A Ricardian Trade Structure in CGE: Modeling Eaton-Kortum Based Trade with GTAP

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We develop an Eaton-Kortum based calibrated computational model, incorporating the basic structure of the GTAP version 7 Model. The underlying trade and associated gravity equations in GTAP are modified from a standard Armington structure with love-of-variety by country of origin to Eaton-Kortum structure with comparative advantage differences within sectors. We describe the theory and model code changes necessary for implementation, and the underlying theory of the appropriate estimating equations for gravity estimation of the trade parameters. We then compare the simulation results derived from GTAP models with Armington and Eaton-Kortum specifications. The impact of trade policy experiments on real income is very similar with marginal differences being driven by the transportation sector. The terms of trade impact of tariff changes is not uniformly larger in Eaton-Kortum relative to the Armington specification. Finally, the impact of trade cost changes on trade volumes are smaller in the Eaton-Kortum specification than in the Armington specification due to the identical price-by-source-country property in the former model.

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Computable general equilibrium (CGE) models have been used since the 1980s for ex-ante analyses of trade policies (Shoven and Whalley, 1984). Since the publication of seminal works by Eaton and Kortum (2002) and Anderson and van Wincoop (2003), trade economists have developed additional tools to analyse the impact of counterfactual trade policy experiments. Anderson and van Wincoop (2003) develop a structural gravity approach with an Armington structure to model international trade. In terms of counterfactual experiments, they deviate from existing CGE models by calibrating the baseline to fitted values from the gravity estimation instead of actual values as in CGE models.

The Eaton and Kortum (2002) model international trade is based on comparative advantage differences between countries. They extend the existing model of comparative advantage with a continuum of varieties and two countries by Dornbusch et al. (1977) to a model of comparative advantage with multiple countries and a continuum of varieties based on a stochastic specification for productivity. Employing an Eaton Kortum trade structure, Dekle et al. (2008) introduce the so-called exact hat algebra (EHA) to solve for the ratio of new to old values of endogenous variables in response to a counterfactual experiment.

Structural gravity models and models in the spirit of Eaton and Kortum (2002) and Dekle et al. (2008) have been called new quantitative trade (NQT) models in the literature. Some scholars employing these models have criticized CGE models. For example, Caliendo and Parro (2015) state: “These models have been criticized for their complexity, lack of transparency and analytical foundations, and the arbitrary choice of the value of key parameters.”

This raises the question how do the NQT models differ from CGE models? Five differences can be identified (see also Bekkers (2019a)). First, the baseline calibration is different. Structural gravity models calibrate the baseline to predicted values, whereas both CGE models and NQT models applying EHA calibrate the baseline to actual, observed values. Second, NQT models place more emphasis on the importance of structural estimation, which can be defined as an approach in which one model and one data set are used to both estimate the parameters of the model and conduct counterfactual experiments. Scholars employing CGE models are more flexible in employing estimated parameters from the literature, and tend to attach more importance on the validity of employed parameters. ¹ Third, the so-

¹ A risk of the former approach is that overly restrictive Cobb Douglas nests are employed (for example for consumer demand between different sectors) since no explicit parameter is visible (the substitution elasticity between sectors is 1 in this case) and thus the claim of structural estimation can be maintained. An example is the comparison of Costinot et al. (2016) and Gouel and Laborde (2021). While the former emphasizes the importance of
ution methods differ. CGE models in levels (typically coded in the GAMS software (Bussieck and Meeraus, 2004)) and structural gravity models solve for the baseline and counterfactual equilibrium in levels. CGE models in relative changes (Dixon et al., 1982; Hertel, 1997) coded in the GEMPACK software (Harrison and Pearson, 1996) log linearize the model equations and calculate percentage changes in endogenous variables. Models employing EHA solve the equilibrium equations for ratios of new to old values of endogenous variables. Fourth, CGE models tend to employ a “big data science” build-on-prior-research approach, whereas NQT models oftentimes develop a new model and collect data in a new or different way than beforehand. The former approach involves the risk that established models are applied in an inappropriate way to new questions. The latter comes with the risk of errors in data collection and counterfactual characteristics of models which only appear after studies have been published and policy advice communicated to policy makers. Fifth and finally, the structures of the models tend to be different. CGE models and databases tend to be more extensive, including detailed information on intermediate and value added demand by industries, as do detailed household, government and investment demands, transfers between agents, link between savings and investment, trade and transport margins, and a host of domestic and international tax/subsidy wedges. NQT models put more emphasis on parsimony by using compact models emphasizing the importance of explaining the empirical regularities under study with the most compact model possible. Oftentimes, NQT models omit investment, including it instead as part of consumption demand (see for example Costinot and Rodriguez-Clare (2014)).

Against this background, we incorporate the Eaton-Kortum trade structure in the GTAP CGE model and compare the results with GTAP’s default Armington specification. For a fair comparison of the two types of models, we compare the non-nested version of the Armington model with the Eaton-Kortum model and calibrate both model specifications to the same trade elasticity: the elasticity of the value of trade with respect to iceberg trade costs. Arkolakis et al. (2012) have shown that under certain conditions the welfare gains from trade in the Eaton-Kortum model are identical to an Armington model. However, there may be differences between the two modeling specifications if implemented in a more elaborate general equilibrium model.

We contribute to the literature by developing an Eaton-Kortum calibrated com-

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2 Hertel et al. (1992) confirm that the same CGE model solved in levels and relative changes produce the same results.

3 We have benefited from conversations with David Hummels and Tom Hertel in developing this point.
putational model, starting with the basic structure of version 7 of the GTAP Model (Corong et al., 2017). GTAP is a widely-used, calibrated computational model (aka computable general equilibrium or CGE model) allowing for analysis of detailed trade policies (e.g. domestic and export taxes, import tariffs and transport margins). It includes expressions for both trade volumes and values, in contrast to new quantitative trade (NQT) models that employ exact hat algebra expressions only for the value of trade. The recursive dynamic version of GTAP also facilitates long run projections and includes capital accumulation, differences in sectoral productivity growth and cost-neutral preference shifters. In this paper, we describe in detail how the static GTAP model with non-homothetic private household preferences is modified from Armington to Eaton-Kortum Ricardian trade structure. We also conduct trade policy experiments to identify differences in results between Armington and Eaton-Kortum specifications in the GTAP model.

The import demand and importer price equations are similar in the Armington and Eaton-Kortum specification in that they follow a constant elasticity of substitution (CES) structure. This similarity implies that the macroeconomic effects (e.g., real income) of trade policy shocks will be similar in both models when calibrated to the same empirically-estimated trade elasticity, or even identical under a set of restrictions as shown by Arkolakis et al. (2012). However, the landed price paid by importers is different. In the Armington specification, the import price is source-specific—i.e., source-specific exporter price plus the various trade taxes (export tax and import tariff) and mode-specific international transport margins for each commodity and bilateral country pair. In the Eaton-Kortum model, a commodity’s import price is equal to the average price from all sources. This is because the import price is an average of the prices over a continuum of varieties within a sector. Moreover, the set of imported commodities from a source region adjusts to changes in costs (extensive margin adjustment) in such a way that the average price from each country of origin is identical. For example, an exporting country with high trade costs compared to another country with moderate trade costs will export varieties. Since the high cost varieties are not exported, the average sectoral price paid by importers will be the same regardless of origin.

This property of identical import prices from different sources has implications for changes in traded quantities and prices in the Armington and Eaton-Kortum model. In particular, the terms of trade effects associated with trade policies are different. In general, an import tariff imposed by a large country drives down the pre-tariff price thus generating terms of trade gains. In the Eaton-Kortum model this effect is more prominent, because of the extensive margin adjustment. Higher commodity tariffs imposed on a source country imply that more expensive varieties will no longer be exported by that source country. As a result the tariff-inclusive import price will not increase as much as in the Armington model. Hence, the expectation is that the terms of trade gains from introducing import tariffs should be larger. However, in a general equilibrium setting this feature of the Eaton-Kortum
model does not necessarily imply that importers incur larger terms of trade gains from imposing tariffs in this model compared to the Armington model. Changes in the terms of trade are also determined by changes in export prices and these also change less in the Eaton-Kortum model.

For purposes of illustration, we run four trade policy experiments to compare the Armington and Eaton-Kortum specifications as implemented in the GTAP framework: (i) global tariff liberalization; (ii) global elimination of export taxes; (iii) global reduction in iceberg trade costs; and (iv) increase in tariffs by a single country vis-a-vis its trading partners. The fourth experiment is included to evaluate differences in terms of trade effects between the two models. In this context, the Armington model has been criticized for its large terms of trade effects from trade policy experiments, due to the love-of-variety by country of origin feature. For example, Brown (1987) argues that the Armington specification displays strong terms of trade effects due to “monopoly power implicit in national product differentiation” which implies a strong incentive to introduce tariffs to exploit terms of trade gains.4

In the comparison, we calibrate the parameter determining responses of trade to trade policy shocks to the same empirically observable trade elasticity: the elasticity of the value of trade to changes in iceberg trade costs. Hence, the Armington elasticity of substitution between domestic and imported varieties and between imported varieties are equalized (“non-nested or collapsed Armington elasticities”) and set equal to one plus the dispersion parameter of the Frechet productivity distribution in the Eaton-Kortum model.5

The simulations generate three main insights. First, the impact of trade policy experiments on real income is very similar between the two models, in line with theory (Arkolakis et al. (2012)). The marginal differences between the two models are driven by the presence of the transportation sector. Second, the impact of trade cost changes on the volume of trade is smaller in the Eaton-Kortum model than in the Armington model. This result is driven by the fact that the elasticity of trade volumes to trade cost changes in Eaton-Kortum is the same as the elasticity of trade values due to the identical-price-by-source-country property of the model, and thus smaller than in the Armington model. Third, the terms of trade gains from increasing tariffs are not uniformly larger in the Eaton-Kortum model. Reductions in import prices are larger, thus generating larger terms of trade gains on the import side. However, increases in export prices are also smaller in the Eaton-

4 See also Shoven and Whalley (1984); Shiells and Reinert (1993); Lloyd and Zhang (2006).
5 Technically, trade elasticities are usually estimated from trade value data combined with tariff data. However, the tariff elasticity and the iceberg elasticity vary by one in both the CES-based Armington and Eaton-Kortum based versions of the gravity equation. Their role is different though, as the value and volume elasticities are identical in the Eaton-Kortum framework, whereas they are different in the Armington model. This is discussed formally below.
Kortum model. Simulations show that the projected changes in terms of trade are larger under Eaton-Kortum in some countries/regions and larger under Armington in others. Hence, our comparison shows that the Eaton-Kortum model does not uniformly generate larger or smaller terms of trade effects, because terms of trade changes are determined by changes in both import and export prices.

This paper has three main contributions. First, we introduce an Eaton-Kortum specification in the GTAP version 7 model (Corong et al., 2017) calibrated to the standard GTAP Data Base (Aguiar et al., 2023). This should enable CGE-modellers in general and GTAP researchers in particular to use the Eaton-Kortum specification for GTAP-based applications. It also provides a bridge between CGE models and the NQT literature, which tends to employ the Eaton-Kortum structure of trade. Second, we develop an Eaton-Kortum specification in quantities, making it useful for applications requiring explicit quantity terms such as the modelling of greenhouse gas emissions. Third, our paper provides a detailed comparison of results between the Eaton-Kortum and Armington specification, exploring differences in the impact on prices, quantities and terms of trade.

The fact that CGE models are usually defined in terms of values, quantities and prices instead of just values and prices as in NQT models requires significant changes to the GTAP model code in order to implement the EK model in this context. First, the market equilibrium is redefined in terms of values. Second, the expressions for traded prices used in the equations for tariff and export tax revenues have to be reformulated starting from landed average prices. Third, update statements for values of imports by source, total imports, and and domestic sales values by end user all have to be reformulated. Fourth, as shown in the Annex section, expressions for aggregate trade values, prices, and quantities, used in the calculation of the terms of trade and GDP, have necessarily been modified as well.

This paper builds on Bekkers et al. (2018), who incorporate an Eaton-Kortum structure in a recursive dynamic CGE model to study the global trade impacts of opening the Northern Sea Route. Bekkers et al. (2018) use the Eaton-Kortum gravity equations to structurally estimate the parameters for their model. As such, their model provides an example of a structurally estimated, calibrated computational model fully incorporating the GTAP database structure. In this paper, we provide more extensive detail on the actual implementation of the Eaton-Kortum trade structure in the GEMPACK-based GTAP model code. We also provide a comparison of Eaton-Kortum and Armington trade structures and their associated results. As in Bekkers et al. (2018) we also derive the underlying gravity estimating equation for the Eaton-Kortum model, though we go further in the comparison to the standard Armington framework.

The paper is organized as follows. The next section describes the changes from Armington to the Eaton-Kortum specification. Section 3 presents and explains the results of several trade policy experiments, while Section 4 concludes. Appendix Section 5 maps out the modifications to the GTAP model code which uses an Arm-
ington trade structure.

2. Model

We modify the trade structure of the standard GTAP version 7 model (Corong et al., 2017) from Armington to Eaton-Kortum specification. GTAP is a widely-used CGE model of the global economy with multiple countries, activities, commodities, factors, intermediate linkages, non-homothetic preferences and agent-specific (firms, government, household and investment) demands.\(^6\) Savings are a fixed share of income and are used to finance investments through a global bank which then allocates investment across countries. A technical description of the Eaton-Kortum model structure which this paper incorporates into the GTAP framework is provided in Bekkers et al. (2018).

An important feature of the Eaton-Kortum model is that the landed bilateral import price does not vary by source country, because of the extensive margin adjustment: from a region with higher costs less of the most expensive varieties are imported such that the average sector price is identical to the price from other source countries. This also implies that the cif, fob and export prices are determined based on the landed price. This structure is opposite to the Armington model in which the export price determines the cif, fob and landed import price.

2.1 Setup

We follow the structure of Eaton and Kortum (2002) to model international trade within each commodity\(^7\). Our goal is to derive expressions for import and domestic prices and quantity demand for each commodity \(c\). Commodities \(c\) destined for country \(d\) are demanded by four groups of agents \(ag\): firms \(fi\), private households \(pr\), government \(go\) and investors \(in\). We assume an identical constant elasticity of substitution (CES) utility function across the continuum of goods for each of these four groups:

\[
q^{ag}_{cd} = \left( \int_0^{\infty} q_{cd}(\omega) \omega^{\sigma_c-1} d\omega \right)^{\frac{1}{\sigma_c-1}} \quad (1)
\]

To sell commodity \(c\) with variety \(\omega\) from source \(s\) to destination \(d\), firms charge the following cif-inclusive (cost of insurance and freight) price:

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\(^6\) GTAP can also be operated in recursive dynamic mode, see for example (GTAP-RD (Aguiar et al., 2019b), WTO Global Trade Model (Aguiar et al., 2019a) and GDyn (Ian-chovichina and McDougall, 2000)). These models allow for capital accumulation and facilitate economic projections.

\(^7\) We use commodity, goods and sector interchangeably throughout the paper.
\[ p_{\text{cif}}^{\text{csd}}(\omega) = \frac{t_{\text{prod}}^{\text{csd}} c_{\text{csd}}^{\text{exp}} + \gamma_{\text{csd}} p_{\text{csd}}^{\text{d}}}{z_{\text{csd}}(\omega)} \]  

where:  
- \( c_{\text{csd}} \) is the total cost of inputs associated with producing commodity \( c \) in exporting country \( s \),  
- \( t_{\text{prod}}^{\text{csd}} \) is the production tax or subsidy on production of commodity \( c \) in exporting country \( s \), and  
- \( t_{\text{exp}}^{\text{csd}} \) is the destination-specific export tax. The price of transport services is represented by \( p_{\text{ts}}^{\text{csd}} \) and \( \gamma_{\text{csd}} \) which is the share of international transport margin services in the cif-value of imports. All taxes are expressed in power of the tax format, i.e. as one plus the ad-valorem tax rate.

Productivity \( z_{\text{csd}}(\omega) \) follows the Frechet distribution with technology parameter \( \lambda_{\text{cs}} \) and dispersion parameter \( \theta_{c} \) as:

\[ F_{\text{csd}}(z) = \exp \left( - \left( \frac{z}{\lambda_{\text{cs}}} \right)^{-\theta_{c}} \right) \]  

where:  
- \( \lambda_{\text{cs}} \) is a technology parameter and determines the country- and sector-specific location of productivity, while \( \theta_{c} \) is a sector-specific dispersion parameter. A smaller \( \theta_{c} \) corresponds to a more dispersed productivity distribution within each sector.

