splitShr(acIni,ac,acIniP,acp) = splitShr(acIni,ac,acIniP,acp)/sumSplit(acIni,acIniP);

```

As a second step, the user can provide mapping relations to GTAP factors, activities and commodities via the EXCEL sheet, and indicate in two sets which activities and commodities are to split based on the GTAP data. If one of two sets is non-empty, the GTAP data base is processed, and split factors are derived.

```

set asplit_gtap(a),csplit_gtap(c);
$$GDXIN "reg\%regGtapName%\%samXLS%.gdx"
  $$LOAD asplit_gtap,csplit_gtap
$$GDXIN

$$setglobal useGtap off
$$ife card(asplit_gtap)>0 $$setglobal useGtap on
$$ife card(csplit_gtap)>0 $$setglobal useGtap on

$$iftheni.useGtap "%useGtap%"=="on"

```

We skip here the code which defines different vectors from the GTAP data base used for splitting. Based on this information, split shares specific to GTAP $p_splitShr2$ are derived and overwrite split shares in $p_splitShr$ as loaded from EXCEL. Not here that split share loaded from the EXCEL file (indicated by the indicator parameter *splitShrOri*) are multiplied with the GTAP derived factors. This can have hard to control consequences and it is generally not recommended to mix the two approaches.

```

acs(ac) $ sum(acp, gtapUse(ac,acp)) = YES;

--- calculate protosam with disaggregations based on GTAP

splitShr(acIni,ac,acIniP,acp) $ ((map_acIni_ac(acIni,ac) and map_acIni_ac(acIniP,acp)) $ (not splitShr(acIni,ac,acIniP,acp))) = 1;

splitShr2(acIni,ac,acIniP,acp) $ (map_acIni_ac(acIni,ac) and map_acIni_ac(acIniP,acp) $ splitShrOri(acIni,ac,acIniP,acp))
= splitShr(acIni,ac,acIniP,acp)*gtapShare(ac,acIni,acp,acIniP);

splitShr2(acIni,ac,acIniP,acp) $ (map_acIni_ac(acIni,ac) and map_acIni_ac(acIniP,acp) $ (not splitShrOri(acIni,ac,acIniP,acp)))
= gtapShare(ac,acIni,acp,acIniP);

splitShr2(acIni,acS,acIniP,aS) $ ((map_acIni_ac(acIni,acS) and map_acIni_ac(acIniP,aS)) $ (NOT splitShr2(acIni,acS,acIniP,aS))) = eps;

splitShr(acIni,ac,acIniP,acp) $ splitShr2(acIni,ac,acIniP,acp) = splitShr2(acIni,ac,acIniP,acp);

```

The combined split factors then define the dis-aggregated proto-SAM which after scaling and cleansing of small values enters the balancing framework.

```

sam(ac,acp,"dis") = sum((acIni,acIniP) $ (map_acIni_ac(acIni,ac) and map_acIni_ac(acIniP,acp)),sam(acIni,acIniP,"ini")*splitShr(acIni,ac,acIniP,acp));
sam(ac,acp,"dis") $ (sam(ac,acp,"dis") = eps) = 0;

```

Note that in opposite to factor taxes, use taxes from GTAP are not processed as the structure of the taxes in MANAGE SAM can deviate substantially from the way taxes are defined in GTAP. In order to use the information from GTAP to split tax accounts, a file "sam_corr.inc" can be generated and placed in the folder for the region.

Note here the following:

1. If split shares are derived from summation (such as for total use of a commodity as in the case for commodity taxes, see first two lines), the weights are not the summed-up split-shares, but the related targets.
2. Weights should be scaled before being used as split shares.
3. Empty split shares from GTAP (= real missing values) are flagged with eps. A division by eps provokes a math trap error. Use dollar operated to avoid this.

Bounds on split cells

The controls shown above under “Split Control for activities” and “Split control for products” allow to restrict for groups of split cells the introduction of bounds relative to their targets. This is useful to control large differences to target values and to easier find problematic split shares. The settings are used as follows in the code where an upper limit of 100 is treated as an infinite bound.

```
*
* --- reflect general bounds on split from interface
*
v_splitCell.lo(a,c) $ (sam(a,c,"dis") > 0) = sam(a,c,"dis") * %lowBoundMakeMatrix%;
v_splitCell.up(a,c) $ ((sam(a,c,"dis") > 0) $ (%UpBoundMakeMatrix% ne 100)) = sam(a,c,"dis") * %UpBoundMakeMatrix%;

v_splitCell.lo(f,a) $ (sam(f,a,"dis") > 0) = sam(f,a,"dis") * %lowBoundFacDemand%;
v_splitCell.up(f,a) $ ((sam(f,a,"dis") > 0) $ (%UpBoundFacDemand% ne 100)) = sam(f,a,"dis") * %UpBoundFacDemand%;

v_splitCell.lo(c,a) $ (sam(c,a,"dis") > 0) = sam(c,a,"dis") * %lowBoundIntDemand%;
v_splitCell.up(c,a) $ ((sam(c,a,"dis") > 0) $ (%UpBoundIntDemand% ne 100)) = sam(c,a,"dis") * %UpBoundIntDemand%;

