

# Approximating Terms of Trade Effects in Single Country CGE Models

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*Single country computable general equilibrium (CGE) models often assume price taking behavior in world markets that may miss potentially important terms of trade effects in trade-exposed sectors. In this paper, we assess numerical evidence for modeling large open economies and develop a methodology for parameterizing a reduced form approximation of international trade linkages from a multi-regional global economy model. Simulated export demand and import supply elasticities suggest that assuming price taking behavior (e.g., small open economy assumption) may miss important impacts in export markets and some commodity import markets. We show that a reduced form approach to capturing terms of trade effects can perform well relative to an analogous multi-regional static model with explicit trade linkage. We also illustrate how the calibration procedure can be extended to a dynamic model using U.S. EPA's SAGE model. Our modeling scenarios demonstrate the relative importance of the large open economy assumption in non-trade policy applications, which can be significant.*

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## 1. Introduction

The computable general equilibrium (CGE) modeling community frequently confronts the issue of dimensionality. The trade-offs involved in modeling multiple regions, sectors, time periods, households, etc. can be numerous and typically lead to CGE models that are designed for specific policy applications with more detailed representations for some features of the economy and less detailed rep-

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representations for others. When analyzing policies outside of those directly related to international trade (e.g., environmental or energy applications), one simplifying assumption typical of single country CGE models is price taking behavior in the world market. This assumption, referred to as the small open economy (SOE) assumption, may be reasonable in many cases, but when the modeled country is large enough or has a significant share of its economy concentrated in trade exposed sectors, it can assume away potentially large terms of trade effects (i.e., changes in international prices). However, many studies employing single country CGE models for non-trade policies continue to assume away terms of trade effects (e.g., [Marten, Garbaccio, and Wolverson \(2019\)](#); [Brown et al. \(2024\)](#)). The SOE assumption is typically driven by a lack of information on behavioral parameters, limited methods for modeling price sensitivity in the world market, or both. In this paper, we generate numerical evidence on the appropriateness of the SOE assumption by sector and region and suggest a simple approach for implementing a large open economy (LOE) representation in single country CGE models when the evidence suggests that world prices can be affected by changes in the domestic economy.

The potential importance of capturing the large open economy assumption in a CGE model can be salient in many non-trade applications. The few studies that explicitly examine the importance of international price responsiveness in non-trade applications generally find that the terms of trade effect can make up a non-trivial share of estimated welfare impacts. For instance, [Böhringer, Rutherford, and Schneider \(2021\)](#) find that a small open economy assumption sometimes overstated the economic costs of a carbon pricing policy in Germany relative to a multi-regional model. [Böhringer and Rutherford \(2002\)](#) decompose the relative role that terms of trade effects have on domestic welfare changes in international carbon pricing scenarios finding that international spillovers can represent a significant component to domestic welfare impacts. Recognizing these potentially important welfare channels in domestic policy modeling, the U.S. Environmental Protection Agency's (EPA's) Science Advisory Board (SAB) recommended that the EPA relax its initial assumption of a small open economy and instead represent the United States economy as a large open economy in its peer review of EPA's CGE model of the United States called SAGE (SAGE is an Applied General Equilibrium).<sup>1</sup> The panel noted that, within the context of modeling the economy-wide effects of environmental regulation, "...doing so will help ensure that the model is able to capture regulatory impacts on traded goods." ([SAB, 2020](#)).

There is limited empirical evidence on export demand and import supply elasticities that can be readily adapted to a multi-sector single country CGE model. While there is general recognition among economists that policy changes in larger economies, such as the United States, may impact world commodity prices, pa-

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<sup>1</sup> The SAB review of EPA's SAGE model was conducted by a panel of experts selected for their experience in CGE modeling and economics of environmental policies.