### 2.2 Price distribution and import share probability

To get to the group-specific landed price \( p_{\text{agg}}^{\text{csd}}(\omega) \) of variety \( \omega \), we multiply the cif-price \( p_{\text{cif}}^{\text{csd}}(\omega) \) by one plus the import tariff \( t_{\text{imp}}^{\text{csd}} \), general iceberg trade costs \( \tau_{\text{csd}}^{\text{g}} \), group- and source-specific iceberg trade costs \( \tau_{\text{cd}}^{\text{ag},g} \) and taxes \( t_{\text{cd}}^{\text{ag},g} \). We also identify commodity sources as domestic and imported (so = dom, imp), and agents such as government, private households, firms and investors (ag = go, pr, fi, in):

\[ p_{\text{csd}}(\omega_{c}) = \left( t_{\text{prod}}^{\text{csd}} c_{\text{csd}}^{\text{exp}} + \gamma_{\text{csd}} p_{\text{csd}}^{\text{d}} \right) t_{\text{csd}}^{\text{imp}} \tau_{\text{cd}}^{\text{ag},g} t_{\text{cd}}^{\text{ag},g} z_{\text{csd}}(\omega) \]  

As the price \( p_{\text{csd}} \) is a function of productivity \( z \), a Frechet distribution for productivity \( z \) implies a Frechet distribution of import prices \( p \) for commodity \( c \) traded from \( s \) to \( d \) (derivation in 5.6):

\[ G_{\text{agg}}^{\text{csd}}(p) = 1 - \exp \left\{ - \left( \frac{t_{\text{prod}}^{\text{csd}} c_{\text{csd}}^{\text{exp}} + \gamma_{\text{csd}} p_{\text{tr}}^{\text{d}}}{\lambda_{\text{csd}}} t_{\text{csd}}^{\text{imp}} \tau_{\text{cd}}^{\text{ag},g} t_{\text{cd}}^{\text{ag},g} \right) \right\}^{-\theta_{c}} \]  

Next we derive the price distribution of commodity \( c \) in the importing country \( d \) in \( G_{\text{agg}}^{\text{csd}}(p) \), using the result that the probability that a price in importer \( d \) is lower than \( p \) is equal to one minus the probability that none of the exporters \( s \) delivers a price
lower than \( p \). So we have:

\[
G^{ag}_{cd}(p) = 1 - \prod_{r=1}^{R} (1 - G_{csd}(p))
\]

(6)

Combining equations (5)-(6) leads to the following price distribution:

\[
G^{ag}_{cd}(p) = 1 - e^{-\Phi^{ag}_{cd}p_c}
\]

(7)

With

\[
\Phi^{ag}_{cd} = \sum_{r=1}^{R} \left( \frac{\prod_{s=1}^{R} c_{cd}^{exp} + \gamma_{csd}p_{csd}}{\lambda_{cs}} \right)^{-\theta_c}
\]

(8)

where: \( \Phi^{ag}_{cd} \) determines the average price of commodity \( c \) in country \( d \) and is a function of unit costs and the level of technology in all its trading partners, \( c_{cs} \) and \( \lambda_{cs} \), and the trade costs for importing from its trading partners.

The probability \( \pi^{ag}_{csd} \) that goods \( c \) in country \( d \) by group \( ag \) are imported from trading partner \( s \) is equal to the probability that the price in country \( s \) is lower than the price in all the other trading partners:

\[
\pi^{ag}_{csd} = P(p_{csd} \leq \min\{p_{csd'} u \neq s\}) = \int_0^{\infty} \prod_{u \neq s} (1 - G^{ag}_{csd}(p)) \, dG^{ag}_{csd}(p)
\]

(9)

Substituting equation (5) and elaborating leads to:

\[
\pi^{ag}_{csd} = \left( \frac{\prod_{r=1}^{R} c_{cd}^{exp} + \gamma_{csd}p_{csd}}{\lambda_{cs}} \right)^{-\theta_c}
\]

(10)

2.3 Quantity imported

We derive the commodity sold from \( s \) to \( d \) based on the quantity share sourced from \( s \) in \( d \). The quantity share is equal to the probability that goods are bought from source country \( s \), with price \( \pi^{ag}_{csd} \) times the average quantity bought from \( s \), \( q_{csd} \), divided by the total quantity \( c \) bought from all other trading partners:

\[
\frac{q^{ag}_{csd}}{q^{ag}_{cd}} = \frac{\pi^{ag}_{csd}q^{ag}_{csd}}{\sum_{u=1}^{R} \pi^{ag}_{csd}q^{ag}_{csd}}
\]

(11)

We use the property that the distribution of prices of goods sourced from country \( s \) and imported by country \( d \) is given by the same distribution as the general distribution of prices in country \( d \), \( G^{ag}_{cd}(p) \). This is Property b on page 1748 of Eaton and Kortum (2002) and is also derived in the Appendix.\(^8\) The implication is

\(^8\) As Eaton and Kortum (2002) point out, this follows from calculating the distribution of
that the average quantity purchased is the same for each source country $s$:

$$
\bar{q}_{csd}^{ag} = \int_{0}^{\infty} p^{-\sigma_c} (p_{cd})^{-\sigma_c} q_{cd}^{ag} dG_{cd} (p) = q_{cd}^{ag}
$$

Substituting equation (12) into equation (11) and using the fact that the sourcing probabilities add up to 1 gives rise to the result that the quantity share sourced from country $s$ is equal to the probability that goods are sourced from country $s$,

$$
\pi_{csd}^{ag} = \frac{\left(\frac{\pi_{csd}}{\phi_{cd}}\right)^{\lambda_{cs}}}{\pi_{cd}^{ag}} q_{cd}^{ag} (12)
$$

We then derive total imports of each commodity $c$ from source $s$, $q_{csd}$, as the sum over imports of the four agents in country $d$:

$$
q_{csd} = \sum_{ag \in \{pr, go, fi, in\}} q_{csd}^{ag} = \sum_{ag \in \{pr, go, fi, in\}} \pi_{csd}^{ag} q_{cd}^{ag}
$$

The probability of country $d$ importing from country $s$, $\pi_{csd}^{ag}$, can be written as the probability of country $d$ importing in $s$, $\pi_{cd}^{ag, imp}$ times the probability of importing from source $s$ conditional upon importing, $\pi_{csd}$:

$$
\pi_{csd}^{ag} = \pi_{cd}^{ag, imp} \pi_{csd}
$$

Substituting (15) into (14) leads to:

$$
q_{csd} = \pi_{csd} q_{cd}^{imp}
$$

With $q_{cd}^{imp}$ being the sum of import demands by the four groups of agents:

$$
q_{cd}^{imp} = \sum_{ag \in \{pr, go, fi, in\}} \pi_{cd}^{imp, ag} q_{cd}^{ag} = \sum_{ag \in \{pr, go, fi, in\}} q_{cd}^{imp, ag}
$$

The probability of importing from source $s$ conditional upon importing, $\pi_{csd}$, prices of goods sourced from $s$ in country $d$ given that goods are actually sourced from country $s$.

$^9$ Eaton and Kortum (2002) use this property to argue that average expenditure does not vary by source. The reasoning is identical for average quantity and average expenditure. Both are determined by prices. With a price distribution not varying by source average quantity and average expenditure do not vary by source.
can be calculated as follows:
\[
\pi_{csd} = \frac{\pi_{csd}^{ag}}{\pi_{csd}^{imp,ag}} = \frac{\pi_{csd}^{ag}}{\sum_{u} \pi_{usi}^{ag}} = \frac{\left(\left(\frac{\pi_{csd}^{ag}}{\pi_{imp}^{ag}}\right)\right)^{-\theta_c}}{\Phi_{cd}^{imp}} (18)
\]

Hence the expression for the conditional probability, \(\pi_{csd}\) in equation (18) indicates that the group index \(ag\) does not play a role. Therefore, we can first sum import demand for the four groups of agents and then allocate total import demand across different sourcing countries. Substituting equation (18) into equation (16) thus leads to an expression for imports from country \(s\), \(q_{csd}\):
\[
q_{csd} = \frac{\left(\frac{\pi_{csd}^{ag}}{\pi_{imp}^{ag}}\right)^{-\theta_c}}{\Phi_{cd}^{imp}} q_{cd}^{imp} (19)
\]

With \(\Phi_{cd}^{imp}\) being a function of the cost of production from different sources:
\[
\Phi_{cd}^{imp} = \sum_{r} \left(\frac{\left(\frac{\pi_{csd}^{ag}}{\pi_{imp}^{ag}}\right)^{-\theta_c}}{\Phi_{cd}^{imp}} q_{cd}^{imp}\right) (20)
\]

Note that \(q_{cd}^{imp}\) is defined in equation (17) and \(q_{cd}^{imp,ag}\) can further be elaborated as follows:
\[
q_{cd}^{imp,ag} = \left(\frac{\pi_{csd}^{ag}}{\pi_{imp}^{ag}}\right)^{-\theta_c} \Phi_{cd}^{imp} q_{cd}^{imp} (21)
\]

Similar to import demand, we can define demand for domestic goods as:
\[
q_{cd}^{dom,ag} = \sum_{ag \in \{pr,go,fi,ln\}} q_{cd}^{dom,ag} (22)
\]

With \(q_{cd}^{dom,ag}\) equal to:
\[
q_{cd}^{dom,ag} = \pi_{cd}^{dom,ag} q_{cd}^{ag} = \left(\frac{\pi_{cd}^{dom,ag}}{\pi_{cd}^{ag}}\right)^{-\theta_c} \Phi_{cd}^{imp} q_{cd}^{imp} + \left(\frac{\pi_{cd}^{dom,ag}}{\pi_{cd}^{ag}}\right)^{-\theta_c} q_{cd}^{ag} (23)
\]
2.4 Price index

To obtain the import price index for each commodity \( c \), we integrate over the price distribution for each of the four groups of agents, \( a_g = pr, go, fi, in: \)

\[
p_{ag}^{cd} = A_c \left( \left( \frac{l_{cd}^{imp,ag}}{\tau_{cd}^{imp,ag}} \right)^{\theta_c} \Phi_{cd}^{imp} + \left( \frac{l_{cd}^{dom,ag} c_{cd}}{\lambda_{cd}} \right)^{-\theta_c} \right) - \frac{1}{\sigma_c} \tag{24}
\]

With \( A_c = \left( \Gamma \left( \frac{\theta_c - c - 1}{\theta_c} \right) \right)^{\frac{1}{\sigma_c}}. \)

The fact that preferences are homothetic and the expression for \( p_{ag}^{cd} \) in equation (24) is a proper price index, we can write expenditure \( x_{ag}^{cd} \) as price times quantity, which will prove useful below:

\[
x_{ag}^{cd} = p_{ag}^{cd} q_{ag}^{cd} \tag{25}
\]

We do change the expressions for import shares used in the definitions of import price indices. As mentioned above, sectoral prices for imports do not vary by origin in the Eaton-Kortum model. Therefore we have to use quantity shares instead of value shares. This also follows from hat differentiating equation (24). The share coefficients then correspond with quantity import and domestic shares respectively in equations (21) and (23). The same logic holds for the expression for the aggregate import price, \( \Phi_{cd}^{imp} \), in equation (20).

2.5 Goods market equilibrium

In the standard GTAP model based on the Armington specification, the goods market equilibrium is formulated based on quantities with identical goods sold to different destinations. This implies that the pre-export tax prices are identical for all destinations, whereas the post-export tax prices vary depending on destination-specific export taxes. In the Eaton-Kortum model, the landed prices are identical for all sources—i.e., prices in destination markets are source-independent. From the perspective of an exporting country, the price of goods sold to different destinations is different. Therefore, we have to reformulate the goods market equilibrium in the GTAP model using values instead of quantities.

In particular, we equalize the value of gross output \( x_{cd}^{prod} \) with the value of import demand from the different trading partners. Since we need the value of import demand net of export and import taxes and payments to the transport sector, we divide the import value by the power of the different taxes and subtract payments to the transport sector. The import value for the different groups of agents \( a_g \) is equal to the total value of demand in destination country \( d \), \( x_{cd}^{imp} \), times the probability that goods are imported from country \( s \), \( \pi_{csd}^{ag} \). In sectors producing goods used for global transport services, we also add the value of transport services exported,
Therefore, we have the following equilibrium condition:

\[
x_{cs}^{prod} = \sum_{ag \in \{pr, go, fi, in\}} \frac{\pi_{cs}^{ag} x_{cs}^{ag} \text{pop}_{pr}}{\pi_{cs}^{ag} x_{cs}^{ag} \text{pop}_{pr}} + \sum_{d} \left( \pi_{cd}^{ag} x_{cd}^{ag} \text{pop}_{imp} - \frac{p_{tr}^{ag} x_{cd}^{ag} \text{pop}_{exp}}{t_{cd}^{ag} t_{csd}^{ag}} \right) + c_{cs} t_{csd}^{m} \tag{26}
\]

To rewrite equation (26), we substitute equations (15), (18) and (25) and the fact that \(x_{cs}^{prod} = c_{cs} t_{cs}^{prod} q_{cs}^{prod}\) and \(q_{so}^{ag} = \pi_{so}^{ag} q_{so}^{ag} \text{pop}_{so} \). The first equation expresses that the value of gross output is equal to the price of gross output \(c_{cs}\) times the quantity of gross output \(q_{cs}^{prod}\), taking into account the production tax \(t_{cs}^{prod}\). The second equation follows from equation (23). After some steps we get the following equilibrium condition:

\[
c_{cs} t_{cs}^{prod} q_{cs}^{prod} = \sum_{ag \in \{p, g, f, i\}} p_{cs}^{ag} q_{cs}^{ag} t_{csd}^{ag} + c_{cs} t_{csd}^{m} + \sum_{d} \left( \frac{c_{cs} t_{cd}^{ag} + \gamma_{csd} t_{csd}^{ag}}{\lambda_{csd}} \right)^{-\theta_{cs}} p_{cd}^{ag} q_{cd}^{ag} t_{cd}^{ag} - \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{cd}^{ag} q_{cd}^{ag} t_{cd}^{ag}}{t_{cd}^{ag}} \right)
\tag{27}
\]

### 2.6 Tax revenues

To calculate import and export tax revenues, we have to take into account that the distribution of prices is independent of the country of origin. This implies that the sectoral landed price for agent group \(ag\) in destination \(d\) is independent of the source country \(s\). In other words, the expression for \(p_{cd}^{ag}\) in (24) gives the landed price at the sectoral level for goods from any origin country. This implies that we start from the value of trade and prices inclusive of all taxes and divide by the various taxes to get to the appropriate base value on which the tax is applied. We start with revenues from the group \(ag = pr, go, fi, in\) and source \(so = dom, imp\) specific import tariffs, defined as \(t_{csd}^{ag}\):

\[
tr_{cd}^{so,ag} = \frac{t_{cd}^{so,ag} - 1}{t_{cd}^{so,ag}} \pi_{cd}^{so,ag} x_{cd}^{so,ag} \text{pop}_{so} = \frac{t_{cd}^{so,ag} - 1}{t_{cd}^{so,ag}} \pi_{cd}^{so,ag} x_{cd}^{so,ag} \text{pop}_{so} \tag{28}
\]

Tariff revenues on purchases by group \(ag\) on goods sourced from origin \(so\), \(tr_{cd}^{imp}\), are equal to the tariff rate \((t_{cd}^{so,ag} - 1)\) times total expenditures by group \(ag\) times the probability that goods are bought from source \(so\). We divide by the power of the tariff rate \(t_{cd}^{so,ag}\) to get the value traded net of tariffs. In the second equality we use equation (25) and \(q_{cd}^{so,ag} = \pi_{cd}^{so,ag} q_{cd}^{ag} \text{pop}_{so}\).

To calculate tariff revenues \(tr_{csd}^{imp}\) on imports from country \(s\), we multiply the
value of trade in cif-terms by the tariff rate \( t_{csd}^{imp} - 1 \) and sum over the three groups of agents:

\[
tr_{csd}^{imp} = \sum_{ag \in \{pr, go, fi, in\}} \frac{ \left( t_{csd}^{imp} - 1 \right) \pi^{ag, imp}_{csd} x_{ag}^{cd} }{ t_{csd}^{imp} \tau_{cd}^{csd} } \tag{29}
\]

Applying equations (15), (18) and (25) leads to the following expression for import tariff revenues:

\[
tr_{csd}^{imp} = \frac{ t_{csd}^{imp} - 1 }{ t_{csd}^{imp} } \left( \frac{ \left( c_{cs}^{exp} t_{csd}^{exp} + \gamma_{csd} p_{tr}^{csd} \right) t_{csd}^{imp} }{ \lambda_{cs} } \right)^{-\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{ p_{cd}^{ag, imp, ag} q_{cd}^{imp, ag} }{ t_{csd}^{imp} t_{cd}^{ag, imp} } \tag{30}
\]

It is not possible to further elaborate equation (30) into an expression based on \( q_{cd}^{imp} \). The reason is that we cannot split up the terms \( p_{cd}^{ag, imp, ag} q_{cd}^{imp, ag} \) in the summation. This impossibility also implies that we cannot derive a price index corresponding to \( q_{cd}^{imp} \).