v_splitCell.lo(gt,a) $ (sam(gt,a,"dis") > 0) = sam(gt,a,"dis") * %lowBoundIndFirmTaxes%;
v_splitCell.up(gt,a) $ ((sam(gt,a,"dis") > 0) $ (%UpBoundIndFirmTaxes% ne 100)) = sam(gt,a,"dis") * %UpBoundIndFirmTaxes%;

v_splitCell.up(gt,a) $ (sam(gt,a,"dis") < 0) = sam(gt,a,"dis") * %lowBoundIndFirmTaxes%;
v_splitCell.lo(gt,a) $ ((sam(gt,a,"dis") < 0) $ (%UpBoundIndFirmTaxes% ne 100)) = sam(gt,a,"dis") * %UpBoundIndFirmTaxes%;

v_splitCell.lo(c,fin) $ (sam(c,fin,"dis") > 0) = sam(c,fin,"dis") * %lowBoundFinDemand%;
v_splitCell.up(c,fin) $ ((sam(c,fin,"dis") > 0) $ (%UpBoundFinDemand% ne 100)) = sam(c,fin,"dis") * %UpBoundFinDemand%;

v_splitCell.lo(c,w) $ (sam(c,w,"dis") > 0) = sam(c,w,"dis") * %lowBoundTrade%;
v_splitCell.up(c,w) $ ((sam(c,w,"dis") > 0) $ (%UpBoundTrade% ne 100)) = sam(c,w,"dis") * %UpBoundTrade%;

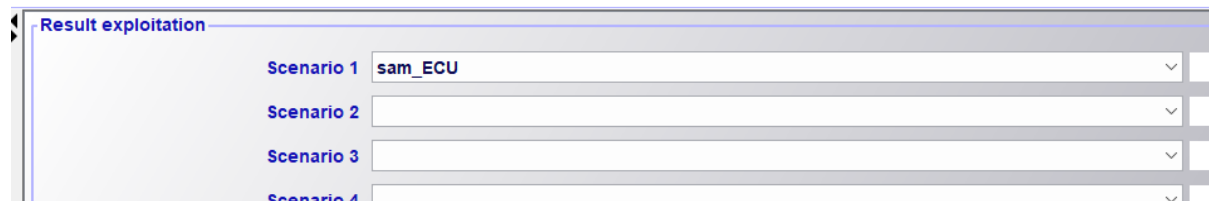
v_splitCell.lo(w,c) $ (sam(w,c,"dis") > 0) = sam(w,c,"dis") * %lowBoundTrade%;
v_splitCell.up(w,c) $ ((sam(w,c,"dis") > 0) $ (%UpBoundTrade% ne 100)) = sam(w,c,"dis") * %UpBoundTrade%;

v_splitCell.lo(gt,c) $ (sam(gt,c,"dis") > 0) = sam(gt,c,"dis") * %lowBoundIndProdTaxes%;
v_splitCell.up(gt,c) $ ((sam(gt,c,"dis") > 0) $ (%UpBoundIndProdTaxes% ne 100)) = sam(gt,c,"dis") * %UpBoundIndProdTaxes%;