parameterizing the rest of world demand for exports and supply of imports remains a challenge. Several studies have empirically estimated export demand elasticities for narrowly defined commodity groupings and usually for bilateral trade with specific trading partners. For instance, there are studies that estimate export demand elasticities for U.S. crops across major international trading partners. Estimates tend to be highly variable depending on the estimation framework and modeled crop type and have shown to range from relatively inelastic to elastic (Duffy, Wohlgenant, and Richardson, 1990; Reimer, Zheng, and Gehlhar, 2012).<sup>2</sup> For smaller countries, Riedel (1988) finds that price and supply conditions make it difficult to isolate precise elasticity estimates. Other work has focused on national-level aggregate export demand elasticities across all sectors in the economy (Goldstein and Khan, 1978; Bahmani-Oskooee and Niroomand, 1998; Senhadji and Montenegro, 1999; Narayan and Narayan, 2010).<sup>3</sup> Empirical evidence for the average import supply elasticity across all trading partners is largely unavailable. However, Goldstein and Khan (1978) suggest that a perfectly elastic import supply function may be reasonable if aggregating to the rest of the world (and not characterizing trade bilaterally with individual countries).

Generally, adapting trade elasticities from the empirical literature to incorporate behavioral changes in the rest of world in response to domestic changes is challenging because estimates typically lack sectoral coverage, consistent behavioral assumptions with the CGE model, and are rarely updated. For these reasons, this paper considers numerical evidence from the Global Trade and Analysis Project (GTAP) modeling framework using GTAPinGAMS as an alternative source of information for elasticity estimates (Aguiar, Narayanan, and McDougall, 2016; Lanz and Rutherford, 2016). GTAP is a well supported and peer reviewed international economy framework that is updated regularly with the best available information on economy structure and behavior. The effect of capturing the global market impacts of a large open economy is implicit within models of the world economy. We make these effects explicit by simulating export demand and import supply price elasticities based on the global model's calibrated structure. While much of this paper is geared toward the United States, the approach for computing elasticities can generally lend evidence as to the appropriateness of the SOE assumption for any given sector-country pairing in the database.

In instances where capturing the rest of world behavioral response to domestic policy shocks is important in single country models, a modeler has two choices, barring building a world economy model from scratch: (1) link the single country

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<sup>2</sup> Reimer, Zheng, and Gehlhar (2012) note that comparisons with other studies is also challenging due to different definitions of export demand in the literature. This is one reason why adapting empirical estimates to a CGE model can be challenging.

<sup>3</sup> These papers are primarily concerned with commodity markets. However, Feyrer and Shambaugh (2012) show that the United States acts as a large open economy in international capital markets as well.

model to an existing international model such as GTAP (e.g. [Caron, Rausch, and Winchester \(2015\)](#)), or (2) build reduced-form mechanisms that approximate trade responses without explicitly modeling the physical process of trade (e.g. [Yuan et al. \(2019\)](#); [Rutherford and Tarr \(2003\)](#)). Both of these approaches have advantages and disadvantages. In the first case, the model would capture the full richness of the international framework. However, this also requires the single country model conform with the larger international framework, both in terms of import and export quantities and prices and may require additional information on subnational trade if the single country model has multiple sub-regions. Furthermore, if the single country model incorporates complicated dynamics (like intertemporal dynamic optimization), the international model may need to be altered and the additional computational complexity could require reduced detail in the domestic model. This option would also require maintaining both the single country and international modeling frameworks, which would increase time and resource burdens. Conversely, a reduced-form approach may have limited potential in terms of its ability to adequately capture in detail all the terms of trade impacts. However, it could significantly simplify the representation of the rest of world and reduce the burdens of maintenance and parameter uncertainty accompanying the first option. To our knowledge, this potential trade off of the reduced form approach has never been explicitly evaluated.