Finally, we discuss calculation of export tax revenues, \( tr_{csd}^{exp} \). To get the base value for the export tax we need the fob-value of trade which follows from subtracting the transport value from the cif-value as defined in equation (30). Multiplying by the export tax rate divided by the power of the export tax, we get for \( tr_{csd}^{exp} \):

\[
tr_{csd}^{exp} = \frac{ t_{csd}^{exp} - 1 }{ t_{csd}^{exp} } \left( \frac{ \left( c_{cs}^{exp} t_{csd}^{exp} + \gamma_{csd} p_{tr}^{csd} \right) t_{csd}^{imp} }{ \lambda_{cs} } \right)^{-\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{ p_{cd}^{ag, imp, ag} q_{cd}^{imp, ag} }{ t_{csd}^{imp} t_{cd}^{ag, imp} } - p_{csd}^{fs} s_{csd} \right) \tag{31}
\]

3. Simulations: Comparing GTAP model with Armington and Eaton-Kortum

To explore the differences between the Eaton-Kortum model and Armington model, we conduct a series of experiments, calibrating both models to the same baseline data and behavioral parameters. The GTAP Data Base, Version 10, is aggregated to 10 regions, 10 sectors, and 5 factors of production. To make the two models comparable we set the substitution elasticity between imports and domestic goods equal to the substitution elasticity between imports from different source countries in the GTAP model with the Armington specification. Furthermore, we calibrate the model to the same trade elasticity in the estimated gravity model, implying that \( \theta_c = \zeta_c - 1 \) with \( \zeta_c \) the substitution elasticity between goods from different source countries. We conduct four sets of experiments:

1) Global tariff liberalization: Eliminate tariffs in all regions (i.e., set tariffs to zero).
2) Global iceberg trade cost reduction: 5% decrease in iceberg trade costs in all regions.

3) Global export tax liberalization: Eliminate export taxes and subsidies (i.e., set them to zero).

4) Unilateral tariff increases: Ten experiments with each region increasing (the power of) tariffs by 10% vis-a-vis other regions.

We conduct policy experiments and focus on three different trade policy instruments (tariffs, export taxes, and iceberg trade costs) to compare results between the GTAP model with Eaton-Kortum and Armington specifications. We also examine the impact on real income, trade volumes and trade values, real GDP, the components of real GDP, and terms of trade. The fourth experiment is included to explore how terms of trade effects of tariffs differ between the two models and to test the hypothesis that the terms of trade gains of raising tariffs are larger in the Eaton-Kortum specification, because pre-tariff prices can be driven down more since the tariff-inclusive price changes less and exporters thus pay a larger part of tariff increases.

Before exploring the simulation results, we first discuss the numerical solution of the model. Since the Eaton-Kortum code is highly non-linear a large number of steps is required in GEMPACK to get an accurate solution and make Walras slack marginal in the model. More specifically, we solve the model with Euler 100-300-500 steps. However, we also show that solutions are virtually identical with a smaller number of steps in the experiments conducted. This implies that in experiments with a larger number of countries and/or sectors the model can be solved with a smaller number of steps, for example Euler 9-11-13 steps.

Figure 1 displays the percentage change in real income (the variable u corresponding to the percentage change in utility of the regional household) in the 10 regions for the four experiments. The figure makes clear that the difference in real income effects is very small, as expected from the theory.

Figure 2 displays changes in export volumes by region. We see that changes in trade volumes are smaller in the Eaton-Kortum model than in the Armington model. The reason is straightforward: the elasticity of trade volumes with respect to trade costs is smaller in the Eaton-Kortum model, \( \theta_c = \zeta_c - 1 \), compared to \( \zeta_c \) in the Armington model. This difference is based on the way we compare the two models and a feature of the Eaton-Kortum model. First, the trade elasticity, defined as the elasticity of the value of trade with respect to iceberg trade costs, is calibrated to be identical in the two models since this is what is empirically identifiable based on gravity estimation. Second, in the Eaton-Kortum model the elasticity of the volume and value of trade with respect to iceberg trade costs are identical, as discussed in the theoretical section, because the price of imports is identical across all sourcing regions. These two facts imply that trade volumes are less responsive to changes in trade costs in the Eaton-Kortum model.

Figure 3 shows the percentage change in real GDP (the variable qgd) in the two
Figure 1. Projected per cent changes in real income under four scenarios

Notes: The figure displays the per cent change in the variable $u$ in response to the four experiments: (i) global elimination of tariffs; (ii) global elimination of export taxes/subsidies; (iii) global reduction in iceberg trade costs by 5%; (iv) unilateral increase in the power of tariffs by 10% in one country vis-a-vis all its trading partners. The variable $u$ measures real income. The per cent change in real income can also be interpreted as the per cent change in equivalent variation.

Source: Simulations with GTAPV7 and GTAPV7-EK
Figure 2. Projected per cent changes in real exports under four scenarios

Notes: The figure displays the per cent change in the variable $q_{xwreg}$ in response to the four experiments: (i) global elimination of tariffs; (ii) global elimination of export taxes/subsidies; (iii) global reduction in iceberg trade costs by 5%; (iv) unilateral increase in the power of tariffs by 10% in one country vis-a-vis all its trading partners. The variable $q_{xwreg}$ is a measure for regional real exports, calculated as the trade value weighted average of bilateral exports by sector.

Source: Simulations with GTAPV7 and GTAPV7-EK
Figure 3. Projected per cent changes in real GDP under four scenarios

Notes: The figure displays the per cent change in the variable $q_{gdp}$ in response to the four experiments: (i) global elimination of tariffs; (ii) global elimination of export taxes/subsidies; (iii) global reduction in iceberg trade costs by 5%; (iv) unilateral increase in the power of tariffs by 10% in one country vis-a-vis all its trading partners. The variable $q_{gdp}$ is a measure for real GDP, defined as the sum of the nominal value consumption plus investment, plus government expenditures plus exports minus imports, divided by a GDP price index, which is a value weighted average of the prices of each of the components of GDP.

Source: Simulations with GTAPV7 and GTAPV7-EK
Figure 4. Projected per cent changes in the terms of trade, aggregate import prices, and aggregate export prices under unilateral tariff liberalization.

Notes: The figure displays the per cent change in the variables $tot$, $pdw$, and $psw$ respectively in the three panels from top to bottom in response to the fourth experiment: unilateral increase in the power of tariffs by 10% in one country vis-a-vis all its trading partners.

Source: Simulations with GTAPV7 and GTAPV7EK
models. The change in GDP differs more between the two models across the different regions, because the change in real exports and real imports differs between the two models.

Finally, we evaluate the changes in the terms of trade in the fourth scenario, testing the hypothesis that the terms of trade gains of raising tariffs are larger in the Eaton-Kortum than in the Armington model. In the Eaton-Kortum model landed prices are identical regardless of the country of origin. Because of this feature, an increase in tariffs on imports will not increase landed (tariff-inclusive) prices as much as in the Armington model and so most of the adjustment will happen on the exporting side. Basically, higher tariffs imply that a share of firms will stop importing. This extensive margin adjustment will imply that the highest cost varieties will drop out thus reducing the pre-tariff import price. Because of the extensive margin adjustment the pre-tariff price will fall more in the Eaton-Kortum model than in the Armington model.

However, this does not necessarily mean that raising tariffs generates larger terms of trade gains in a general equilibrium model in Eaton-Kortum than in the Armington specification, because the imposition of tariffs also implies that the price level in the importing country increases leading to higher export prices. However, the increase in export prices is also smaller in the Eaton-Kortum than in the Armington specification because landed prices are identical for all sourcing countries. Therefore, the projected change in the terms of trade can be either smaller or larger in Eaton-Kortum compared to the Armington specification depending on whether the difference in the changes in import or export prices is bigger.

We illustrate the above by evaluating the changes in terms of trade for the fourth experiment, i.e. 10 experiments increasing the power of tariffs in each of the regions by 10 per cent vis-a-vis all other regions. Figure 4 shows that in some regions the terms of trade improvement in the Armington model is larger, such as in SouthAsia, NAmerica, and LatinAmer, whereas in other regions the terms of trade improvement is larger in the Eaton-Kortum model, such as Oceania, EastAsia, and SeaAsia. However, we observe that the reduction in (pre-tariff) import prices is larger in Eaton-Kortum than in the Armington specification, while at the same time the increase in export prices is smaller in the Eaton-Kortum specification. In regions where the terms of trade improvement is larger in the Eaton-Kortum specification, such as Oceania, the larger import price reduction dominates the smaller export price increase.

4. Concluding remarks

In this paper we have modified the trade structure of the GTAP model, changing it from one with an Armington structure with love of variety by country of origin to one with an Eaton-Kortum structure with comparative advantage differences within sectors based on a stochastic distribution of productivities.

An important feature of the Eaton-Kortum specification is that prices from each
origin country within a sector are identical, implying that both the value and volume share of trade from a source country are equal to the probability that a country sources goods from the source country. This probability, equal to the import share, has the same shape as in the Armington framework.

Because of the property that import prices are identical for each source country, incorporating the Eaton-Kortum framework in the GTAP-model is a complicated undertaking. A distinction has to be made between bilateral costs and bilateral prices with the former determined by the costs of production and tariffs and the latter determined by average price levels. Hence, the code is structured such that first the price index is determined based on costs and then tariffs, transportation margins, and export taxes are subtracted to calculate bilateral prices.

The simulations generate three main insights. First, the real income effects are virtually identical in the Eaton-Kortum and Armington versions of the GTAP model. Second, changes in the volume of trade are smaller in response to trade cost changes in the Eaton-Kortum than in the Armington specification, whereas changes in the value of trade are (virtually) identical. Third, the terms of trade gains of imposing tariffs differ with with pre-tariff import prices driven down more in the Eaton-Kortum specification with export prices also rising less. This implies that terms of trade effects are larger under Eaton-Kortum for some countries and smaller for other countries, when compared to the terms of trade effects in the Armington specification.

The work in this paper can be extended in at least three directions. First, projections of the impact of counterfactual experiments on volumes of trade and trade prices can be compared with empirical estimates of this response in the data. This might give insight into the question as to whether the Armington structure might be a better description of actual patterns of trade than the Eaton-Kortum structure in some sectors whereas it could be opposite in other sectors. Also other models such as the Ethier-Krugman and the Melitz model could be included in the comparison exercise. (See for example Bekkers and Francois (2018) for Ethier-Krugman-Melitz variations within the GTAP framework.) Second and more specifically, a test of the validity of the Eaton-Kortum model can be conducted based on its prediction that tariff-inclusive bilateral import prices are identical across all source countries. This prediction can be tested using estimates on the response of bilateral trade prices to changes in trade costs.\footnote{Depending on the outcome of such specification tests, one could imagine implementing nested versions of Armington, Eaton-Kortum, and Ethier-Krugman-Melitz, with different specifications deemed appropriate for different sectors.} Third, the Eaton-Kortum model can be employed in recursive-dynamic applications to evaluate whether long-run projections are different in the Armington and Eaton-Kortum frameworks. Because of the structural interpretation of the technology parameter in the Eaton-Kortum model, the framework can be extended with endogenous changes in technology, as
is done in the work by Góes and Bekkers (2023) on decoupling.

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References


5. Appendix: Implementation in GEMPACK

To convert the GTAP model code based on nested Armington preferences to Eaton-Kortum, we have to make five sets of changes. All these changes are related to the fact that sectoral prices do not vary by origin in the Eaton-Kortum model. That is, the average price for a destination country at the sector level is the same for goods from any country of origin. The reason is that a country of origin displaying higher trade costs will export fewer varieties to a specific country of destination. As a result, the increase in the average bilateral sector price due to higher costs if offset by the reduction in the average bilateral sector price because the country would export fewer varieties for which it has a low productivity.

The five sets of changes are mapped out in the next five subsections. We start with the change in trade elasticities, followed by the change in the expressions for import demand and price indices. Then we describe changes in the expressions for tax revenues. Next we point out changes in the goods market equilibrium condition. Other changes in the model code (calculating the terms of trade, trade components of GDP, and trade indices) are presented in 5.6. In the model code, we continue to use variables like $p_{ci}$ and $p_{fob}$, but these variables cannot be interpreted as prices in the model. Instead they are a measure of costs, used to determine the quantity imported and the cost in an importing country. We use the usual conventions for notation in the GEMPACK code, i.e. subscripts $c$ and $a$ are used to indicate commodities and activities (or generally, sectors) and $s$ and $d$ to indicate source and destination countries. Variable $x(c,a,s,d)$ indicates flows from sector $c$ to $a$ from country $s$ to $d$.

5.1 Values and update statements

We first discuss the various update statements before turning to the expressions for import demand, tax revenues and goods market equilibrium in the model code. Given the same pattern of GTAP model code for various agents, we only discuss in this paper the changes made to household import and domestic flows at basic and purchaser’s prices and their associated equations.11 Both VMPP and VMPB flows are still used to calculate tax revenues and are updated based on values—i.e., $V=P^*Q$—as in the Armington specification.

### Listing 1. Value of household expenditure and update statements

```plaintext
Variable (all,c,COMM) (all,r,REG) # private consumption price for commodity c in region r #;
Variable (all,c,COMM) (all,r,REG)
```

11 In the GTAP model code, P denotes private household, G for government, I for Investment, and F for firms, with sources identified as D for domestic and M for imported. For example, the value flows for government domestic and imported expenditures at basic prices are $VDGB$, $VMGB$ with corresponding price variables $pgd$, $pgm$ and quantity variables $qgd$ and $qgm$. 
Variable (all,c,COMM)(all,r,REG)
qpm(c,r)  # private household demand for imported commodity c in region r #;

Variable (all,c,COMM)(all,r,REG)
tpd(c,r)  # power of tax on domestic c purchased by private hhld in r #;

Variable (all,c,COMM)(all,r,REG)
tpm(c,r)  # power of tax on imported c purchased by private hhld in r #;

!< gtapv7-ek: define domestic and import price variables >!

Variable (orig_level=1.0)(all,c,COMM)(all,r,REG)
ppdek(c,r)  # price of domestic c purchased by household in r, net of tax #;

Variable (orig_level=1.0)(all,c,COMM)(all,r,REG)
ppmek(c,r)  # price of imported c purchased by household in r, net of tax #;

Coefficient (ge 0)(all,c,COMM)(all,r,REG)
VMPP(c,r)  # private hhld expenditure on imp. c in r at producer prices #;

Read
VMPP from file GTAPDATA header "VMPP";

!< gtapv7-ek: Modify update statement by changing price from ppm to ppa >!
Update (all,c,COMM)(all,r,REG)
VMPP(c,r) = ppa(c,r) * qpm(c,r);

Coefficient (ge 0)(all,c,COMM)(all,r,REG)
VMPB(c,r)  # private household expenditure on imp. c in r at basic prices #;

Read
VMPB from file GTAPDATA header "VMPB";

!< gtapv7-ek: Modify update statement from pms to ppmek >!
Update (all,c,COMM)(all,r,REG)
VMPB(c,r) = ppmek(c,r) * qpm(c,r);

!< Expenditures at producer prices have a uniform price in the EK-model >!
Coefficient (ge 0)(all,c,COMM)(all,r,REG)
VMPPEK(c,r)  # private hhld expenditure on domestic c in r at purchaser’s prices, EK #;

!< Update based on quantity shares >!
Formula (initial)(all,c,COMM)(all,r,REG)
VMPPEK(c,r) = VMPP(c,r);

Update (all,c,COMM)(all,r,REG)
VMPPEK(c,r) = qpm(c,r);

Coefficient (ge 0)(all,c,COMM)(all,r,REG)
VMPBEK(c,r)  # prv hhld expenditure on imported c in r at basic prices, EK #;

Formula (initial)(all,c,COMM)(all,r,REG)
VMPBEK(c,r) = VMPB(c,r);

Update (all,c,COMM)(all,r,REG)
VMPBEK(c,r) = qpm(c,r);

Equation E_ppmek
# EK household consumption prices for imported com. c, net of tax #
(all,c,COMM)(all,r,REG)
ppmek(c,r) = ppa(c,r) - tpm(c,r);

Equation E_ppdek
# EK household consumption prices for domestic com. c, net of tax #
(all,c,COMM)(all,r,REG)
ppdek(c,r) = ppa(c,r) - tpd(c,r);

For the Eaton-Kortum specification, the update statements for values at purchaser’s price, VMPP and VDPP, use the composite price, ppa, for both imported and domestic goods multiplied by their appropriate quantity variables qpm and qpd. The update statements for values at basic prices, VMPB and VDPB, use a new variable ppmek and ppdek which are calculated based on the household composite
price, \( ppa \), diminished by the power of tax \((1+\text{rate})\) for imported \((tpm)\) and domestic \((tpd)\) goods. We also introduce new coefficients in the Eaton-Kortum specification \((VMPPEK, VMPBEK, VDPPEK, VDPBEK)\), for use in the market clearing equation.