v_splitCell.up(gt,c) $ (sam(gt,c,"dis") < 0) = sam(gt,c,"dis") * %lowBoundIndProdTaxes%;
v_splitCell.lo(gt,c) $ ((sam(gt,c,"dis") < 0) $ (%UpBoundIndProdTaxes% ne 100)) = sam(gt,c,"dis") * %UpBoundIndProdTaxes%;
*
```

5.3.2 Exploring the results

The parameter p_SAM which tracks the SAM over the different steps can be loaded into the explorer.



The code offers the following views:

Column and row sums
 SAM, before split
 SAM, split
 SAM, split split cells, only
 Macro SAM
 SAM Statistics

They also allow to compare different versions as shown below:

	target	fixes	solution	% diff	Abs diff
a-cbcc-0 c-cbcc-0		384.72		385.83	-4.91
a-cbcc-0 c-wtra-0		4.91		4.91	0.00
a-cbcc-0 total0		389.63		370.75	-4.85
a-cere-0 c-cere-0		75.68		70.91	-6.30
a-cere-0 c-wtra-0		0.43		0.43	-0.04
a-cere-0 total0		76.11		71.34	-4.27

By sorting based on the “% diff” or “Abs diff” columns, potentially outliers can be tracked easily.

By, for instance, moving the SAM columns into a box and adding statistics, the maximal and minimal deviations for all non-zero rows for a column become immediately visible.

	target	fixes	solution	% diff	Abs diff
Mean	3.40	0.22	3.29	3.40	0.22
Median	0.32	0.02	0.30	0.32	0.02
min	-4.17	0.00	-3.18	-4.17	0.00
max	66.25	2.10	64.15	66.25	2.10
c-ptfs-0	0.11		0.10	-3.21	0.00
c-feed-0	6.76		6.45	-4.63	0.31
c-text-0	0.18		0.17	-3.27	0.01

A similar report is available for the macro-SAM:

	Solution	% diff	Abs diff
DEBT ENT	1684780.00	1684780.00	
DEBT GOV	9077280.00	9077280.00	
DEBT WORLD	8594530.00	8594530.00	
act com	178421392.00	178421392.00	
com act	77549824.00	77549824.00	
com com	15498018.00	15498018.00	
com HH	65291712.00	65291712.00	
com GOV	15583493.00	15583493.00	
com INV	20306584.00	22106694.00	8.14
com INVPUB	7947122.00	7917122.00	-0.38
com STKC	-247240.00	-2017348.88	87.74
com WORLD	24210996.00	24210994.00	-0.00
lab act	40328004.00	40328004.00	
cap act	59491160.00	59491160.00	
HH lab	40328004.00	40328004.00	
HH cap	35737100.00	35737100.00	
HH ENT	1190154.25	1190155.25	0.00
HH GOV	242376.69	242376.69	

Equally, some key statistics can be inspected:

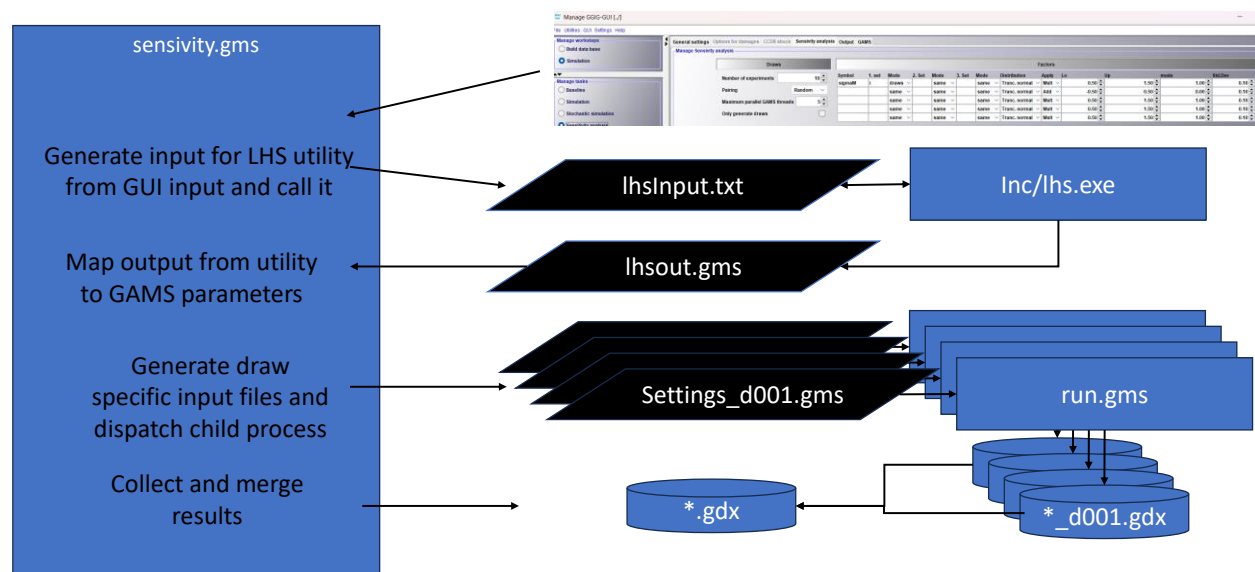
SAM Statistics [0]		
SAM columns		
stat		
target		solution
stat		sam_ECU
nonZent	3953.00	3948.00
Mean	18.45	18.49
Max	1962.08	2058.84
min	-201.73	-201.73
StdDev	93.86	95.20
1DevLoCnt	4.00	4.00
1DevHiCnt	126.00	125.00
2DevLoCnt	1.00	1.00
2DevHiCnt	69.00	66.00

6. Sensitivity analysis

The sensitivity analysis builds on the simulation task of MANGE-WB by adding a Latin-Hypercube Sampling (LHS) based approach to conduct stratified random sampling of user defined model parameters or exogenous shock elements. The use of LHS gives a good coverage of probability distributions with limited number of draws, important, as each draw requires a separate model run.

An overview on the process is given below. The user defines the parameters subject to sensitivity analysis and their distribution on an interface control detailed below. This input serves enters the LHS utility which will define the draws for each individual experiments based on this information. The output from the utility is transformed by the GAMS code of the mother process **sensitivity.gms** into GAMS statements which are inputted into a run-specific include file read by **run.gms**. The mother process will start individual simulation runs, on demand in parallel. Each run will store its results into a separate GDX container of which the name comprises a field referring to the experiment number, such as **d001** in the example below. Once all simulations are run, the mother process will collect and merge the results from all runs into one single GDX container which the user can then explore with the reporting backend.

Figure 6.1: Overview on sensitivity analysis



The additional GUI control for the sensitivity analysis, see Figure 6.2, comprises on the left-hand side a panel with options which refer to the overall set-up of the analysis and not the individual parameter. The *number of experiments* define the number of simulations runs to perform and is equal to number of quantiles covered by LHS. LHS will guarantee that there is exactly one draw from the distribution of each random variable in each quantile. The start number allows to run the sensitivity analysis in blocks such that repeated runs of the utility don't overwrite previous results. Using this requires however an additional program if results from multiple runs of the utility need to be combined.