Some well-known single country models in the United States include a reduced form representation of a large open economy but offer limited information on how it is calibrated ([Yuan et al., 2019](#); [Koopman et al., 2002](#)).<sup>4</sup> We note, however, that there is an extensive CGE modeling literature stemming from [Dixon et al. \(1982\)](#) that incorporates commodity-specific export demand schedules into single country models in other parts of the world, even using GTAP as a primary data source ([Dixon and Rimmer, 2002](#)).<sup>5</sup> While there are several ways to calibrate the LOE assumption into a single country model, assumed elasticity estimates used to parameterize the assumption drive the reduced form approach's ability to adequately capture rest of world price responsiveness. Though existing work has relied on the GTAP database to calibrate export demand responses, we are unaware of any study that reports sector or region specific elasticity estimates as evidence for its potential importance in economy-wide modeling applications nor compares the efficacy

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<sup>4</sup> Applications of the USAGE model that incorporate export demand and import supply elasticities to approximate terms of trade effects in the United States can be found in [Giesecke \(2011\)](#) and [Dixon, Rimmer, and Waschik \(2017\)](#).

<sup>5</sup> For instance, [Dixon and Rimmer \(2010\)](#) assess optimal tariffs for automobiles in Australia and [Liu, Giesecke, and Nassios \(2023\)](#) study the general equilibrium implications of an international student levy similarly in Australia. Notably, in work not directly related to the ORANI/MONASH modeling ecosystem but that implement a large open economy representation for exports in another region of the world, [He et al. \(2022\)](#) study labor productivity impacts from COVID-19 in China.

of the reduced form approach relative to a consistently calibrated global economy model. Furthermore, there are relatively few papers that demonstrate the relative importance of modeling a large open economy in non-trade policy applications. This paper aims to fill these gaps in the literature. Using the estimates simulated from the GTAP modeling platform, we develop a method to calibrate the reduced form approach in [Markusen \(1995b\)](#) and [Yuan et al. \(2019\)](#) to directly test how well it captures trade related impacts relative to an internally consistent multi-country model of the world economy. We also use this environment to uncover impacts that would have otherwise been missed given a small open economy assumption with a single price of foreign exchange. We compare these three frameworks using the static GTAPinGAMS model by simulating the trade and welfare impacts for a suite of illustrative environmental regulations modeled as sector-specific productivity shocks in the United States. The results suggest that the reduced form approach approximates the impacts of the global model almost exactly and that the small open economy model can under- or overestimate welfare cost depending on the direction of terms of trade effect.

We then demonstrate how the reduced-form approach can be extrapolated to the SAGE model, a large-scale intertemporal CGE model of the United States that features multiple sub-regions, sectors, and households. We illustrate how the static calibration procedure can translate to a dynamic framework. In a dynamic model without an explicit rest of world representation, choices on how trade elasticities are projected into the future implicitly assume relative growth trajectories between the modeled country and rest of world. We use the static GTAPinGAMS framework to inform the direction and magnitude of economic growth on export demand and import supply elasticities and run several simulations in SAGE to quantify its potential importance. Our results generally suggest that single country models that assume a small open economy may miss important price impacts in the export market but may do a reasonable job on modeling imports. Furthermore, we find that these impacts are not significantly affected by differential growth effects between the modeled country and the rest of the world.

The remainder of the paper is organized as follows. In Section 2, we present an overview of the assumptions made in a SOE and LOE single country CGE model and provide numerical evidence on the magnitude of reduced form elasticities. We then present the reduced form methodology from [Yuan et al. \(2019\)](#) and our strategy for calibrating the model with simulated elasticities from GTAP in Section 3. Results from the comparative exercises using GTAPinGAMS and SAGE are presented in Section 4 and we conclude in Section 5.

## **2. The Small vs. Large Open Economy Assumption**

Single country small-open economy CGE models assume fixed world prices. Applications of this approach commonly use a single price of foreign exchange in the domestic economy to clear the aggregate trade market (summed across ex-

ported and imported commodities). These applications often include a fixed level for the balance of payments between exports and imports determined by the reference dataset used to calibrate the model. The price of foreign exchange captures changes in domestic prices relative to fixed world prices and adjusts to keep the aggregate trade balance in equilibrium. For example, if the price of foreign exchange increases due to a policy shock, then domestic producers would generally seek to export more and import less, all else equal.