5.2 Trade elasticities

In this section, we discuss changes to \( ESUBD(c) \) and \( ESUBM(c) \) parameters which are used in the import demand equation. In the Eaton-Kortum model the coefficient in import demand on prices is the dispersion parameter \( \theta_c \) (omitting the country subscript). We henceforth change each agent’s domestic-import demand equations from \( ESUBD(c) \) to \( THETA(c) \) and also in the import demand equation \( E_{qxs} \) from \( ESUBM(c) \) to \( THETA(c) \).

To calibrate \( THETA \) we can use gravity estimates on the tariff elasticity from an external source or the estimated elasticities in the GTAP Data Base. In the gravity equation based on the Armington model the substitution elasticity, \( \sigma_c \), is equal to one plus the tariff elasticity \( \eta_{tar}^c \), \( \sigma_c = 1 + \eta_{tar}^c \), if the gravity equation is estimated using tariff-inclusive values. In the Eaton-Kortum model the tariff elasticity (also estimated based on tariff-inclusive values) is equal to the dispersion parameter, so we have \( \theta_c = \eta_{tar}^c \):  

\[
\begin{align*}
T(c,r) & = ESUBM(c,r) - 1; \\
\end{align*}
\]

Effectively, this means that the trade elasticity parameter changes from \( ESUBM \) into \( THETA = ESUBM - 1 \).

<table>
<thead>
<tr>
<th>Coefficient (parameter) (all, c, COMM) (all, r, REG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>THETA(c,r)</td>
</tr>
<tr>
<td># region-specific dispersion parameter of Frechet distr. of productivity #;</td>
</tr>
<tr>
<td>Formula (initial) (all, c, COMM) (all, r, REG)</td>
</tr>
<tr>
<td>THETA(c,r) = ESUBM(c,r) - 1;</td>
</tr>
</tbody>
</table>

5.3 Import demand and price index

We also change the expressions for each agent’s import demand and price index. The original model is characterized by a two-level nested structure of import demand—i.e., second level CES function for commodity sourcing of imports by

\[
\begin{align*}
T(c,r) & = ESUBM(c,r) - 1; \\
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

In the Eaton-Kortum model the import share in quantities and values is equal, implying an equal coefficient on prices and tariffs in respectively the quantity and value (gravity) equation.
region of origin and top-level CES function for domestic and imported demand for the four groups of end users: firms, private households, the government and investors. Although in the new model we work with identical trade elasticities between domestic and imported and between imported goods from different sources, we stick with the nested structure of import demand, adding up import demand by the four groups of end users to determine total import demand. The reason is that import demand shares vary across the groups of end users, whereas import shares from different exporters are identical across end users.

We introduce five equations in this section: (1) bilateral import demand; (2) aggregate import demand for the different end users; (3) importer cost; (4) aggregate price index for the different end users; and (5) total domestic demand and total import demand. Since expressions for each of the end users are identical, we only show the model code for one of the end users: private households. We will also do so in the remainder of the text.

The expression for bilateral import demand is given in equation (19). To turn this equation into GEMPACK-code we first define the bilateral cost, $c_{csd}$ and the aggregate import cost, $c_{imp}^{cd}$. Substituting the expression for $\Phi_{cd}^{imp}$ in equation (20) gives:

$$ c_{csd} = \left( \frac{t_{csd}^{prod} e_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{\lambda_{csd}} \right)^{\frac{1}{\gamma_{csd}}} $$

$$ c_{imp}^{cd} = \left( \frac{\Phi_{cd}^{imp}}{\lambda_{csd}^{\gamma_{csd}}} - 1 \right) $$

Note our use of the term bilateral cost and aggregate import cost, as they denoted costs and not prices. As discussed earlier, import prices are not varying by country of origin with the bilateral price equal to the aggregate price index. Therefore, we refer to entities defined in equations (A.2)-(A.3) not as prices but as costs in the Eaton-Kortum specification.

Equation (19) can now be written as:

$$ q_{csd} = \left( \frac{t_{csd} e_{csd}}{c_{cd}^{imp}} \right)^{-\theta_c} q_{cd}^{imp} $$

Writing this equation in relative changes leads to the expression for $q_{xs}$ which is similar to the expression in the Armington specification except for a different trade elasticity, $THETA (c)$. In the code we stick to the use of the variable names of the Armington specification, although $pmds$ and $pms$ should now be seen as costs (i.e., not prices) in the Eaton-Kortum specification.

We now turn to domestic and import demand for the different end users. We focus on import demand since the equation for domestic private consumption demand is identical. Equation (21) can be rewritten as follows, substituting equations...
(24) and (A.3):
\[
q_{imp,ag}^{cd} = \left( A_c \frac{\tau_{cd}}{\tau_{cd} p_{cd}^{ag}} \right)^{-\theta_c} q_{cd}^{ag} \tag{A.5}
\]

Hat differentiating equation (A.5) results in the following expression for private import demand, \(q_{pm}\) as a function of aggregate private demand \(q_{pa}\), the price of private consumption, \(ppa\), and the consumption tax inclusive cost of import demand, \(ppm = pms + tpm\):

Listing 3. Aggregate Import and private household demand

```
Equation E_qxs
# regional demand for disaggregated imported commodities by source #
(all, c, COMM) (all, s, REG) (all, d, REG)
qxs(c,s,d) = -ams(c,s,d) + qms(c,d) - THETA(c,d) * [pmds(c,s,d) - ams(c,s,d) - pms(c,d)];
```

```
Equation E_qpd
# private consumption demand for domestic goods #
(all, c, COMM) (all, r, REG)
qpd(c,r) = qpa(c,r) - THETA(c,r) * [ppd(c,r) - ppa(c,r)];
```

```
Equation E_qpm
# private consumption demand for aggregate imports #
(all, c, COMM) (all, r, REG)
qpm(c,r) = qpa(c,r) - THETA(c,r) * [ppm(c,r) - ppa(c,r)];
```

Third, the importer cost level is defined in equation (A.3). Hat differentiating this expression gives:

\[
\overline{c}_{cd}^{imp} = \frac{\left( c_{cd} \tau_{cd}^{imp} \right)^{-\theta_c}}{\sum_u \left( c_{usi} \tau_{usi}^{imp} \right)^{-\theta_c}} \left( \overline{c}_{cd}^{imp} + \overline{\tau}_{cd}^{imp} \right) = \frac{q_{cd}^{imp}}{q_{cd}^{ag}} \left( \overline{c}_{cd}^{imp} + \overline{\tau}_{cd}^{imp} \right) \tag{A.6}
\]

The second step follows from equation (19). Hence, the relative change in the importer price level is a weighted average of the relative changes in bilateral prices with as weights quantity shares. This comes from the quantity share and value share being identical, because the price is not varying by country of origin in the Eaton-Kortum specification.

Equation (A.6) is written like in the Armington specification in the code. However, the calculation for import share sourced from region \(r\), MSHRS, will have to be modified since quantity shares are employed.

Listing 4. Import cost equations

```
Coefficient (parameter) (all, c, COMM) (all, s, REG) (all, d, REG)
VMSBEK(c,s,d)
# initial value of imports of c from s to d at domestic (basic) prices #;
Formula (initial) (all, c, COMM) (all, s, REG) (all, d, REG)
VMSBK(c,s,d) = VMSB(c,s,d);
```
As shown in the codes above, the share MSHRS is initialized using the value share in the data. This is consistent with the Eaton-Kortum specification, since value shares are equal to quantity shares. The share is then updated using quantity shares with qmsn defined as the negative of qms.

We write the expression for the aggregate price index of end user ag in equation (24) in relative changes, substituting the expression for $\Phi_{imp}^{cd}$ from equation (A.3):

$$\hat{p}_{cd}^{ag} = \left( \frac{d_{imp}^{ag} - d_{cd}}{d_{cd}} \right)^{-\theta_c} \left( \frac{c_{imp}^{ag} - c_{cd}}{c_{cd}} \right)^{-\theta_c} \left( \frac{d_{dom}^{ag} - d_{cd}}{d_{cd}} \right)^{-\theta_c} \left( \frac{c_{dom}^{ag} - c_{cd}}{c_{cd}} \right)^{-\theta_c}$$

$$= \frac{q_{imp}^{ag} - q_{cd}}{q_{cd}} \left( \frac{c_{imp}^{ag} - c_{cd}}{c_{cd}} \right)^{-\theta_c} + \frac{q_{dom}^{ag} - q_{cd}}{q_{cd}} \left( \frac{c_{dom}^{ag} - c_{cd}}{c_{cd}} \right)^{-\theta_c} \lambda_{cd}$$

The second line follows from the expressions for $d_{cd}^{imp}$ and $d_{cd}^{dom}$ in equations (21) and (23) and written in the code as in the Armington specification. However, the expression for the import share, PMSHR, should be changed since quantity shares are used.

**Listing 5. Household composite prices**

```plaintext
Zerodivide default RNREG;
!< Composite Tradeables >!
Coefficient (all,c,COMM)(all,s,REG)(all,d,REG)

MSHR(c,s,d) # share of imports from s in imp. bill of r at basic prices #;
Formula (initial)(all,c,COMM)(all,s,REG)(all,d,REG)

MSHR(c,s,d) = VMSBEK(c,s,d) / sum{ss,REG, VMSBEK(c,ss,d)};
Update (all,c,COMM)(all,s,REG)(all,d,REG)

MSHR(c,s,d) = qxs(c,s,d) * ams(c,s,d) * qmsn(c,d);
Update (explicit)(all,c,COMM)(all,s,REG)(all,d,REG)

VMSB(c,s,d) = MSHRS(c,s,d) * VMB(c,d);

Equation E_pms
# price for aggregate imports #
(aall,c,COMM)(all,d,REG)

pms(c,d) = sum{s,REG, MSHRS(c,s,d) * [pmds(c,s,d) - ams(c,s,d)]};

Equation E_qmsn
# negative of aggregate imports of c in region r, basic price weights #
(aall,c,COMM)(all,r,REG)

qmsn(c,r) = -1 * [qms(c,r)];
```
VMPPEK is updated only with the change in the corresponding quantity as discussed in Subsection 5.1. For the share expressions in quantities such as VMPPEK we could have continued to use value shares, because they are based on prices which are not source-specific, for example VMPP = PPA * QPM. We have chosen not to do so, because the quantity shares are also defined in goods market equilibrium below.

Finally, we also discuss implementation in the code of the expression of total import demand in equation (17), omitting the expression for total domestic demand which is completely analogue. Hat differentiating equation (17) gives:

\[
\frac{\dot{q}_{cd}^{\text{imp}}}{q_{cd}^{\text{imp}}} = \sum_{ag \in \{pr, go, fi, in\}} \frac{q_{cd}^{\text{imp},ag}}{q_{cd}^{\text{imp}}} - \frac{\dot{q}_{cd}^{\text{imp},ag}}{q_{cd}^{\text{imp}}} \tag{A.8}
\]

In the Armington specification, the volume share, \(q_{cd}^{\text{imp},ag}\), can be turned into a value share by multiplying the numerator and denominator by the same sales price of a uniform imported good. However, in the Eaton-Kortum specification, the sales price is different because goods imported by different end users are not uniform. Hence, the shares should be updated based on quantities and not values. Equation (A.8) is thus implemented as follows in the code:\(^{13}\)

Listing 6. Import market clearing

```plaintext
1 !< gtapv7-ek: calculate import shares using quantity-updated coefficients >!
2 Coefficient (all,c,COMM)(all,r,REG) VMBEK(c,r) # value of aggregate imports of commodity c in r #;
3 Formula (all,c,COMM)(all,r,REG) VMBEK(c,r) = sum(a,ACTS, VMFBEK(c,a,r)) + VMIBEK(c,r) + VMPBEK(c,r) + VMGBEK(c,r);
4 Coefficient (all,c,COMM)(all,a,ACTS)(all,r,REG) FMCSHREK(c,a,r) # share of import c used by act. a in r #;
5 Formula (all,c,COMM)(all,a,ACTS)(all,r,REG) FMCSHREK(c,a,r) = VMFBEK(c,a,r) / VMBEK(c,r);
6 Coefficient (all,c,COMM)(all,r,REG) PMCSHREK(c,r) # share of import c used by priv. hhlds in r #;
7 Formula (all,c,COMM)(all,r,REG) PMCSHREK(c,r) = VMPBEK(c,r) / VMBEK(c,r);
8 Coefficient (all,c,COMM)(all,r,REG) GMCSHREK(c,r) # share of import c by gov’t in r #;
9 Formula (all,c,COMM)(all,r,REG) GMCSHREK(c,r) = VMGBEK(c,r) / VMBEK(c,r);
10 Coefficient (all,c,COMM)(all,r,REG) IMCSHREK(c,r) # share of import c by investment in r #;
11 Formula (all,c,COMM)(all,r,REG) IMCSHREK(c,r) = VMIBEK(c,r) / VMBEK(c,r);
```

\(^{13}\) We have only included expressions for VMPBEK and PMSHREK, the expressions for the values and shares of the other end users are defined identically.
### 5.4 Tax revenues

We also change the expressions for (relative changes in) import and export tax revenues to total income. The reason is that the import price is independent of the origin country. This implies that we cannot for example calculate the cif-value (used as base value for import taxes) as import quantity times cif-price as in the code with the Armington specification. To calculate the cif-value, we have to use the landed price per agent $ag$ and divide by both $ag$-specific import taxes and normal import taxes. This proper cif-price is then multiplied by the quantity imported.

#### 5.4.1 Tax revenues in the GTAP model

Before discussing changes made to various tax calculations, we briefly discuss the tax revenue calculations in the model code which are specified based on marginal (a.k.a. ordinary change in GEMPACK) rather than relative changes in taxes since many taxes could be zero or negative in the case of subsidy. Relative changes cannot be calculated for variables with initial values equal to zero (division by zero) and to prevent percentage change variables from changing signs when for example taxes are turned to subsidies. Denoting tax revenues for tax $k$ by $T_k$, income by $Y$ and the ratio of tax revenues to income for tax $k$ by $R_k$, we have:

$$ R_k = \frac{T_k}{Y} \quad (A.9) $$

Differentiating equation (A.9) gives:

$$ dR_k = \frac{dT_k}{Y} - \frac{T_k}{Y^2} dY \quad (A.10) $$

Reorganizing equation (A.10) leads to:

$$ YdR_k + T_k y = dT_k \quad (A.11) $$

$y$ is the relative change in income, $y = \frac{dY}{Y}$. We use equation (A.11) to calculate the ratio of tax revenues to income for the different types of taxes. The tax revenue changes are added up to calculate the relative change in household income, as further elaborated in Corong et al. (2017). Table A1 provides an overview of the different types of taxes in the model for which calculations have been changed.
net of taxes we should use ppa specific power of taxes for domestic and imported prices. Hence to get to the value pds instead of ppa base. The reason is that we use the composite price for the other three agents. We use pdek made for private household tax revenues since the changes follow a similar pattern of change in tax revenues gives:

\[
d tr_{cr}^{so,ag} = \frac{ag_{cr}^{so,ag}}{t_{cr}^{so,ag}} \left( \frac{t_{cr}^{so,ag} - 1}{t_{cr}^{so,ag}} + \frac{p_{cr}^{ag} q_{cr}^{ag}}{t_{cr}^{so,ag}} + q_{cr}^{ag} \right)\]

\[
= \frac{ag_{cr}^{so,ag}}{t_{cr}^{so,ag}} \left( \frac{t_{cr}^{so,ag} - 1}{t_{cr}^{so,ag}} + \frac{p_{cr}^{ag} q_{cr}^{ag}}{t_{cr}^{so,ag}} + q_{cr}^{ag} \right)\]

\[
= \frac{ag_{cr}^{so,ag}}{t_{cr}^{so,ag}} + \left( \frac{ag_{cr}^{so,ag}}{p_{cr}^{ag} q_{cr}^{ag}} - \frac{ag_{cr}^{so,ag}}{t_{cr}^{so,ag}} \right) \left( \frac{ag_{cr}^{so,ag}}{t_{cr}^{so,ag}} + q_{cr}^{ag} \right)\]

(A.12)

We implement the code changes for each agent, but only discuss the changes made for private household tax revenues since the changes follow a similar pattern for the other three agents. We use ppdek = ppa – tpd and ppmek = ppa – tpm instead of pds and pms to calculate the change in the domestic and imported tax base. The reason is that we use the composite price ppa then net out the source-specific power of taxes for domestic and imported prices. Hence to get to the value net of taxes we should use ppa – tpm instead of pms.