The option *"Restricted"* for pairing will let the LHS utility target a zero-co-variance matrix between each pair of random vars by reshuffling the drawn random numbers. The success of this will depend on the number of random variables and experiments, with a higher number of random variables and a lower number of experiments increasing the chance to larger non-zero co-variances. *"Random"* pairing will not control for the co-variances, resulting in a faster execution of the LHS utility.

The *"Maximum parallel GAMS threads"* controls how many MANAGE-WB simulations are run in parallel. Once this maximum is reached, the mother process waits until one of the runs is finished before starting another one. It is recommended to use less than the available physical cores available on the CPU as the maximum. The parallel execution can be switched off by setting the control to 1. Equally, as solvers such as CONOPT4 or PATH can use parallel cores, the GAMS run parameter threads should be set to 1 if multiple simulation runs are performed in parallel.

The option *"Only generate draws"* will report the drawn values for the random vars in the listing file, but not start the actual simulation runs. This is useful to control the set-up of the sensitivity analysis before the potential time-consuming actual analysis is started.

Figure 6.2: GUI control for overall set-up of sensitivity analysis.

The right-hand side of the tab comprises a user-edited table, see Figure 6.3. The field *"Symbol"* defines the parameter or variable to disturb (non-existing ones in the GAMS code will be defined as an additional parameter). The follow up elements refer to the names of the sets found on up to three dimensions of the parameter (parameters with more dimensions are not supported), followed by mode field which can be set to *"draws"* or *"same"*. Using *"draws"* implies that each element on the set will be treated like a

separate random variable where “same” will use the same realization for all set elements. For instance, in the example below, the one-dimensional parameter $p_ghgRedRate$ will have different realizations for the different time point as depicted by the set $grcIt$ in each run. If “same” would be used instead, for a single run, the same realization would be found for all time points. The other three random variables are scalars and the name for all sets is left empty.

The follow up control for each parameter defines the type of distribution to use (truncated normal, uniform, triangular), if the random realization should be added to or multiplied with the given parameter value from the base line or found in the shock file, and the lower and upper limit for the distribution, its mode and the standard deviation, the latter only for the truncated normal distribution.

Figure 6.3: GUI control to define parameters as random vars and related distributions

Factors													
Symbol	1. set	Mode	2. Set	Mode	3. Set	Mode	Distribution	Apply	Lo	Up	mode	Std.Dev	
p_salesTaxRecl		same		same		same	Uniform	Mult	0.10	1.90	1.00	0.50	
p_factTaxRecl		same		same		same	Uniform	Mult	0.10	1.90	1.00	0.50	
p_dirTaxRecl		same		same		same	Uniform	Mult	0.10	1.90	1.00	0.50	
p_ghgRedRate	grcIt	draws		same		same	Trunc. normal	Mult	0.50	5.00	1.01	1.00	
		same		same		same	Trunc. normal	Mult	0.50	1.50	1.00	0.10	

Results shown from a sensitivity analysis will show simultaneously the outcomes from all runs as seen below:

Overview [0]

Regions

Items

Quantity

Draws

base

tza_ccdr

2019

2020

2021

2022

2023

2024

2025

2026

2027

2028

2029

2030

2031

Real GDP at market prices

53.19

54.11

56.56

59.51

62.84

66.55

71.29

76.22

81.34

86.53

91.96

97.64

103.56

Real GDP, p.a. change

1.73

4.52

5.22

5.59

5.91

7.12

6.91

6.72

6.38

6.27

6.18

6.06

By using a pivot which shows the different runs in the rows of a table, their outcomes can be directly compared:

Overview [0]

Regions

tza

Items

Quantity

Products

Real GDP at market prices

tza_ccdr

	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036
base	53.19	54.11	56.56	59.51	62.84	66.55	71.29	76.22	81.34	86.53	91.96	97.64	103.56	111.13	119.22	127.56	136.15	145.10
d1	53.19	54.11	56.56	59.51	62.84	66.55	71.29	76.21	81.33	86.52	91.94	97.61	103.50	111.07	119.15	127.48	136.07	145.02
d2	53.19	54.11	56.56	59.51	62.84	66.55	71.29	76.21	81.34	86.53	91.96	97.64	103.56	111.18	119.26	127.58	136.16	145.11
d3	53.19	54.11	56.56	59.51	62.84	66.55	71.29	76.22	81.34	86.53	91.95	97.62	103.52	111.12	119.19	127.52	136.10	145.05
d4	53.19	54.11	56.56	59.51	62.84	66.55	71.28	76.20	81.31	86.48	91.88	97.52	103.37	110.89	118.96	127.28	135.84	144.79
d5	53.19	54.11	56.56	59.51	62.84	66.55	71.29	76.21	81.33	86.52	91.95	97.63	103.55	111.14	119.23	127.57	136.15	145.10