(a) Modeled Country's Exports into the World Market (b) World Imports into the Modeled Country's Market

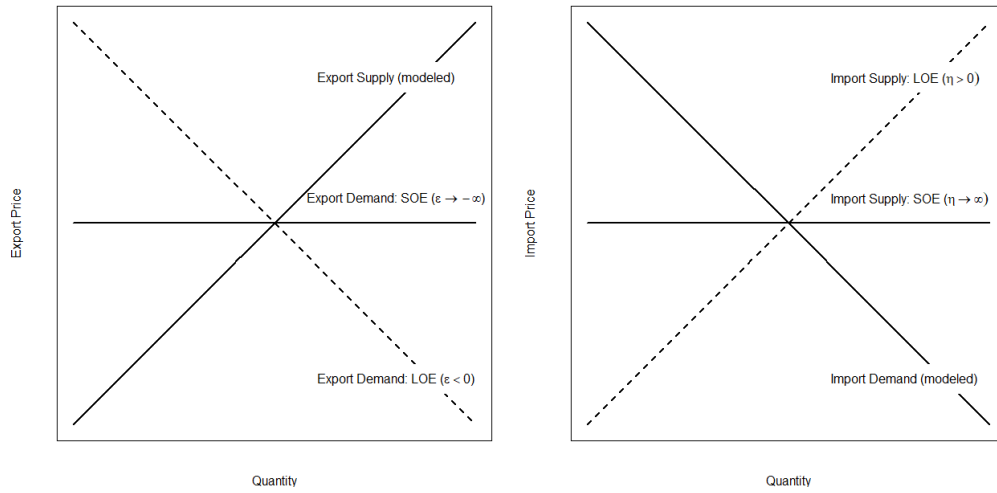


Figure 1. Open Economy Trade Assumptions

This simplifying assumption does not allow for behavioral responses in the rest of the world economy. In a CGE modeling context, the SOE assumption is consistent with a perfectly elastic export demand and import supply curve for each modeled commodity. Figure 1 presents a simplified linear representation of this assumption. A typical implementation of the SOE assumption in a CGE model is reflected by the solid lines. For exports depicted in Figure 1a, a single country open-economy CGE model includes a function that allocates domestic production between domestic and foreign markets (e.g., constant elasticity of transformation conditional on benchmark shares), which produces an upward sloping supply curve for the modeled country's exports into the world market. Under the SOE assumption world prices are constant, such that the rest of world demand (ROW) curve for the modeled country's exports is flat. We define the price elasticity of ROW demand for the modeled country's export of good  $i$  as:

$$\frac{\partial x d_i^{row}}{\partial p e_i} \frac{p e_i}{x d_i^{row}} = \epsilon_i \quad (1)$$

where  $p e_i$  is the relative price of the modeled country's exports and  $x d_i^{row}$  is the

ROW demand for good  $i$  produced in the modeled country. The SOE assumption is equivalent to assuming that  $\epsilon_i$  tends toward  $-\infty$  for each commodity. Under an LOE assumption, the demand curve for the modeled country's exports would slope downward (e.g.,  $\epsilon_i < 0$ ) - depicted as a dotted line - resulting in changes to both the export price and the quantity produced.

Figure 1b presents the analogous case for imports. In an open-economy CGE model, import demand is typically handled through a function that distinguishes demand for goods from domestic and international markets (e.g., an Armington aggregator constant elasticity of substitution function). The SOE assumption is consistent with the rest of world supply curve for imports into the modeled country as flat. We define the price elasticity of rest of world supply of imports into the modeled country's commodity market for good  $i$  as:

$$\frac{\partial ms_i}{\partial pm_i} \frac{pm_i}{ms_i} = \eta_i \quad (2)$$

where  $pm_i$  is the relative price of imports from the rest of the world and  $ms_i$  is the supply of imports into the modeled country's commodity market for good  $i$ . The SOE assumption assumes that  $\eta_i \rightarrow \infty$  for every good. The LOE assumption for imports - depicted as a dotted line - introduces an upward sloping import supply function (e.g.,  $\eta_i > 0$ ) of goods produced in the ROW and supplied into the modeled country's commodity market.