Listing 7. Tax revenue to regional income

```plaintext
1  Equation E_del_taxrpc
2  # change in ratio of private consumption tax payments to regional income #
3  (all, r, REG)
4  100.0 * INCOME(r) * del_taxrpc(r) + TAXRPC(r) * y(r)
5      - sum(c, COMM,
6          VDPP(c, r) * tpd(c, r) + DITAX(c, r) * [ppdek(c, r) + qpd(c, r)])
7      + sum(c, COMM,
8          VMPP(c, r) * tpm(c, r) + MPTAX(c, r) * [ppmek(c, r) + qpm(c, r)]);
```
5.4.3 Import tariff revenues to regional income ratio

We now turn to the marginal change in import tax (tariff) revenues. Substituting equations (A.2)-(A.3), equation (30) for import tariff revenues can be rewritten as follows:

\[
\Delta t_{imp}^{csd} = \left( t_{imp}^{csd} - 1 \right) \left( \frac{c_{csd}^{imp} c_{cd}^{imp}}{c_{cd}^{imp}} \right)^{-\theta_c} \sum_{ag \in \{p, g, f\}} \frac{p_{ag}^{imp} q_{ag}^{imp}}{t_{imp}^{csd} t_{cd}^{imp}} (A.13)
\]

Totally differentiating equation (A.13) gives:
\[ dtr_{imp}^{csd} = \left( \frac{c^{csd}}{c^{cd}} \right)^{-\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag, imp}^{cd}}{t_{ag, imp}^{cd}} \]
\[ + \left( 1 - \frac{1}{t_{imp}^{csd}} \right) \left( \frac{c^{csd}}{c^{imp}} \right)^{-\theta_c} \sum_{ag \in \{p, s, f\}} \frac{p_{ag, imp}^{cd}}{t_{ag, imp}^{cd}} \]
\[ \times \left( -\theta_c \left( \frac{c^{csd}}{c^{cd}} - \frac{c^{imp}}{c^{cd}} \right) + \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag, imp}^{cd}}{t_{ag, imp}^{cd}} \right) \]
\[ + \sum_{ag' \in \{pr, go, fi, in\}} \frac{p_{ag, imp}^{cd}}{t_{ag, imp}^{cd}} \left( \frac{p_{ag}^{cd}}{t_{imp}^{cd} t_{cd}^{cd}} + q_{imp, ag}^{imp, ag} \right) \]

(A.14)

To write equation (A.14) into GEMPACK code we need to determine expressions for the coefficients in the three lines of the equation. The coefficient on the relative change in import tariffs, \( t_{imp}^{csd} \), in the first line of equation (A.14) is equal to the tariff-inclusive value of imports, \( VMSB \). Next, we observe that the coefficient in the second line of equation (A.14), is equal to the tariff inclusive value of trade, \( VMSB \), minus the cif-value of imports, \( VCIF \), also defined as the value of tariff revenues in the code, \( MTAX = VMSB - VCIF \). The share of the value imported by private household \( pr \) is given by the following expression:

\[ \frac{p_{pr, imp, pr}^{cd}}{t_{imp, pr}^{cd} t_{cd}^{cd}} = PMCSHR(c, d) = \frac{VMPB(c, d)}{VMB(c, d)} \]  

(A.15)

Finally, the relative changes in the third line of equation (A.14) are easily written in GEMPACK code as follows:

\[ \theta_c \left( \frac{c^{csd}}{c^{cd}} + \frac{c^{imp}}{c^{cd}} - \frac{c^{imp}}{c^{cd}} \right) = THETA(c) \ast \left[ pmds(c, s, d) - ams(c, s, d) - pms(c, d) \right] \]

(A.16)

And:

\[ \frac{p_{pr}^{cd}}{t_{imp, pr}^{cd} t_{cd}^{cd}} = ppmcif(c, s, d) \]  

(A.17)

We write equation (A.14) in the GTAP model code as:

Listing 8. Import tariff revenue to regional income

```plaintext
Equation E_del_taxrimp
# change in ratio of import tax payments to regional income #
(all, d, REG)
100.0 * INCOME(d) * del_taxrimp(d) + TAXRIMP(d) * y(d)
- sum(c, COMM, sum(s, REG, VMSB(c, s, d) * [tm(c, d) + tms(c, s, d)])
```
We now explain the various coefficients and variables defined in the new code, starting with the expression for the tariff-inclusive value of imports, $V_{MBS}$. The coefficient in the first line of equation (A.14) shows that the tariff inclusive value of imports, $V_{MBS}$, is equal to the bilateral import share, $MSHRS$, times the value of imports, $V_{MB}$:

$$V_{MBS}(c,s,d) = MSHRS(c,s,d) \times V_{MB}(c,d)$$  \hfill (A.18)

The value of imports at basic (i.e., tariff-inclusive) prices consists of the sum of the value of imports by the four end users as in the code based on the Armington specification. The bilateral share of imports from source $s$, $MSHRS$, is equal to the quantity share as explained and defined in Section 5.3. $V_{MBS}$ is initialized using basedata values and updated based on an explicit update statement:

Listing 9. Update statements for Imports at basic prices

To determine the value of imports at cif-value, $VCIF$, we start from the value inclusive of tariffs, $V_{MBS}$, and divide by the value of import tariffs. As explained before, the reason is that the import price is independent of the country of origin. Hence, we start with the tariff inclusive price and subtract tariff rates to arrive at the tariff exclusive price. This is implemented as follows with an explicit update statement like before:

Listing 10. Update statements for Imports at cif prices

$$V_{CIF}(c,s,d) = \frac{MSHRS(c,s,d) \times V_{MB}(c,d)}{MTAXR(c,s,d)} \quad \text{(# tax ratio on imports of good $c$ from source $s$ to dest. $d$ #)};$$
The coefficient \( MTAX \) is defined as before, i.e., the difference between \( VMSB \) and \( VCIF \). \( VMSBEK \) is defined in Section 5.3 and \( VCIFEK \) is defined analogous to the initial value of \( VCIF \). Finally, \( ppmcifek \) is defined as the price of private household goods, \( ppa \) (identical for imported and domestic goods), exclusive of import taxes and tariffs.

### 5.4.4 Export taxes

We derive the marginal change in export taxes by reformulating the expression for export taxes in equation (31) as follows:

\[
tr_{csd}^{\text{exp}} = (t_{csd}^{\text{exp}} - 1) \left( \left( \frac{c_{csd}^{\text{exp}}}{c_{csd}^{\text{imp}}} \right)^{-\theta_c} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} - p_{tr}^{\text{exp}} t_{csd}^{ts_{csd}} \right) \tag{A.19}
\]

Totally differentiating equation (A.19) gives:

\[
dtr_{csd}^{\text{exp}} = \left( \left( \frac{c_{csd}^{\text{exp}}}{c_{csd}^{\text{imp}}} \right)^{-\theta_c} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} - p_{tr}^{\text{exp}} t_{csd}^{ts_{csd}} \right) \tag{A.19}
\]

\[
= \left( \left( \frac{c_{csd}^{\text{exp}}}{c_{csd}^{\text{imp}}} \right)^{-\theta_c} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} - p_{tr}^{\text{exp}} t_{csd}^{ts_{csd}} \right) \tag{A.19}
\]

\[
\frac{\left( c_{csd}^{\text{exp}} \right)^{-\theta_c}}{c_{cd}} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} - p_{tr}^{\text{exp}} t_{csd}^{ts_{csd}}
\]

\[
\left( \frac{c_{csd}^{\text{exp}}}{c_{csd}^{\text{imp}}} \right)^{-\theta_c} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} - p_{tr}^{\text{exp}} t_{csd}^{ts_{csd}} \tag{A.19}
\]

\[
\times \left( -\theta_c \left( \hat{c}_{csd} + \bar{c}_{csd} \right) - \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag,\text{imp}}}{t_{csd} t_{cd}^{\text{exp}} t_{cd}^{\text{imp}}} \left( \frac{p_{ag}^{\text{exp}}}{t_{csd}^{\text{imp}}} + \frac{p_{ag}^{\text{imp}}}{t_{tr}^{\text{exp}} t_{csd}^{\text{imp}}} \right) \right) \tag{A.20}
\]

Similar to import tariffs, we need to determine expressions for the coefficients and variables in equation (A.20). The coefficient in the first line is the fob value of trade, i.e. the cif value of trade minus the value of transport services, \( VFOB \). The coefficient in the second line is the value of exports inclusive of export taxes.
minus the value of exports exclusive of export taxes, equal to export tax revenues, $VFOB - VXSB = XTAXD$. The coefficient in the third line is equal to the ratio of the cif value of trade divided by the fob value of trade, $\frac{VFOB}{VXSB}$. The relative change variables and shares in the fourth line are defined as in the expression for import tariffs, except for the following term which is the change in the landed price by end user net of export taxes:

$$\frac{p_{cd}^{ag}}{t_{cd}^{imp,ag}} = pgmcifx(c,s,d) \quad (A.21)$$

The fifth line is the share of the value of transportation services divided by the fob value of trade, $\frac{VTFSD}{VXSB}$, multiplied by the relative change in the value of transportation services:

$$\frac{p_{cd}^{tr}}{t_{cd}^{exp,ts}} = ptrans(c,s,d) + tx(c,s) + txs(c,s,d) + qtmfsda(c,s,d) \quad (A.22)$$

The model code changes consistent with Equation (A.20) are as follows:

**Listing 11.** Export tax revenue to regional income ratio

```plaintext
Equation E_del_taxrexp
# change in ratio of export tax payments to regional income #
(all,s,REG)
100.0 * INCOME(s) * del_taxrexp(s) + TAXREXP(s) * y(s)
- sum{c,COMM, sum{d,REG, VFOB(c,s,d) * [tx(c,s) + txs(c,s,d)]
+ XTAXD(c,s,d)
+ [V CIF(c,s,d) / VFOB(c,s,d)] * 
[- THETA(c,d) * [pmds(c,s,d) - ams(c,s,d) - pms(c,d)]
+ GMCSHR(c,d) * [pgmcifxek(c,s,d) + qgm(c,d)]
+ PMCSHR(c,d) * [ppmcsifxek(c,s,d) + qpm(c,d)]
+ sum{a,ACTS, FMCSHR(c,a,d) * [pfmcifxek(c,a,s,d) + qfm(c,a,d)]
+ IMCSHR(c,d) * [pimcsifxek(c,s,d) + qim(c,d)]}]
- [VTFSD(c,s,d) / VFOB(c,s,d)]
* [ptrans(c,s,d) - tx(c,s) - txs(c,s,d) + qtmfsdek(c,s,d)]
));
```

The other terms in the equation defining the marginal change in export tax revenue, \( \text{del\_taxrexp} \), are the agent-specific shares—e.g., \( \text{PMCSHR} \)—have been defined in the section defining the marginal change in import tariff revenues.

In the code we define and update the different values as follows. The fob-value of trade, \( VFOB \), is initialized from the data and updated as the difference between the cif-value of trade, \( VCIF \) (defined in Section 5.3), and the value of transportation services, \( VTFSD \) (coded as in the Armington model). The export value of trade, \( VXSB \), is initialized from the data and updated as the difference between the cif-value \( VCIF \) and the value of transport services \( VTFSD \), multiplied by the power of (inverse) export taxes, \( XTAXR \):
Listing 12. Update statements for export value flows

1. \texttt{Update \{explicit\} (all,c,COMM)(all,s,REG)(all,d,REG)}
2. \texttt{VFOB(c,s,d) = VCIF(c,s,d) - VTFSD(c,s,d);}
3. \texttt{Update \{explicit\} (all,c,COMM)(all,s,REG)(all,d,REG)}
4. \texttt{VXSB(c,s,d) = VFOB(c,s,d) / XTAXR(c,s,d);}
5. \texttt{Coefficient \{all,c,COMM\} \{all,r,REG\} \{all,s,REG\}}
6. \texttt{XTAXR(c,r,s)} # tax ratio on exports of good c from source s to dest. d #
7. \texttt{Formula \{initial\} (all,c,COMM)(all,s,REG)(all,d,REG)}
8. \texttt{XTAXR(c,s,d) = VFOBEK(c,s,d) / VXSBEK(c,s,d);}
9. \texttt{Update \{all,c,COMM\} \{all,s,REG\} \{all,d,REG\}}
10. \texttt{XTAXR(c,s,d) = tx(c,s) * txs(c,s,d);}

The value of export taxes, \(XTAXD\), is defined as the difference between \(VCIF\) and \(VXSB\). \(VXSBEK\) and \(VFOBEK\) are the initial values of respectively \(VXSB\) and \(VXSB\) and defined as an analogue of \(VMSBEK\) and \(VCIFEK\). Finally, the variables defining the landed price for private household (the code changes for other agents are identical), net of export taxes is calculated as follows:

Listing 13. household import prices net of export taxes

1. \texttt{Variable \{all,c,COMM\} \{all,s,REG\} \{all,d,REG\}}
2. \texttt{ppmcifxek(c,s,d)} # price of hhld imports in d net of imp tax and export tax by s #
3. \texttt{Equation E_ppmcifxek}
4. \texttt{# price of private household imports in d net of imp tax and export tax by s #}
5. \texttt{(all,c,COMM)(all,s,REG)(all,d,REG)}
6. \texttt{ppmcifxek(c,s,d) = ppmcifek(c,s,d) - tx(c,s) - txs(c,s,d);}

5.5 Goods market equilibrium condition

Finally, we have to change the market clearing condition. In the standard GTAP model based on the Armington specification, the goods market equilibrium is formulated with quantities. In the Eaton-Kortum specification, the goods market equilibrium is reformulated in terms of values—i.e., prices times quantities. The reason is that prices in destination markets are only destination specific and thus source-independent. That implies that the price of goods sold to different destinations is different. The market closure condition is given in equation (27) and can be written more concisely as follows using the expressions for bilateral costs and importer cost in equations (A.2)-(A.3) and writing \(c_{ps}^{p} = c_{ps}^{prod}\):

\[
\begin{aligned}
\frac{p_{cs}}{c_{ps}}^{prod} &= \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cs}q_{cs}}{P_{cs}^{dom,ag}} + c_{cs}t_{cs}^{s} \\
&+ \sum_{s} \left( \frac{c_{cs}t_{cs}^{exp}}{t_{cs}^{exp}} \right)^{-\theta_{q}} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cs}^{imp,ag}}{t_{cs}^{imp,exp}} - \frac{p_{cs}^{imp,exp}}{t_{cs}^{imp,exp}} \right) 
\end{aligned}
\] (A.23)
Log differentiating equation (A.23) gives:

\[
\begin{align*}
\tilde{p}_{c_s} + \tilde{q}_{c_s}^{\text{prod}} &= \sum_{ag \in \{pr, go, fi, in\}, c_s} \frac{ag \text{ dom}_ag \ p_{c_s} \ q_{c_s}^{\text{prod}}}{c_s q_{c_s}^{\text{prod}}} + \sum_{ag' \in \{pr, go, fi, in\}} \frac{ag' \text{ dom}_ag' \ p_{c_s} \ q_{c_s}^{\text{prod}}}{c_s q_{c_s}^{\text{prod}}} \\
&\quad \times \left( \frac{c_{c_s} + t_{c_s} - c_{cd}}{c_{c_s} q_{c_s}^{\text{prod}}} + \sum_{ag' \in \{pr, go, fi, in\}} \frac{ag' \text{ imp}_ag' \ p_{c_s} \ t_{c_s}}{\text{ exp}_ag' \ t_{c_s}} \right) \\
&\quad - \frac{c_{c_s} + t_{c_s} - c_{cd}}{c_{c_s} q_{c_s}^{\text{prod}}} + \sum_{ag' \in \{pr, go, fi, in\}} \frac{ag' \text{ imp}_ag' \ p_{c_s} \ t_{c_s}}{\text{ exp}_ag' \ t_{c_s}} \\
&\quad + \frac{p_{c_s}^{\text{exp}} t_{c_s}}{c_{c_s} q_{c_s}^{\text{prod}}} \left( \tilde{p}_{c_s} + \tilde{q}_{c_s}^{\text{prod}} \right)
\end{align*}
\]

(A.24)

To write equation (A.24) in GEMPACK code we need to define various shares not defined in the previous subsection in taxes. First, the share of output sold domestically and as exports to destination s in the first and second line of (A.24) are equal respectively to \(DSSHR = \frac{VDB}{VCB}\) and \(XSSHR = \frac{VXS}{VCB}\). \(VDB\) and \(VCB\) are defined as in the Armington specification and the expression for \(VXS\) in the code is derived in the previous subsection. The share of the domestic value sold to end users, say private household \(p\) (similar for agent groups \(go\) and \(fi\) and \(in\)), \(PDCSHR\), is defined analogous to the import share sold to end user \(pr\), \(PMCSHR\), in equation (A.15). The coefficient in the third line is equal to the ratio of the cif value of trade divided by the fob value of trade, \(\frac{VCI}{VFOB}\), and the expressions in the code of \(VCIF\) and \(VFOB\) have already presented in the previous subsection. The relative change variables and the share of import demand sold to end user \(pr\) in the fourth line have been discussed in the previous subsection. The ratio of the transport-value of trade to the fob-value of trade in the fifth line is equal to \(\frac{VTFSD}{VFOB}\) with the expression in the code for \(VTFSD\) the same as in the Armington model, while the expression for \(VFOB\) has been presented in the previous subsection. Finally, the share of
sales to the global transport sector total sales in the sixth and last line is equal to \( STSHR = \frac{VST}{VCB} \).