Note here that the run “base” is one where nothing is added the parameters subject to sensitivity analysis in mode “add”. Accordingly, for mode “mult”, these parameters are multiplied with unity such that are kept unchanged from their original value in both cases. By adding relative or absolute changes against the outcome of run “base”, the changes provoked by the different random realizations of the parameters can be directly seen:

Overview [0]

Regions

2x

Items

Quantity

Products

Real GDP at market prices

Percentage diff.

Draws

View type

Table

	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050
Mean	81.34	86.53	91.96	97.64	103.56	111.12	119.21	127.55	136.14	144.94	153.98	163.27	172.81	182.58	192.52	202.61	212.75	222.99	233.67	244.70	256.12	267.87	279.88	292.06
%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.01%	-0.01%	-0.01%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.00%	-0.01%	-0.01%	-0.01%	-0.01%
min	81.31	86.48	91.88	97.52	103.37	110.89	118.96	127.28	135.84	144.61	153.62	162.89	172.39	182.13	192.04	202.10	212.19	222.38	232.89	243.95	255.28	266.92	278.81	290.86
%	-0.04%	-0.07%	-0.10%	-0.13%	-0.18%	-0.22%	-0.22%	-0.22%	-0.22%	-0.23%	-0.24%	-0.24%	-0.25%	-0.25%	-0.26%	-0.26%	-0.26%	-0.26%	-0.26%	-0.31%	-0.33%	-0.36%	-0.39%	-0.42%
max	81.38	86.60	92.06	97.79	103.78	111.37	119.49	127.86	136.49	145.32	154.40	163.74	173.32	183.14	193.12	203.27	213.47	223.78	234.54	245.66	257.18	269.03	281.16	293.47
%	0.05%	0.07%	0.11%	0.15%	0.21%	0.21%	0.23%	0.24%	0.25%	0.26%	0.27%	0.28%	0.29%	0.30%	0.31%	0.32%	0.33%	0.33%	0.35%	0.37%	0.41%	0.43%	0.45%	0.48%

It is also possible to add statistics such as “mean”, “max” and “min” to a table, as depicted above.

The sensitivity analysis is hence a tool to assess result robustness or to improve the scenario definition by testing alternative parameterization of shock files. The set-up can be in most cases directly defined via the GUI without further coding efforts, and the reporting tool allows to easily derive statistics on reported model outcomes.

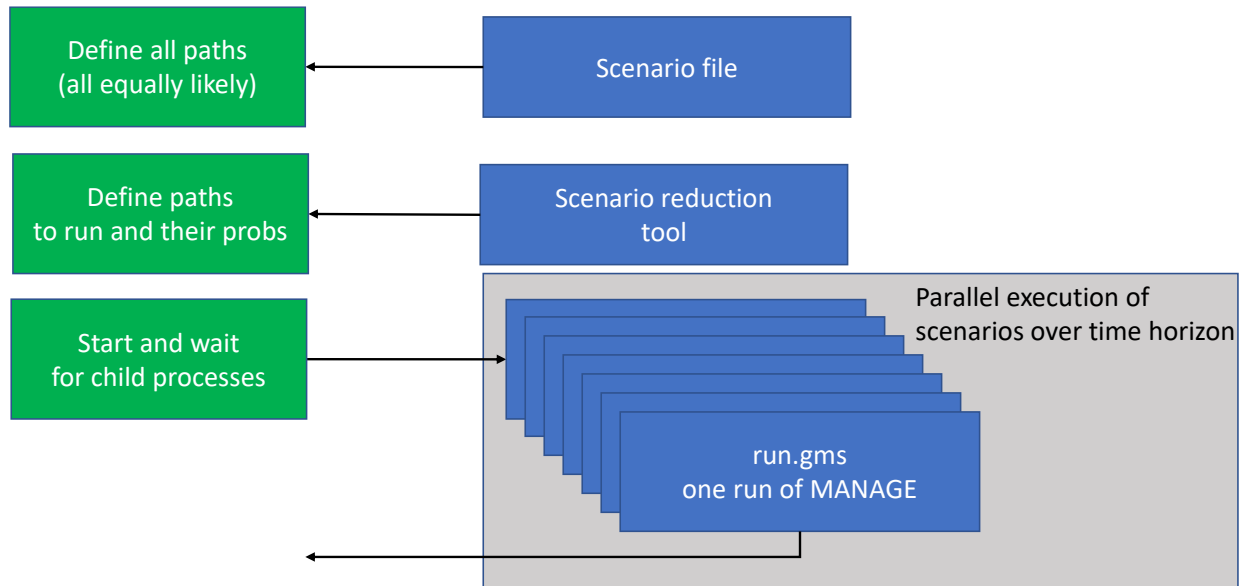
7. Stochastic simulation runs

The stochastic simulation runs are similar to the sensitivity analysis as they also perform a pre-defined number of simulations run. However, whereas sensitivity analysis uses LHS which results in a set-up where each single simulation is equally likely, the stochastic simulation runs are suitable for cases where also small probability high impact shocks are analyzed and their matters. Typical examples are catastrophic weather-related events such as tropical storms or floods which combine low probabilities with high damages. Their so-called Expected Annual Damages (EAD) are often rather low as high damage events have often return periods as high as 100 or 1000 years. In order to capture an event with a 1000-year return period (= 0.001% probability to occur in any year), a LHS approach would require already 1.000 quantiles. If the probably for a storm or flood occur is low, most of these 1000 draws would run a no-change experiment compared to baseline where no storms or floods are considered if the sensitivity analysis would be used. The stochastic simulation run utility therefore uses a specific algorithm, a so-called “scenario reduction” algorithm¹⁴, which combines similar outcomes from a large number of Monte-Carlo draws into one experiment. If there are many similar ones Monte-Carlo draws (such as no or quite limited storms or floods occurring), these are summarized by one run with a higher probability, only. Compared to LHS, this leaves more room for runs referring to less likely outcomes.

The following 7.1 gives an overview on the stochastic extension of MANAGE-WB. The user will define in its shock file the Monte-Carlo draws. Each time series with the random variables, typically damages, drawn, defines one so-called path from the base to last simulation year. The scenario reduction tool will analyze all these paths and average similar one into actual runs to perform. The GAMS mother will run these are individual MANAGE-WB simulation in parallel, as in sensitivity analysis, and collect and merge the results into one GDX container.

¹⁴ H. Heitsch and W. Römisch. Scenario tree reduction for multistage stochastic programs. Computational Management Science, 6:117-133, 2009

Figure 7.1: Overview on sensitivity analysis



In opposite to the sensitivity analysis, the use of the stochastic extension requires a specific set-up of the shock file, in the example used below, disasters which destroy a part of the capital stocks are modelled by random shocks to the depreciation rate. The shock files need to be organized as follows:

1. Prepare for the scenario tree reduction in your shock file