### 2.1 Numerical Evidence

We consider a structural model of the global economy to investigate the limitations of the small open economy assumption in single country CGE models. We use GTAP version 9, a comprehensive international modeling platform (data from 2011 with 140 regions and 57 sectors per region) that features regular updates to both its reference country level social accounting matrices and estimates of trade, production and consumption elasticities (Aguilar, Narayanan, and McDougall, 2016). Because GTAP is a comprehensive modeling framework that covers the global economy and specifies bilateral trade between regions, it implicitly captures the rest of world response to changes in a domestic economy.<sup>6</sup>

We simulate export demand and import supply elasticities for every sector-region pairing in the GTAP v9 database using the static model in Lanz and Ruther-

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<sup>6</sup> The GTAP model relies on the Armington assumption to characterize the demand and supply of traded commodities between countries. While the GTAP model is used extensively in the peer reviewed literature, there are some papers suggesting that Armington elasticities are potentially biased downward due to challenges in the estimating frameworks (McDaniel and Balistreri, 2003). The numerical evidence for large open economies produced in this paper may be subject to this potential limitation because simulated export demand and import supply elasticities would similarly be biased downward (in absolute value) if we think that the Armington elasticities in GTAP should be larger.



ford (2016).<sup>7</sup> There are several methods for calculating implicit local elasticities in a general equilibrium model. The approach taken in this paper is to perturb reference export and import tax rates in the model to approximate changes in the cost of supplying or demanding goods to and from foreign markets.<sup>8</sup> We set the tax perturbation to be  $\delta = 10^{-4}$ , reflecting a marginal change to existing tax rates to generate the elasticities.

This procedure for calculating estimates of own-price elasticities is described programmatically in Listing 1, which provides relevant code snippets to describe the experiments. The code initializes two additional parameters to facilitate the tax perturbation experiments (`tperb(g, r)` and `tmperb(i, r)`), which are incorporated in the model from Lanz and Rutherford (2016) in the production and import aggregation portions of the MPSGE code. The former perturbs the reference tax rate on exports collected by the exporting country and the latter acts as an addition to benchmark import tariffs collected by the importing country. To simplify post processing calculations, we define an additional reporting variable, `MD(i, s, r)`, which represents equilibrium bilateral trade flows of commodity  $i$  from country  $s$  into country  $r$ . We iterate over the model to calculate export demand and import supply elasticities. Following the relevant tax perturbation, export demand elasticities are calculated as the percent change in imports into the ROW with respect to a percent change in the modeled country's export price. Conversely, import supply elasticities are calculated as a percent change in imports into the modeled country with respect to a percent change in the ROW's export price.

**Listing 1.** Simulating Trade Elasticities (code snippets from `multi_region_model.gms`)

```
$title multi-regional gtap model (2 country)

* -----
* set options
* -----

$if not set country $set country usa
```

---

<sup>7</sup> Model code and data to reproduce all results in this paper are available for download in the supplemental materials.