The changes to goods market equilibrium in equation (A.24) are implemented in the GTAP model code as follows:

**Listing 14. Market clearing condition for domestic goods**

```
1 Equation E_pds
2 # assures market clearing for commodities #
3 (all, c, COMM) (all, r, REG)
4 pds(c,r) + qc(c,r)
5 - DSSH(c,r) * [GDCSHR(c,r) * [pgdek(c,r) + qgd(c,r)]
6 + PDCSHR(c,r) * [ppdek(c,r) + qpd(c,r)]
7 + sum[a,ACTS, FDCSHR(c,a,r) * [pfdek(c,a,r) + qfd(c,a,r)]]
8 + IDCSHR(c,r) * [pidek(c,r) + qid(c,r)] ]
9 + sum[d,REG, XSSHR(c,r,d) * [[VCIF(c,r,d) / VFOB(c,r,d)] *
10 [- THETA(c,d) * [pmds(c,r,d) - ams(c,r,d) + pms(c,d)]]
11 + GMCSHR(c,d) * [pgmcifxek(c,r,d) + qgm(c,d)]
12 + PMCSHR(c,d) * [ppmcifxek(c,r,d) + qpm(c,d)]
13 + sum[a,ACTS, FMCSHR(c,a,d) * [pfmcifxek(c,a,r,d) + qfm(c,a,d)]]
14 + IMCSHR(c,d) * [pimcifxek(c,r,d) + qim(c,d)]]
15 - [VTFSD(c,r,d) / VFOB(c,r,d)] *
16 + [ptrans(c,r,d) - tx(c,r) - txs(c,r,d) + qtmfsdek(c,r,d)]
17 + IF[c in MARG, STSHR(c,r) * [pds(c,r) + qst(c,r)]]
18 + tradslack(c,r));
```

### 5.6 Other Changes to the model code

The GTAP model code also calculates a whole range of additional measures like the terms of trade, GDP, trade indices and other economic variables. Since the first three measures are based on import and export prices, we also modify their associated equations in the GTAP model code with Eaton-Kortum specification.

### 5.7 Terms of Trade

The value of exports in fob-terms, \( x_s^{exp} \), can be obtained by multiplying the second line of the expression for goods market equilibrium, equation (A.23), by \( t^{imp}_{csd} \), summing over sectors \( c \) and export destinations \( d \):

\[
x_s^{exp} = \sum_c \sum_d \left( c_{csd} \frac{\alpha_{csd} \alpha} {\alpha^{imp}_{csd} \alpha^{imp}_{csd} t^{exp}_{csd} t^{exp}_{csd}} - \theta_c \right) \sum_{a \in \{pr,go,fi,in\}} \frac{p^{ag}_{cd} q^{ag}_{cd}} {t^{imp}_{csd} t^{imp}_{csd}} - \frac{p^{tr}_{csd} t^{exp}_{csd}} {t^{exp}_{csd}} S^{exp}_{csd} \tag{A.25}
\]

We rewrite equation (A.25), including the fob-cif-margin as a separate term, to consider changes in margins when calculating the fob-price changes:

\[
x_s^{exp} = \sum_c \sum_d \left( c_{csd} \frac{\alpha_{csd} \alpha} {\alpha^{imp}_{csd} \alpha^{imp}_{csd} t^{exp}_{csd} t^{exp}_{csd}} - \theta_c \right) \sum_{a \in \{pr,go,fi,in\}} \frac{p^{ag}_{cd} q^{ag}_{cd}} {t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd} t^{imp}_{csd}} \tag{A.26}
\]
With $t_{\text{csd}}^{\text{ciffof}}$ the cif-fob margin:

$$
t_{\text{csd}}^{\text{ciffof}} = \frac{(c_{\text{csd}} \tau_{\text{cr}}^{\text{imp}}) - \theta_c}{\left(\frac{c_{\text{csd}} \tau_{\text{cr}}^{\text{imp}}}{c_{\text{cr}}^{\text{imp}}}\right)^{\theta_c}} \sum_{ag' \in \{pr, go, fi, in\}} \frac{p_{ag' \text{imp} \text{ag}}^{\text{imp}} q_{c_{ag' \text{imp} \text{ag}}}^{\text{imp}}}{t_{\text{csd}}^{\text{ciffof}} t_{\text{csd}}^{\text{imp}}} - p_{\text{tr}}^{\text{tr}} t_{\text{csd}}^{\text{ciffof}} \tag{A.27}
$$

The fob price for end user $ag$ is given by the landed price $p_{ag}^{\text{ag}}$ divided by the tariffs $t_{\text{csd}}^{\text{imp}}$ and $t_{\text{csd}}^{\text{imp} \text{ag}}$, the fob-cif-margin $t_{\text{csd}}^{\text{ciffof}}$, and iceberg trade costs $\tau_{\text{csd}}$. Next we calculate the change in the fob-price, differentiating LHS and RHS with respect to price terms. We take into account the changes in tariffs and the fob-cif-margin in calculating the change in the average fob-price:

$$
x_r^{\text{exp}} p_r^{\text{exp}} = \sum_{c} \sum_{s} \left(\frac{c_{\text{csd}} \tau_{\text{cr}}^{\text{imp}}}{c_{\text{cr}}^{\text{imp}}}\right)^{\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag}^{\text{ag} \text{imp} \text{ag}} q_{c_{ag} \text{imp} \text{ag}}^{\text{imp}}}{t_{\text{csd}}^{\text{ciffof}} t_{\text{csd}}^{\text{imp}}} - p_{\text{tr}}^{\text{tr}} t_{\text{csd}}^{\text{ciffof}} - \tau_{\text{csd}} \right) \tag{A.28}
$$

Hat differentiating the expression for $t_{\text{csd}}^{\text{ciffof}}$ in equation (A.27), we can rewrite equation (A.28) as follows:

$$
x_r^{\text{exp}} p_r^{\text{exp}} = \sum_{c} \sum_{s} \left(\frac{c_{\text{csd}} \tau_{\text{cr}}^{\text{imp}}}{c_{\text{cr}}^{\text{imp}}}\right)^{\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag}^{\text{ag} \text{imp} \text{ag}} q_{c_{ag} \text{imp} \text{ag}}^{\text{imp}}}{t_{\text{csd}}^{\text{ciffof}} t_{\text{csd}}^{\text{imp}}} - p_{\text{tr}}^{\text{tr}} t_{\text{csd}}^{\text{ciffof}} \right) \left\{ \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag}^{\text{ag} \text{imp} \text{ag}} q_{c_{ag} \text{imp} \text{ag}}^{\text{imp}}}{t_{\text{csd}}^{\text{imp}}} \tau_{\text{csd}}^{\text{imp}} - \tau_{\text{csd}} \right\} \tag{A.29}
$$

To write this expression in GEMPACK, we use $x_r^{\text{exp}} = VXWREGION$ (r). Furthermore, we observe that the coefficient between brackets in the first line is equal to the fob value of exports, $VFOB$; the coefficient in the second line is equal to the
share import purchases by end user, e.g., \( PMCSHR \), for private households; the ratio in the third line is equal to the value of transport services divided by the FOB value of exports, \( VTFSD/VFOB \); and the coefficient in the last line is again equal to the share of import purchases by end users. Equation (A.29) is thus written in the code as follows:\(^{14}\)

Listing 15. Regional export price indices

```plaintext
Variable (orig_level=1.0)(all,r,REG)

psw(r) # index of prices received for tradeables produced in r #

Equation E_psw

# estimate change in index of prices received for tradeables produced in r #

VXWREGION(r) * psw(r)

- sum(c,COMM, sum(d,REG, VFOB(c,r,d) * [
[ GMCSHR(c,d) * pgmcifek(c,r,d)
+ PMCSHR(c,d) * ppmcifek(c,r,d)
+ IMCSHR(c,d) * pimcifek(c,r,d)
+ sum(a,ACTS, FMCSHR(c,a,d) * pfmcifek(c,a,r,d))
+ asm(c,r,d)]
- [VTFSD(c,r,d) / VFOB(c,r,d)] *
  [VTFSD(c,r,d) / VFOB(c,r,d)] *
  ptrans(c,r,d) + qtmfsda(c,r,d)
+ THETA(c,d) * [pmds(c,r,d) - asm(c,r,d) - pms(c,d)]
- GMCSHR(c,d) * pgmcifek(c,r,d)
- PMCSHR(c,d) * ppmcifek(c,r,d)
- IMCSHR(c,d) * pimcifek(c,r,d)
- sum(a,ACTS, FMCSHR(c,a,d) * pfmcifek(c,a,r,d)]
    ]
  ]} * sum(m,MARG, VST(m,r) * pds(m,r));
```

Next we define the CIF-value of trade for each importer \( s \) as follows:

\[
x_{d}^{imp} = \sum_{c} \sum_{s} \tau_{csd} \left( \frac{c_{csd}}{\tau_{csd}} \right)^{-\theta_{c}} \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cd}^{ag}}{t_{imp,cd}^{imp,ag}} \tag{A.30}
\]

Hat differentiating equation (A.30) with respect to price terms on the LHS, \( p_{s}^{imp} \) and RHS, \( p_{cd}^{ag} \) (thus taking into account that \( p_{cd}^{ag} \) is inclusive of iceberg trade costs), gives:

\(^{14}\) The code also accounts for the change in the price of the transport commodity. This was omitted in the derivation to keep equation (A.29) concise.
The GTAP model code also calculates for the changes in GDP value, price and quantity indices. With this, we revise the calculations for changes in fob export revenues, \( x^\text{exp}_s \), which defines \( x^\text{exp}_s \) and cif import expenditures, \( x^\text{imp}_s \). The relative change in the price component of \( x^\text{exp}_s \) was already derived in equation (A.29) to enable the calculation of changes in the terms of trade. In the calculation of changes to GDP, the relative change in the quantity component has to be determined as well.
Hat differentiating equation (A.26) with respect to quantity components gives:

\[
x_s^{\exp} q_s^{\exp} = \sum_c \sum_d \left( \frac{c_{sd} t_{cd}^{\imp} c_{cr}}{c_{cr}^{\imp}} \right)^{\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag}^{imp, ag}}{t_{cd}^{imp}} q_{ag}^{imp, ag} - p_{ag}^{tr} q_{ag}^{cd}
\]

\[
* \left( \frac{\tau_{cd} - \theta_c}{\tau_{cd} + \tau_{cd} - \tau_{cd}^{\imp}} + \sum_{ag \in \{pr, go, fi, in\}} \frac{p_{ag}^{imp, ag}}{t_{cd}^{imp}} q_{ag}^{imp, ag} \right)
\]

\[
(A.32)
\]

We observe that the change in the quantity of exports calculated using equation (A.32) is equal to the fob value of trade weighted change in the quantity of imports, \(q_{cd}\), as defined in equation (A.4). Equation (A.32) leads to the following GEMPACK code:

**Listing 17. Regional export quantity indices**

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Variable (orig_level=1.0)(all,r,REG)</td>
</tr>
<tr>
<td>2</td>
<td>qsw(r) # index of quantity of tradeables produced in r #;</td>
</tr>
<tr>
<td>3</td>
<td>Equation E_qsw</td>
</tr>
<tr>
<td>4</td>
<td># calculate change in index of quantities received for tradbles produced in r #</td>
</tr>
<tr>
<td>5</td>
<td>(all,r,REG)</td>
</tr>
<tr>
<td>6</td>
<td>VXWREGION(r) * qsw(r)</td>
</tr>
<tr>
<td>7</td>
<td>= \text{sum}{c,\text{COMM, sum}{d,REG, VFBOB(c,r,d) * }</td>
</tr>
<tr>
<td>8</td>
<td>[ \text{VCIF}(c,r,d) / \text{VFBOB}(c,r,d) ] *</td>
</tr>
<tr>
<td>9</td>
<td>[- \text{THETA}(c,d) * \text{pmds}(c,r,d) - ams(c,r,d) - pm(c,d)]</td>
</tr>
<tr>
<td>10</td>
<td>+ \text{GMCSHR}(c,d) * \text{qgm}(c,d)</td>
</tr>
<tr>
<td>11</td>
<td>+ \text{PMCSHR}(c,d) * \text{qpm}(c,d)</td>
</tr>
<tr>
<td>12</td>
<td>+ \text{IMCSHR}(c,d) * \text{qim}(c,d)</td>
</tr>
<tr>
<td>13</td>
<td>+ \text{sum}{a,\text{ACTS, FMCSHR}(c,a,d) * \text{qfm}(c,a,d) }</td>
</tr>
<tr>
<td>14</td>
<td>- \text{[VTFSD}(c,r,d) / \text{VFBOB}(c,r,d) } * \text{qmfsda}(c,r,d)</td>
</tr>
<tr>
<td>15</td>
<td>} } + \text{sum}{m,\text{MARG, VST}(m,r) * \text{qst}(m,r) };</td>
</tr>
</tbody>
</table>

In the code, we calculate the change in the value of exports as the sum of the change in the price and quantity. As a check on the correctness of this approach we add up the theoretical expressions in equations (A.29) and (A.32) and calculate the
change in value as follows:

\[
x_s \exp \left( p_s^{\exp} + q_s^{\exp} \right) = \sum_c \sum_d \left( \frac{c_{csd} r_{csd}^{\imp} \tau_{csd}}{c_{cr}^{\imp} \theta_c} \right)^{-\theta_c} \sum_{ag \in \{p, g, f\}} \frac{a_{csd}^{imp, ag} p_{cr}^{ag} q_{cr}^{ag}}{t_{csd}^{imp, ag}} - p_{c^{\imp}d}^{tr} t_{csd}^{tr} \right) * \left\{ \sum_{ag \in \{pr, go, fi, in\}} \frac{a_{csd}^{imp, ag} p_{cr}^{ag} q_{cr}^{ag}}{t_{csd}^{imp, ag}} \sum_{ag' \in \{p, g, f\}} \frac{a_{csd}^{imp, ag} p_{cr}^{ag} q_{cr}^{ag}}{t_{csd}^{imp, ag}} \left( \frac{p_{cd}^{ag}}{t_{csd}^{imp, ag}} + q_{cd}^{imp, ag} \right) \right\}
\]

Merging terms gives:

\[
x_s \exp \left( p_s^{\exp} + q_s^{\exp} \right) = \sum_c \sum_d \left( \frac{c_{csd} r_{csd}^{\imp} \tau_{csd}}{c_{cr}^{\imp} \theta_c} \right)^{-\theta_c} \sum_{ag \in \{pr, go, fi, in\}} \frac{a_{csd}^{imp, ag} p_{cr}^{ag} q_{cr}^{ag}}{t_{csd}^{imp, ag}} - p_{c^{\imp}d}^{tr} t_{csd}^{tr} \right) * \left\{ \sum_{ag \in \{pr, go, fi, in\}} \frac{a_{csd}^{imp, ag} p_{cr}^{ag} q_{cr}^{ag}}{t_{csd}^{imp, ag}} \left( \frac{p_{cd}^{ag}}{t_{csd}^{imp, ag}} + q_{cd}^{imp, ag} \right) \right\}
\]