```
$$include '%incf%/iniStoch.gms'
```

2. Draw randomly realizations of the shock and put into pre-defined parameter *p_randvar*

```
p_randVar("depr",nodes) = sum(t_nodes(tCur,nodes),p_depr(tCur) * max(1,normal(1,0.2)));
p_randVar("depr","n1") = p_depr("%baseYear%");
```

3. Let the tool summarize to the desired # of scenarios to run

```
$$include '%incf%/runStoch.gms'
```

4. And pick the realizations for a specific draw later *run.gms*

```
-----
depr.fx(t) = sum(t_nodes(t,nCur) $ draws_nodes(draws,nCur), p_randVar("depr",nCur));
```

These parts are in the following shown as they would appear in a shock file:

```
-----
depr.fx(t) = sum(t_nodes(t,nCur) $ draws_nodes(draws,nCur), p_randVar("depr",nCur));
```

The following example shows a shock file drawing on the example above:

```

$iftheni.decl "%1"=="decl"

    set randVars / "depr" /;
    $$include '%incf%/iniStoch.gms'

$elseif.decl "%1"=="stoch"

    parameter p_depr(t); p_depr(t) = depr.1(t);
    p_randVar("depr",nodes) = sum(t_nodes(tCur,nodes),p_depr(tCur) * max(1,normal(1,0.2)));
    p_randVar("depr","n1") = p_depr("%baseYear%");

    $$include '%incf%/runStoch.gms'

$else.decl
    depr.fx(t) = sum(t_nodes(t,nCur) $ draws_nodes("d%iScen%",nCur), p_randVar("depr",nCur));
$endif.decl

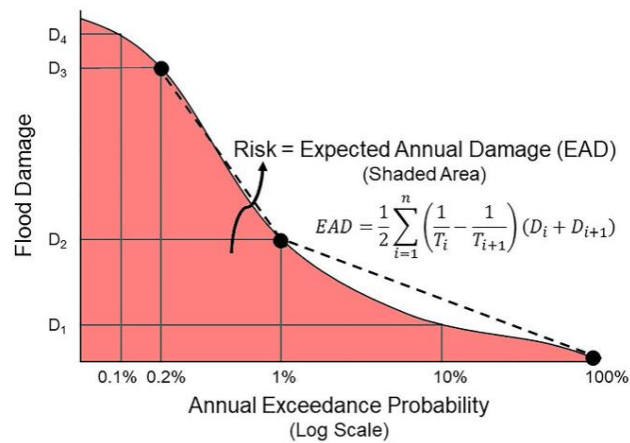
```

Needed for both child and mother:
necessary set definitions

Only used
by mother process:
Define random vars
and apply scenario
reduction

Used in child process ("run.gms") to pick specific realizations
for current scenario %iScen%

A typical application field are damages reported by size for a limited number of selected return period. The expected annual damage is typically calculated by linear interpolation between given point, assuming that the damage size is inversely proportional on these segments:



From: Bilskie, M. V., Angel, D. D., Yoskowitz, D., & Hagen, S. C. (2022). Future flood risk exacerbated by the dynamic impacts of sea level rise along the northern gulf of Mexico. *Earth's Future*, 10(4), e2021EF002414.

The following code uses this approach to define random draws where the GAMS random variable generator for the uniform distribution is used to draw a probability between 0 and 1, and the GAMS code interpolate the damage between the segment encompassing the drawn probabilities.

The input consists of the damage estimates, here an increase in the depreciation rates stored in the table *p_drm*, for different return period. A T5 (= five year return period) event has a probability of 20% and leads in the example to a very modest increase in the depreciation rate of 0.001, when compared to a 100 year event (1% probability) which increases the depreciation rate by 0.04:


```

set retPer "Return periods" / T1,T5,T10,T50,T100,Tend ;;

table      p_Drm "Increase in depreciation for disasters with a certain return period"
      dmg      prob
T1         1.E-10    1
T5         0.001     0.2
T10        0.01      0.1
T50        0.02      0.02
T100       0.04      0.01
Tend       0.04      0.00
;

--- first draw probability from uniform
p_randVar("depr",nodes) = uniform(0,1);