Equation (A.33) can be converted into GEMPACK code, using conversions also
used above, or by simply adding up the linear price and quantity variables:

### Listing 18. Regional export value indices

<table>
<thead>
<tr>
<th>Variable</th>
<th>(all, r, REG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( vsw(r) )</td>
<td>index of value of tradeables produced in r</td>
</tr>
</tbody>
</table>

**Equation** \( E_{vsw} \)

\[
\text{calculate change in index of values received for tradeables produced in r}\]

\[
(vall, r, REG) \]

\[
vsw(r) = psw(r) + qsw(r) \]

Next we need to derive the relative change in the import quantity, \( q_{d,imp}^\prime \). The change in the price component was already calculated in equation (A.31). To calculate the relative change in quantity, we hat differentiate equation (A.30) with respect to \( q_{d,imp} \):

\[
x_{d,imp}^\prime q_{d,imp}^\prime = \sum_c \sum_d \left( \frac{c_{csd}^\prime r_{csd}^\prime}{c_{cr}^\prime} \right) - \theta_c \sum_{a' \in \{pr, go, fi, in\}} \frac{p_{ag'cd}^\prime q_{ag'cd}^\prime}{t_{ag'cd}^\prime} q_{d,imp}^\prime q_{ag'cd}^\prime + \sum_{a' \in \{pr, go, fi, in\}} \frac{p_{ag'cd}^\prime q_{ag'cd}^\prime}{t_{ag'cd}^\prime} q_{d,imp}^\prime q_{ag'cd}^\prime \]

(A.34)

Similar to exports volume, the change in the quantity of imports is calculated using equation (A.34) which is equal to the cif value of trade weighted change in the quantity of imports, \( q_{csd} \), as defined in equation (A.4). Equation (A.34) can be written in GEMPACK code as follows:

### Listing 19. Regional import quantity indices

<table>
<thead>
<tr>
<th>Variable</th>
<th>(all, r, REG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( qdw(r) )</td>
<td>index of quantity of tradeables used in region r</td>
</tr>
</tbody>
</table>

**Equation** \( E_{qdw} \)

\[
\text{estimate change in index of quantities of tradeable products used in r}\]

\[
(vall, r, REG) \]

\[
VMWREGION(r) \times qdw(r) \]

\[
\text{sum}_{c, COMM, \text{sum}_{s, REG, VCI} \{c, s, r\} \times \{ } \]

\[
\text{GMC}_{c, r} \times qgm(c, r) + \text{FM}_{c, r} \times qpm(c, r) + \text{I}_{c, r} \times qim(c, r) + \text{sum}_{a, ACTS, FMCSHR} \{c, a, r\} \times qfm(c, a, r) \]

\[
- \text{ams}(c, s, r) - \text{THETA}(c, r) \times \{pmds(c, s, r) - \text{ams}(c, s, r) - \text{pms}(c, r) \}
\]

}\)

Finally we can also calculate the relative change in the import value, \( x_{d,imp}^\prime \). Hat differentiating equation (A.30) with respect to both quantity and price terms, \( q_{d,imp}^\prime \)
and \( \frac{\Delta \text{imp}_{c_{sd}, t_{cd}}}{\text{imp}_{c_{sd}, t_{cd}}} \), gives:

\[
\Delta x_{d} = \sum_{c} \sum_{d} \left( \frac{c_{sd} \tau_{c_{sd}}}{\text{imp}_{d} \text{imp}_{d}} \right) - \theta_{c} \sum_{ag \in \{pr, go, fi, in\}} \Delta q_{imp, ag} \frac{p_{cd} q_{imp, ag}}{\text{imp}_{d} \text{imp}_{d}}
\]

Equation (A.35) can be concisely written in the GEMPACK code as the sum of the percentage changes in the regional import price and quantity variables:

### Listing 20. Regional import value indices

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(all, r, REG) vdw(r)</td>
<td># index of values for tradeables used in region r #</td>
</tr>
</tbody>
</table>

**Equation E_vdw**

# estimate change in index of quantities of tradeable products used in r #

\( (all, r, REG) \)

vdw(r) = pdw(r) + qdw(r);

### 5.9 Calculating trade indices

The equation for the fob value of exports for commodity \( c \) in country \( r \), is defined in \( \text{vxw}_f \). Using the expression for the value of exports by country \( r \), \( xsw(r) \), we can derive an analogous expression by omitting the summation over sector \( c \):

### Listing 21. Commodity- and region-specific value of exports

<table>
<thead>
<tr>
<th>Equation E_vxwfob</th>
<th># the change in FOB value of exports of c from r #</th>
</tr>
</thead>
<tbody>
<tr>
<td>(all, c, COMM) (all, r, REG) VXW(c, r) * vxwfob(c, r)</td>
<td></td>
</tr>
<tr>
<td>- sum[d, REG, VFOB(c, r, d) *</td>
<td></td>
</tr>
<tr>
<td>[VCIF(c, r, d) / VFOB(c, r, d)] *</td>
<td></td>
</tr>
<tr>
<td>[ - THETA(c, d) * [pmds(c, r, d) - ams(c, r, d) - pms(c, d)] + GMCSR(c, d) * [pqmcifek(c, r, d) + qgm(c, d)] + FMCSR(c, d) * [ppmcifek(c, r, d) + qpm(c, d)] + IMCSR(c, d) * [pimcifek(c, r, d) + qim(c, d)] + sum[a, ACTS, FMCSR(c, a, d) * [pfmcifek(c, a, r, d) + qfm(c, a, d)]] - [VFSD(c, r, d) / VFOB(c, r, d)] * [ptrans(c, r, d) + qtmsda(c, r, d)] ] + IF[c in MARG, VST(c, r) * [qst(c, r) + pds(c, r)]];</td>
<td></td>
</tr>
</tbody>
</table>

Similar adjustments can be made for the fob price of exports and the fob quantity of exports. For the fob price of exports in industry \( c \) in country \( r \), \( pxw \), we use the expression for the price of exports by country \( r \), \( psw \). Omitting the summation over sector \( c \) we get the following expression for \( pxw \):
Listing 22. Commodity- and region-specific export prices

```
VXW(c,r) * pxw(c,r) = sum(d,REG, VFOB(c,r,d) *
  [ GMCSHR(c,d) * pgmcifek(c,r,d)
  + PMCSHR(c,d) * ppmcifek(c,r,d)
  + IMCSHR(c,d) * pimcifek(c,r,d)
  + sum(a,ACTS, FMCSHR(c,a,d) * pfmcifek(c,a,r,d)) + ams(c,r,d)
  ] - [VTFSD(c,r,d) / VFOB(c,r,d) ] *
  [ ptrans(c,r,d) + qtmfsda(c,r,d) ]
  + THETA(c,d) * [ pmds(c,r,d) - ams(c,r,d) - pms(c,d) ]
  - GMCSHR(c,d) * pgmcifek(c,r,d)
  - PMCSHR(c,d) * ppmcifek(c,r,d)
  - IMCSHR(c,d) * pimcifek(c,r,d)
  - sum(a,ACTS,FMCSHR(c,a,d) * pfmcifek(c,a,r,d) ]
) + IF[c in MARG, VST(c,r) * pds(c,r)];
```

The fob quantity of exports for commodity \( c \) in country \( r \), \( qxw \), can be determined based on the expression for quantity by country \( r \), \( qsw \). Omitting the summation over sectors gives the following expression for \( qxw \):

Listing 23. Commodity- and region-specific quantity of exports

```
Equation E_qxw
\[ \text{change in volume of exports of (margin and non-margin) commodity } c \text{ from } r \] = sum(d,REG, VFOB(c,r,d) *
  [ - ams(c,r,d) - THETA(c,d) * [ pmds(c,r,d) - ams(c,r,d) - pms(c,d) ]
  + GMCSHR(c,d) * [pgmcifek(c,s,d) + qgm(c,d)]
  + PMCSHR(c,d) * [ppmcifek(c,s,d) + qpm(c,d)]
  + IMCSHR(c,d) * [pimcifek(c,s,d) + qim(c,d)]
  + sum(a,ACTS,FMCSHR(c,a,s,d) * pfmcifek(c,a,s,d) ]
) + IF[c in MARG, VST(c,r) * pds(c,r)];
```

On the import side we have to calculate the cif value of imports in sector \( c \) to country \( r \), \( vmwcif \). Using the expression for the value of imports by country \( r \), \( xdw( r ) \) above, we can derive an analogous expression as follows by not summing over sector \( c \):

Listing 24. Commodity- and region-specific quantity of imports

```
Equation E_vmwcif
\[ \text{the change in CIF value of imports of commodity } c \text{ into } r \] = sum(s,REG, VCIF(c,s,d) *
  [ GMCSHR(c,d) * [pgmcifek(c,s,d) + qgm(c,d)]
  + PMCSHR(c,d) * [ppmcifek(c,s,d) + qpm(c,d)]
  + IMCSHR(c,d) * [pimcifek(c,s,d) + qim(c,d)]
  + sum(a,ACTS,FMCSHR(c,a,s,d) * pfmcifek(c,a,s,d) ]
  - THETA(c,d) * [ pmds(c,s,d) - ams(c,s,d) - pms(c,d) ]
) ];
```
Similar adjustments can be made for the cif price of imports and the cif quantity of imports. For the cif price of imports in industry $c$ in country $s$, $pmw$, we use the expression for the price of imports by country $s$, $pdw$. Omitting the summation over sector $c$ we get the following expression for $pmw$:

**Listing 25. Commodity- and region-specific CIF price of imports**

```plaintext
Equation E_pmw
# computes % change in CIF price index of imports of c into d #
(all,c,COMM)(all,d,REG)
  VM CIF(c,d) * pmw(c,d)
  - sum(s,REG, VCIF(c,s,d) *
    [ GMCSHR(c,d) * p GMCIF(c,s,d) + PMCSHR(c,d) * ppmcifek(c,s,d) + IMCSHR(c,d) * pimcifek(c,s,d) +
      sum(a,ACTS, FMCSHR(c,a,d) * pfmcifek(c,a,s,d)) ] + ams(c,s,d));
```

The fob quantity of imports for commodity $c$ in country $s$, $qmw$, can be determined based on the expression for quantity by country $s$, $qdw$. Omitting the summation over commodities results in the following expression for $qmw$:

**Listing 26. Commodity- and region-specific quantity of imports**

```plaintext
Equation E_qmw
# change in volume of imports of commodity c into d #
(all,c,COMM)(all,d,REG)
  VM CIF(c,d) * qmw(c,d)
  - sum(s,REG, VCIF(c,s,d) *
    [ GMCSHR(c,d) * qgm(c,d) + PMCSHR(c,d) * qpm(c,d) + IMCSHR(c,d) * qim(c,d) +
      sum(a,ACTS, FMCSHR(c,a,d) * qfm(c,a,d)) ] -
      ams(c,s,d) - THETA(c,d) * [pmds(c,s,d) - ams(c,s,d) - pms(c,d) ]
    );
```
6. Additional derivation equations

6.1 Additional derivations for the theoretical model

This subsection contains additional derivations of some of the equations in the main text and the appendix. The equations are ordered as they appear in the text.

Equation (5) Substituting equation (4) into (3) and making a change of variable from \( z \) to \( p \) gives:

\[
G_{csd}(p) = P(p_{csd} \leq p) = P \left( \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{p_{csd}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \leq p \right) = P \left( \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{p_{csd}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \leq z_{cs} \right) = P \left( z_{cs} \geq \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{p_{csd}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \right) = 1 - P \left( z_{cs} \leq \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{p_{csd}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \right) = 1 - \exp \left\{ - \left( \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{\lambda_{cs}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \right) - \theta_c \right\} p^{\theta_c} \}
\]

Equation (7) Substituting equation (5) into (6) leads to:

\[
C_{cd}^{ag}(p) = 1 - \prod_{i=1}^{N} \exp \left\{ - \left( \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{\lambda_{cs}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \right) - \theta_c \right\} p^{\theta_c} \}
\]

\[
= 1 - \exp \left\{ - \sum_{c} \left( \frac{c_{csd}^{exp} + \gamma_{csd} p_{csd}^{tr}}{\lambda_{cs}} t_{csd}^{imp} t_{csd}^{so}\tau_{csd} \tau_{csd}^{so} \right) - \theta_c \right\} p^{\theta_c} \}
\]

\[
= 1 - e^{-\Phi_{cd}^{ag} \theta_c} \]

Equation (10) The probability that goods in country \( d \) are sourced from country \( c \) is equal to the probability that the price in country \( s \) is lower than the price in all the other trading partners:
\[ \pi_{csd}^{ag} = P \left( p_{csd} \leq \min\{ p_{kcd}; k \neq s \} \right) \]
\[ = \int_0^\infty \prod_{k \neq i} \left( 1 - G_{kcd}^{ag} (p) \right) dG_{csd}^{ag} (p) \]
\[ = \int_0^\infty \left( e^{-\left( \frac{\left( cr_{csd}^{exp} + \gamma csd p_{csd}^{tr} \right) t_{csd}^{imp} t_{csd}^{so} t_{csd}^{so}}{\lambda_{cr}} \right)^{\theta_c}} \right) \prod_{k \neq i} \left( e^{-\left( \frac{\left( cr_{kcd}^{exp} + \gamma kcd p_{kcd}^{tr} \right) t_{kcd}^{imp} t_{kcd}^{so} t_{kcd}^{so}}{\lambda_{ki}} \right)^{\theta_k}} \right) \] 
\[ = \prod_{k \neq i} \left( e^{-\left( \frac{\left( cr_{csd}^{exp} + \gamma csd p_{csd}^{tr} \right) t_{csd}^{imp} t_{csd}^{so} t_{csd}^{so}}{\lambda_{cr}} \right)^{\theta_c}} \right) \]
\[ = \prod_{k \neq i} \left( e^{-\left( \frac{\left( cr_{csd}^{exp} + \gamma csd p_{csd}^{tr} \right) t_{csd}^{imp} t_{csd}^{so} t_{csd}^{so}}{\lambda_{cr}} \right)^{\theta_c}} \right) \]
\[ = \frac{\Phi_{cd}}{\Phi_{cd}} \]

Equations (A.1) We start from equation (19) formulated with values instead of quantities:
\[ x_{csd} = \frac{\left( \frac{\prod_{k \neq i} \left( \frac{\left( cr_{csd}^{exp} + \gamma csd p_{csd}^{tr} \right) t_{csd}^{imp} t_{csd}^{so} t_{csd}^{so}}{\lambda_{cr}} \right)^{\theta_c}}{\Phi_{cd}^{imp}} \right) x_{cd}^{imp}}{\Phi_{cd}^{imp}} \]

Defining the transport margin (in power terms) as one plus the value of transport services divided by the fob value of trade, \( itm_{csd} = 1 + \frac{\gamma_{csd}^{kcd} t_{csd}^{imp}}{t_{csd}^{imp} \gamma_{csd}^{kcd}} \), we can rewrite
equation (B.1) as:

\[ x_{csd} = \frac{\left( \frac{\prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}} \right)^{-\theta_c} \Phi_{cd}^{imp} } {\Phi_{cd}^{imp}} \]  

Writing equation (B.2) in logs, capturing \((\frac{\prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}})\) by an exporter fixed effect \(d_{csd}\), \(\Phi_{cd}^{ag}\) and \(x_{ag}^{cd}\) by an importer-fixed effect \(d_{cd}\) and writing \(\tau_{csd}\) as a function of observable gravity regressors \(gr_{csd}\) and an error term \(\varepsilon_{csd}\) leads to the gravity equation in (A.1) in the main text.