--- map to damage based on trapezoidal rule (i.e. the damage is proportional to the
probability)

p_randVar("depr",nodes) = p_depr("%baseYear%")
+ sum(retPer $ ( (p_randVar("depr",nodes) lt p_drm(retPer-1,"prob")) $ (p_randVar("depr",nodes) ge p_drm(retPer,"prob"))),
      p_drm(retPer-1,"dmg") + (p_drm(retPer,"dmg") - p_drm(retPer-1,"dmg"))
      * (p_drm(retPer-1,"prob") - p_randVar("depr",nodes)) / (p_drm(retPer-1,"prob") - p_drm(retPer,"prob")));

```

Results from the stochastic extension can be analyzed similar to those from a sensitivity analysis, however, the user now needs to reflect that the probability of the different runs will differ. The in-built statistics in reporting tool will consider this automatically. The following screen-shot shows how to add some statistics:

The screenshot shows a software interface with a data table and a 'Set statistics' dialog box.

Data Table:

	2018	2019	2020	2021	2022	2023
d1		0.050000	0.050000	0.054656	0.055163	
d2		0.050000	0.053956	0.050000	0.050000	
d3				0.050000	0.053202	
d4				0.050000	0.054800	
d5				0.055613	0.050000	
d6				0.051071	0.050000	
d7				0.050000	0.056865	
d8				0.050000	0.050000	
d9				0.050418	0.050000	
d10				0.050000	0.050000	
d11				0.050000	0.050000	
d12				0.050102	0.055701	
d13				0.050000	0.050000	
d14				0.053470	0.050000	
d15				0.050000	0.067204	
d16				0.053460	0.050000	
d17				0.056909	0.050000	
d18		0.050000	0.050000	0.050000	0.058732	
d19		0.054217	0.050000	0.050000	0.050000	
d20		0.050000	0.050000	0.050000	0.050000	
d21		0.051058	0.050000	0.050000	0.053206	0.050000

'Set statistics' Dialog Box:

- ☒ Treat zeros as missing values for statistics
- ☐ Only show outliers
- Set maximum percentage of outliers: 2
- Set outlier detection method: No outlier detection
- Select statistics:
 - Sum
 - Nobs
 - Mean
 - Median
 - StdDev
 - q1
 - q3
 - min
 - max
 - minOutlier
 - maxOutlier
 - freeEval
- Free evaluation field: []
- Buttons: ok, Cancel, Update

Which gives:

Capital stock and depreciation [0]

Regions

TUR

Products

Depreciation rate

TUR_shk_drm

	2018	2019	2020	2021	2022	2023	2024	2025	2026
Mean		0.051974	0.051476	0.051959	0.051732	0.051886	0.051728	0.051664	
StdDev		0.005210	0.005158	0.005210	0.005185	0.005200	0.005184	0.005177	
q1		0.050000	0.050000	0.050000	0.050000	0.050000	0.050000	0.050000	
q3		0.052556	0.050000	0.052005	0.052266	0.052189	0.051604	0.051238	
min		0.050000	0.050000	0.050000	0.050000	0.050000	0.050000	0.050000	
max		0.066921	0.068120	0.068789	0.067204	0.068732	0.067254	0.063054	
d1		0.050000	0.050000	0.054656	0.055163	0.052057	0.050000	0.050000	
d2		0.050000	0.053956	0.050000	0.050000	0.054645	0.050000	0.050000	
d3		0.050000	0.050000	0.050000	0.053202	0.050645	0.060437	0.050000	
d4		0.050000	0.050000	0.050000	0.054800	0.058772	0.050000	0.060776	
d5		0.053120	0.050000	0.055613	0.050000	0.050000	0.050000	0.054095	
d6		0.058063	0.050000	0.051071	0.050000	0.055193	0.053813	0.056691	

By solely selecting the statistics, an overview on the distribution can be gained:

Overview [1]

Regions

TUR

TUR_shk_drm

2018

2019

2020

2021

2022

2023

2024

2025

2026

2027