Equation (18) \(\pi_{csd}\), can be calculated as a straightforward conditional probability as follows:

\[ \pi_{csd}^{aq} = \pi_{csd}^{imp,ag} = \frac{\pi_{csd}^{ag}} {\sum_{k \neq j} \pi_{csd}^{ag}} = \frac{\left( \frac{\left( \prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}} \right)^{-\theta_c} \Phi_{cd}^{imp} } {\Phi_{cd}^{imp}} \right)} {\sum_{k \neq j} \left( \frac{\left( \prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}} \right)^{-\theta_c} \Phi_{cd}^{imp} } {\Phi_{cd}^{imp}} \right)} \]  

Equation (17) Summing over the import demands of the groups of agents gives:

\[ q_{cd}^{imp} = \sum_{ag \in \{pr, go, fi, in\}} \pi_{cd}^{imp,ag} \pi_{cd}^{ag, pop_j} = \frac{\sum_{k \neq j} \left( \frac{\left( \prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}} \right)^{-\theta_c} \Phi_{cd}^{imp} } {\Phi_{cd}^{imp}} \right)} {\sum_{k \neq j} \left( \frac{\left( \prod_{cr}^{	ext{exp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}} \prod_{cd}^{	ext{exp}} \prod_{cd}^{	ext{imp}}} {\lambda_{cd}} \right)^{-\theta_c} \Phi_{cd}^{imp} } {\Phi_{cd}^{imp}} \right)} \]  

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**Equation (24)** We derive the price index as follows:

\[
\left( p_{cd}^{ag} \right)^{1-\sigma_s} = \int_0^\infty p^{1-\sigma_s} dG_{cd}^{ag} (p)
\]

\[
= \int_0^\infty p^{1-\sigma_s} \left( 1 - e^{-\Phi_{cd}^{ag} p^{\gamma_c}} \right) dt
\]

\[
= \Phi_{cd}^{ag} \int_0^\infty \left( \frac{t}{\Phi_{cd}^{ag}} \right)^{\frac{\gamma_c}{\theta_c}} \left( 1 - e^{-t} \right) dt
\]

\[
= \Phi_{cd}^{ag} \Gamma \left( \frac{\theta_c - \sigma_s + 1}{\theta_c} \right)
\]

(B.5)

With \( \Gamma (r) \) the gamma function, \( \Gamma (r) = \int_0^\infty t^{r-1} e^{-t} dt \). Hence the price index is defined as:

\[
p_{cd}^{ag} = \left( \Phi_{cd}^{ag} \right)^{-\frac{1}{\frac{\gamma_c}{\theta_c}}} \left( \Gamma \left( \frac{\theta_c - \sigma_s + 1}{\theta_c} \right) \right)^{\frac{1}{\frac{\gamma_c}{\theta_c}}}
\]

(B.6)

Using the expressions for \( \Phi_{cd}^{ag} \) and \( \Phi_{cd}^{imp} \) in respectively equations (8) and (20) we arrive at equation (24):

\[
p_{cd}^{ag} = A_s \left( \sum_i \left( \frac{t_{cr}^{prod} c_{cr}^{exp} + \gamma_{cd} p_{cd}^{fs}}{\lambda_{cr}} \right)^{-\theta_c} t_{cd}^{imp} \tau_{cd} \right)^{-\theta_c} \left( \frac{t_{cd}^{imp} c_{cd}^{imp,ag}}{\lambda_{cd}} \right)^{-\theta_c}
\]

(B.6a)

With:

\[
\Phi_{cd}^{imp} = \sum_i \left( \frac{t_{cr}^{prod} c_{cr}^{exp} + \gamma_{cd} p_{cd}^{fs}}{\lambda_{cr}} \right)^{-\theta_c} t_{cd}^{imp} \tau_{cd} \right)^{-\theta_c}
\]
Equation (25) follows from the fact that demand is homothetic and that $p_{cd}^{ag}$ is the price-index as derived in equation (24):

$$
q_{cd}^{ag} = \frac{1}{\omega_s} \int_0^\infty \left( p^{-\gamma_s} \right) \left( x_{cd}^{ag} \right)^{\frac{\alpha_s-1}{\alpha_s}} d\omega_s
$$

Equation (27) We start with equation (26) and apply $x_{cr}^{prod} = c_{cr}^{prod} q_{cr}^{prod}$ and $x_{cd}^{ag} = p_{cd}^{ag} q_{cd}^{ag}$.

$$
c_{cr}^{prod} q_{cr}^{prod} = \sum_{ag \in \{pr,go,fi,in\}} \frac{\pi_{cd}^{dom,ag} p_{cr} q_{cr}^{ag} p_{pc}}{\sum_{j \neq i} \left( \sum_{ag \in \{pr,go,fi\}} \frac{\pi_{cd}^{ag} p_{cd}^{ag} q_{cd}^{ag}}{t_{cd}^{imp,ag} t_{exp,imp} t_{cd}^{exp} t_{cd}^{imp,exp}} - \frac{p_{cr}^{exp} t_{cd}^{imp}}{t_{cd}^{exp}} t_{cd}^{imp} \right) + c_{im} t_{cd}^{m}}
$$

(B.7)

In the next step we substitute the expression for $\pi_{cd}^{ag}$ in equation (18):

$$
c_{cr}^{prod} q_{cr}^{prod} = \sum_{ag \in \{pr,go,fi,in\}} \frac{\pi_{cd}^{dom,ag} p_{cr} q_{cr}^{ag} p_{pc}}{\sum_{j \neq i} \left( \sum_{ag \in \{pr,go,fi\}} \frac{\pi_{cd}^{ag} p_{cd}^{ag} q_{cd}^{ag}}{t_{cd}^{imp,ag} t_{exp,imp} t_{cd}^{exp} t_{cd}^{imp,exp}} - \frac{p_{cr}^{exp} t_{cd}^{imp}}{t_{cd}^{exp}} t_{cd}^{imp} \right) + c_{im} t_{cd}^{m}}
$$

(B.8)

Substituting $q_{cd}^{so,ag} = \pi_{cd}^{so,ag} q_{cd}^{ag} p_{pc}$ gives:

$$
c_{cr}^{prod} q_{cr}^{prod} = \sum_{ag \in \{pr,go,fi,in\}} \frac{p_{cr} q_{cr}^{ag} p_{pc}}{t_{cd}^{dom,ag}} + \sum_{j \neq i} \left( \sum_{ag \in \{pr,go,fi\}} \frac{\pi_{cd}^{ag} p_{cd}^{ag} q_{cd}^{ag}}{t_{cd}^{imp,ag} t_{exp,imp} t_{cd}^{exp} t_{cd}^{imp,exp}} - \frac{p_{cr}^{exp} t_{cd}^{imp}}{t_{cd}^{exp}} t_{cd}^{imp} \right) + c_{im} t_{cd}^{m}
$$

(B.9)
Finally applying equation (18) again leads to equation (27):

\[
\begin{align*}
\frac{c_{ct}^{prod}}{c_{cr}^{prod}} & = \sum_{ag \in \{p, g, f\}} \frac{p_{ct}^{dom,ag}}{p_{cd}^{dom,ag}} + c_{im} tr_s^m
+ \sum_{j \neq i} \left( \frac{\left( \frac{c_{ct}^{exp} + \gamma_{cd} P_{cd}^{ts}}{\lambda_{cr}} \right)^{t_{cd}^{exp} t_{cd}^{exp}}}{\Phi_{cd}^{imp}} \right) \sum_{ag \in \{pr, go, fi, in\}} \frac{P_{cd}^{imp,ag}}{P_{cd}^{imp,ag}} - \frac{p_{csd}^{tr}}{p_{csd}^{exp} t_{csd}^{exp}} \\
\end{align*}
\]

Equation (30) We can rewrite equation (29) as follows:

\[
\begin{align*}
tr_{csd}^{imp} & = \sum_{ag \in \{pr, go, fi, in\}} \left( t_{cd}^{imp} - 1 \right) \frac{\pi_{cd}^{ag, imp}}{\pi_{cd}^{ag, imp}} \\
& = \sum_{ag \in \{pr, go, fi, in\}} \left( t_{cd}^{imp} - 1 \right) \frac{\pi_{cd}^{ag, imp}}{\pi_{cd}^{ag, imp}} P_{cd}^{ag, ag} \\
& = \left( t_{cd}^{imp} - 1 \right) \frac{\Phi_{cd}^{imp}}{\Phi_{cd}^{imp}} \sum_{ag \in \{p, g, f\}} \frac{P_{cd}^{ag, imp,ag}}{P_{cd}^{ag, imp,ag}} \\
\end{align*}
\]

In the second line we apply equations (15) and (25) and in the third line equation (18). Identical price distribution across sources The price distribution for goods sourced from country \(s\) in country \(d\) is found by integrating the probability that goods are actually sourced from country \(s\) over the price distribution of goods sourced from country \(s\) up to price \(p\), conditional on the probability that goods
are actually sourced from country \( s \):

\[
G_{csd}(p) = \frac{1}{\pi_{csd}} \int \prod_{k \neq i} (1 - G_{ckd}(q)) \, dG_{csd}(q) \, dq
\]

\[
= \frac{1}{\Phi_n} \int \prod_{0 \neq i} \left( e^{-T_s(d_{in}c_i)^{-\theta} q^p} \right) \, dq
\]

\[
t = \theta^p \int \prod_{0 \neq i} \left( e^{-T_s(d_{in}c_i)^{-\theta} t} \right) \, dt
\]

\[
= \Phi_n \int \prod_{0 \neq i} e^{-T_s(d_{in}c_i)^{-\theta} t} \, dq
\]

\[
= \frac{\Phi_n}{\Phi_n} \prod_{0 \neq i} e^{-\Phi_n t} \left| p^p \right|
\]

\[
= 1 - e^{-\Phi_n p^p}
\]

6.2 Additional derivations for the GEMPACK implementation

Here we present additional derivations of equations used in the description of the GEMPACK implementation. Equation (A.14) Totally differentiating equation gives:

\[
dt r_{csd}^{imp} = t_{csd}^{imp} - 1 \left( \frac{t_{csd}^{imp}}{\epsilon_{csd} + \tau_{csd} - \epsilon_{cr}} \right)
\]

\[
+ tr_{csd}^{imp} \sum_{ag \in \{pr, go, fi, in\}} \left( \frac{p_{csd}^{ag} q_{cr}^{ag}}{t_{csd}^{imp} q_{cr}^{ag}} \right) \left( \frac{p_{cr}^{ag} q_{cr}^{ag}}{t_{cr}^{imp} q_{cr}^{ag}} \right) (B.12)
\]

Reorganizing equation (B.12) leads to equation (A.14).
Equation (A.20) Totally differentiating equation (A.19) generates:
\[
dtr_{csd}^{exp} = tr_{csd}^{exp} \frac{t_{csd}^{exp} - t_{csd}^{exp}}{t_{csd}^{exp}}
\]
\[+ t_{csd}^{exp} \sum_{ag \in \{p,g,f\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag}} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
\[+ t_{csd}^{exp} t_{csd}^{imp,ag} \sum_{ag \in \{p,g,f\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag}} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
\[\ast \left( -\theta_c \left( p_{csd}^{imp} + \tau_{csd}^{imp} - p_{cr}^{imp} \right) + \sum_{ag \in \{p,g,f\}} p_{csd}^{imp} q_{cr}^{imp,ag} \sum_{ag' \in \{pr,go,fi,in\}} p_{csd}^{imp} q_{cr}^{imp,ag'} \right)
\]

Equation (A.26) We merge the term in \( p_{csd}^{tr} t_{csd} \) with the other terms (omitting the revenues in the transport sector):
\[
\sum_{c,c \neq s,t} \left( p_{csd}^{imp} - p_{cr}^{imp} \right) \sum_{ag \in \{p,g,f\}} p_{csd}^{imp} q_{cr}^{imp,ag} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
\[= \sum_{c,c \neq s,t} \left( p_{csd}^{imp} - p_{cr}^{imp} \right) \sum_{ag \in \{p,g,f\}} p_{csd}^{imp} q_{cr}^{imp,ag} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
\[= \sum_{c,c \neq s,t} \left( p_{csd}^{imp} - p_{cr}^{imp} \right) \sum_{ag \in \{p,g,f\}} p_{csd}^{imp} q_{cr}^{imp,ag} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
\[= \sum_{c,c \neq s,t} \left( p_{csd}^{imp} - p_{cr}^{imp} \right) \sum_{ag \in \{p,g,f\}} p_{csd}^{imp} q_{cr}^{imp,ag} \sum_{ag' \in \{pr,go,fi,in\}} \frac{p_{csd}^{imp} q_{cr}^{imp,ag'}}{t_{csd}^{exp} \cdot t_{cr}^{imp,ag'}} \left( t_{csd}^{exp} t_{csd}^{imp,ag} + t_{csd}^{imp,ag} \right)
\]
With \( t_{cfd}^{cif-fob} \) the cif-fob margin:

\[
t_{cfd}^{cif-fob} = \left( \frac{p_{cfd}^{imp} r_{cfd}^{imp}}{p_{cfd}^{imp} r_{cfd}^{imp}} \right)^{-\theta_c} \sum_{ag \in \{p, g, f\}} \frac{\sum_{ag} \frac{p_{cfd}^{imp} r_{cfd}^{imp}}{p_{cfd}^{imp} r_{cfd}^{imp}}}{\theta_c} - p_{cfd}^{imp} t_{cfd}^{imp}
\]

**Listing 27.** Value of household expenditure and update statements

```
1 !< gtapv7-ek: define domestic and import price variables >!
2 Variable (orig_level=1.0) (all,c,COMM) (all,r,REG)
3 ppmek(c,r) # price of imported c purchased by household in r, net of tax #

4 Coefficient (ge 0) (all,c,COMM) (all,r,REG)
5 VMPP(c,r) # private hhld expenditure on imp. c in r at producer prices #
6 Read VMPP from file GTAPDATA header "VMPP"
7 !< gtapv7-ek: Modify update statement by changing price from ppm to ppa >!
8 Update (all,c,COMM) (all,r,REG)
9 VMPP(c,r) = ppa(c,r) * qpm(c,r);
10 Coefficient (ge 0) (all,c,COMM) (all,r,REG)
11 VMPB(c,r) # private household expenditure on imp. c in r at basic prices #
12 Read VMPB from file GTAPDATA header "VMPB"
13 !< gtapv7-ek: Modify update statement from pms to ppmek >!
14 Update (all,c,COMM) (all,r,REG)
15 VMPB(c,r) = ppmek(c,r) * qpm(c,r);
16 !< Expenditures at producer prices have a uniform price in the EK-model >!
17 Coefficient (ge 0) (all,c,COMM) (all,r,REG)
18 VMPPEK(c,r) # private hhld expenditure on domestic c in r at purchaser’s prices, EK #
19 !< Update based on quantity shares >!
20 Formula (initial) (all,c,COMM) (all,r,REG)
21 VMPPEK(c,r) = VMPP(c,r);
22 Update (all,c,COMM) (all,r,REG)
23 VMPPEK(c,r) = qpm(c,r);

24 Equation E_ppmek
25 # EK household consumption prices for imported com. c, net of tax #
26 (all,c,COMM) (all,r,REG)
27 ppmek(c,r) = ppa(c,r) - tpm(c,r);
```

**Listing 28.** Import cost equations

```
1 !< gtapv7-ek: define domestic and import price variables >!
2 Coefficient (parameter) (all,c,COMM) (all,s,REG) (all,d,REG)
3 VMSBEK(c,s,d) # initial value of imports of c from s to d at domestic (basic) prices #
4 Formula (initial) (all,c,COMM) (all,s,REG) (all,d,REG)
5 VMSBEK(c,s,d) = VMSB(c,s,d);
6 Coefficient (all,c,COMM) (all,s,REG) (all,d,REG)
7 MSHRS(c,s,d) # share of imports from s in imp. bill of r at basic prices #
8 Formula (initial) (all,c,COMM) (all,s,REG) (all,d,REG)
9 MSHRS(c,s,d) = VMSBEK(c,s,d) / sum{ss,REG, VMSBEK(c,ss,d)};
10 Coefficient (all,c,COMM) (all,s,REG) (all,d,REG)
11 MSHRS(c,s,d) = VMSBEK(c,s,d) / sum{ss,REG, VMSBEK(c,ss,d)};
12 Update (all,c,COMM) (all,s,REG) (all,d,REG)
```
MSHRS(c,s,d) = qx(s,c,s,d) * ams(c,s,d) + qmsn(c,d);

Update (explicit) (all, c, COMM) (all, s, REG) (all, d, REG)
VMSB(c,s,d) = MSHRS(c,s,d) * VMB(c,d);

Equation E_pms
# price for aggregate imports #
(all, c, COMM) (all, d, REG)
pms(c,d) = sum(s, REG, MSHRS(c,s,d) * [pmds(c,s,d) - ams(c,s,d)]);

Equation E_qmsn
# negative of aggregate imports of c in region r, basic price weights #
(all, c, COMM) (all, r, REG)
qmsn(c,r) = -1 * [qms(c,r)];