2028

Real GDP at market prices

Mean

3724.39

3752.33

3820.66

4251.87

4460.08

4681.33

4935.40

5202.95

5492.30

5808.65

6125.00

Median

3724.39

3754.18

3822.06

4254.54

4457.52

4678.26

4935.68

5202.15

5492.93

5809.31

6125.00

min

3724.39

3740.85

3801.73

4232.40

4437.83

4660.37

4907.42

5147.04

5435.26

5756.19

6077.16

max

3724.39

3754.18

3825.51

4261.62

4474.70

4698.44

4956.85

5229.48

5522.33

5831.93

6147.50

Real GDP, p.a. change

Mean

0.75

1.82

11.29

4.90

4.96

5.43

5.42

5.56

5.76

Median

0.80

1.90

11.38

4.96

5.00

5.50

5.50

5.60

5.80

min

0.44

1.27

11.00

4.46

4.67

4.59

4.88

5.21

5.54

max

0.80

1.90

11.40

5.03

5.04

5.56

5.55

5.67

5.90

Real GDP at market prices, per capita

Mean

44975.47

44947.88

45410.86

50154.51

52261.77

54550.63

57215.15

60013.41

63033.98

66332.23

69633.23

Median

44975.48

44970.14

45427.45

50185.98

52231.72

54514.84

57218.33

60004.18

63041.25

66339.79

69633.23

min

44975.48

44810.46

45185.84

49924.91

52001.02

54306.41

56890.78

59368.56

62379.42

65733.24

69633.23

max

44975.48

44970.14

45468.51

50269.55

52433.06

54750.00

57463.80

60319.38

63378.64

66598.05

69633.23

Real Household consumption, per capita

Mean

25456.29

25842.26

27215.93

30048.45

31061.07

32116.13

33458.23

34849.20

36716.66

38761.05

40812.50

Median

25456.29

25838.88

27225.63

30070.02

31078.01

32090.24

33424.31

34864.59

36710.59

38772.35

40812.50

min

25456.29

25838.88

27112.60

29847.35

30917.28

31923.86

33275.66

34583.34

36249.77

38319.55

40812.50

max

25456.29

25865.20

27286.61

30123.59

31210.63

32235.06

33723.14

35016.26

36953.71

39029.92

40812.50

Private consumption, real on real GDP

Mean

56.60

57.49

59.93

59.91

59.43

58.87

58.48

58.07

58.25

58.43

Median

56.60

57.46

59.88

59.85

59.38

58.85

58.41

58.02

58.23

58.40

min

56.60

57.46

59.86

59.78

59.32

58.78

58.35

57.94

58.11

58.30

max

56.60

57.72

60.39

60.14

59.83

59.13

59.28

58.37

58.50

58.64

Government consumption, real, share on real GDP

Mean

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

Median

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

min

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

15.52

max

15.52

15.52

15.52

15.52

15.52

15.52

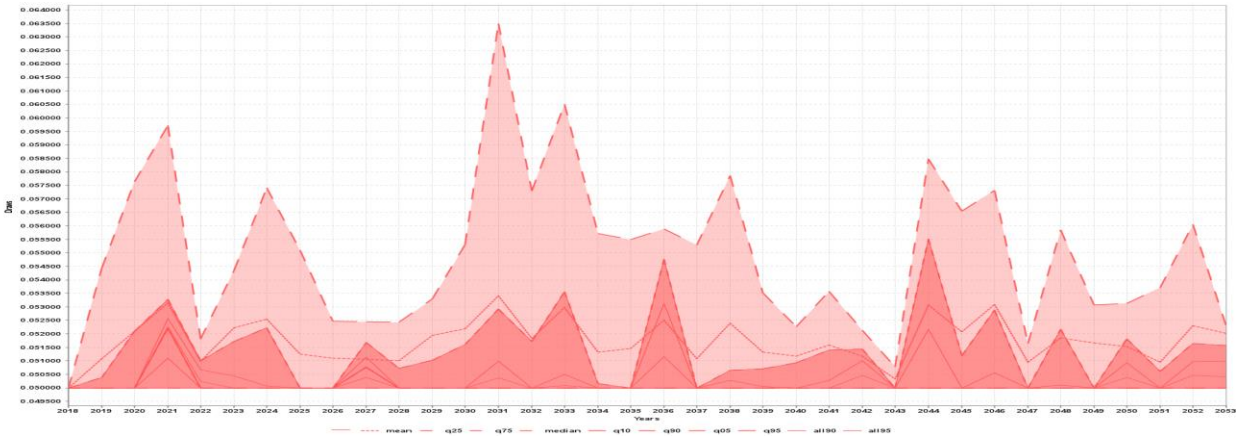
15.52

15.52

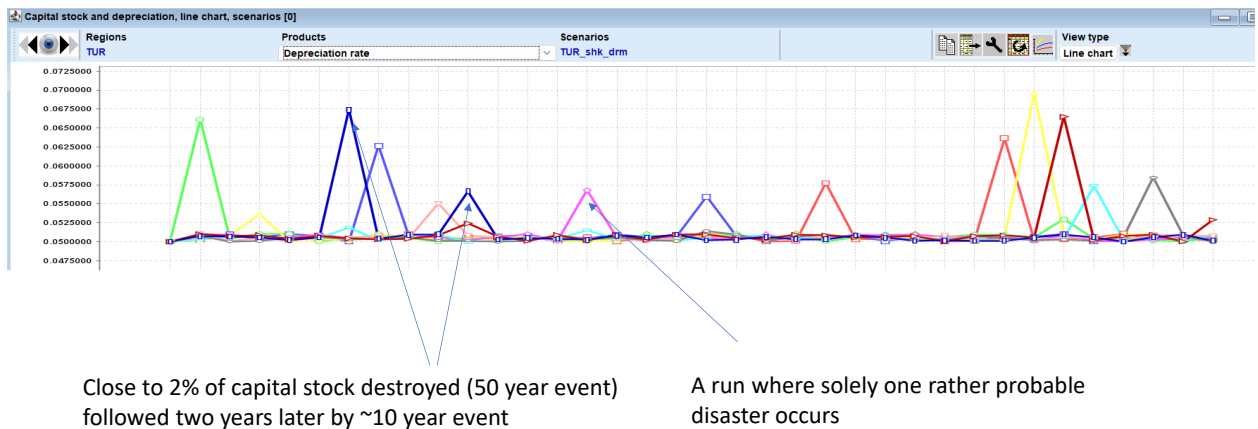
15.52

15.52

A so-called “deviation” graph can also help to analyze outcomes:



The following graph shows the generated draws for 10 scenarios with the damage data shown above:



Summarizing, the stochastic extensions is useful for cases where low probability events are analyzed such that even LHS would require to many runs to capture the distribution. A typical case are damages reported for different higher return periods such as 50, 100, 500 or 1.000 years. The reporting interface support the analyze by automatically calculated probability weighted statistics and by certain graphs to visualize outcomes.

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