<sup>8</sup> We have also alternatively calculated elasticities using a "disequilibrium" approach, which produces similar results. The disequilibrium approach fixes the baseline price of an exported or imported commodity by a perturbation parameter,  $\delta$ , which is a small number and calculates the implied elasticities at the benchmark calibration of the core model. Because the model is solved as a mixed complementarity problem, fixing a price relaxes the associated market clearance condition in the model yielding excess supply in the model. We have found that if the chosen value of  $\delta$  is small enough, then the bias introduced into elasticity estimates is minimal. Code is available for this alternative technique.



```

* -----
* sets
* -----

set
  g(*) Goods plus C - SD - DD -- G and I
  i(g) Goods
  f(*) Factors
  r(*) Regions
  rm(r) Regions included in the model;
alias(r,s);

* -----
* added parameters for calculating implicit elasticities
* -----

* initialize tax perturbations
parameter
  tperb(g,r) export demand tax perturbation,
  tmperb(i,r) import supply tax perturbation;

tperb(g,r) = 0;
tmperb(i,r) = 0;

* -----
* relevant gtap model code snippets
* -----

$ontext

$model:gtap9

$sectors:
  Y(g,r)$ (rm(r)*vom(g,r)) ! Supply
  M(i,r)$ (rm(r)*vim(i,r)) ! Imports

$commodities:
  P(g,r)$ (rm(r)*(vom(g,r)-vxm(g,r))) ! Domestic output price
  PE(g,r)$ (rm(r)*vxm(g,r)) ! Export market
  PM(j,r)$ (rm(r)*vim(j,r)) ! Import price
  PT(j)$vtw(j) ! Transportation services
  PF(f,r)$ (rm(r)*evom(f,r)) ! Primary factors rent
  PS(f,g,r)$ (rm(r)*sf(f)*vfm(f,g,r)) ! Sector-specific primary factors

$consumers:
  GOV(r)$rm(r) ! Government agent

* sectoral output (plus final demand)
$prod:Y(g,r)$ (rm(r)*vom(g,r)) s:0 t:(etadx(g)) m:esub(g,r) va:esubva(g) i.tl(m)
:esubdm(i)
  o:P(g,r) q:(vom(g,r)-vxm(g,r)) a:GOV(r) t:rto(g,r) p:(1-rto(g,r))
  o:PE(g,r) q:vxm(g,r) a:GOV(r) t:(rto(g,r)+tperb(g,r)) p:(1-rto(g,r))
  i:P(i,r) q:vdfm(i,g,r) p:(1+rtfd0(i,g,r)) i.tl: a:GOV(r) t:rtfd(i,g,r)
  i:PM(i,r) q:vifm(i,g,r) p:(1+rtfi0(i,g,r)) i.tl: a:GOV(r) t:rtfi(i,g,r)
  i:PS(sf,g,r) q:vfm(sf,g,r) p:(1+rtf0(sf,g,r)) va: a:GOV(r) t:rtf(sf,g,r)

```

```

i:PF(mf,r) q:vfm(mf,g,r) p:(1+rtf0(mf,g,r)) va: a:GOV(r) t:rtf(mf,g,r)

* import aggregator function
$prod:M(i,r)$ (rm(r)*vim(i,r)) s:(esubm(i)) s.tl:0
o:PM(i,r) q:vim(i,r)
i:PE(i,rm)$vxm(i,rm) q:vxmd(i,rm,r) p:pvxmd(i,rm,r) rm.tl:
+ a:GOV(rm) t:(-rtxs(i,rm,r)) a:GOV(r) t:(rtms(i,rm,r)*(1-rtxs(i,rm,r)))
a:GOV(r) t:tmperb(i,r)
i:PT(j)#(s) q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl: a:GOV(r) t:rtms(i,s,r)

* additional report variable
$report:
v:MD(i,s,r) i:PE(i,s) prod:M(i,r)

...

$offtext
$sysinclude mpsgeset gtap9

* -----
* simulate elasticities to use in single region model case
* -----

* set workspace and iteration limit
gtap9.workspace = 1024;
gtap9.iterlim = 1000;

set
sec_i(i) loop index;

sec_i(i) = yes;

parameter
sim_elasticities own-price elasticities for single region model;

* exports
loop(sec_i$(vxmd(sec_i,'%country%','row')),

tperb(sec_i,'%country%') = 1e-4;
$include gtap9.gen
solve gtap9 using mcp;
sim_elasticities(sec_i,'exports') =
(MD.L(sec_i,'%country%','row')/vxmd(sec_i,'%country%','row')-1) /
(PE.L(sec_i,'%country%')-1+eps);
tperb(sec_i,'%country%') = 0;
);

* imports
loop(sec_i$(vxmd(sec_i,'row','%country%')),

tperb(sec_i,'%country%') = 1e-4;
$include gtap9.gen
solve gtap9 using mcp;
sim_elasticities(sec_i,'imports') =
((MD.L(sec_i,'row','%country%'))/vxmd(sec_i,'row','%country%')-1) /
(PE.L(sec_i,'row')-1+eps);

```

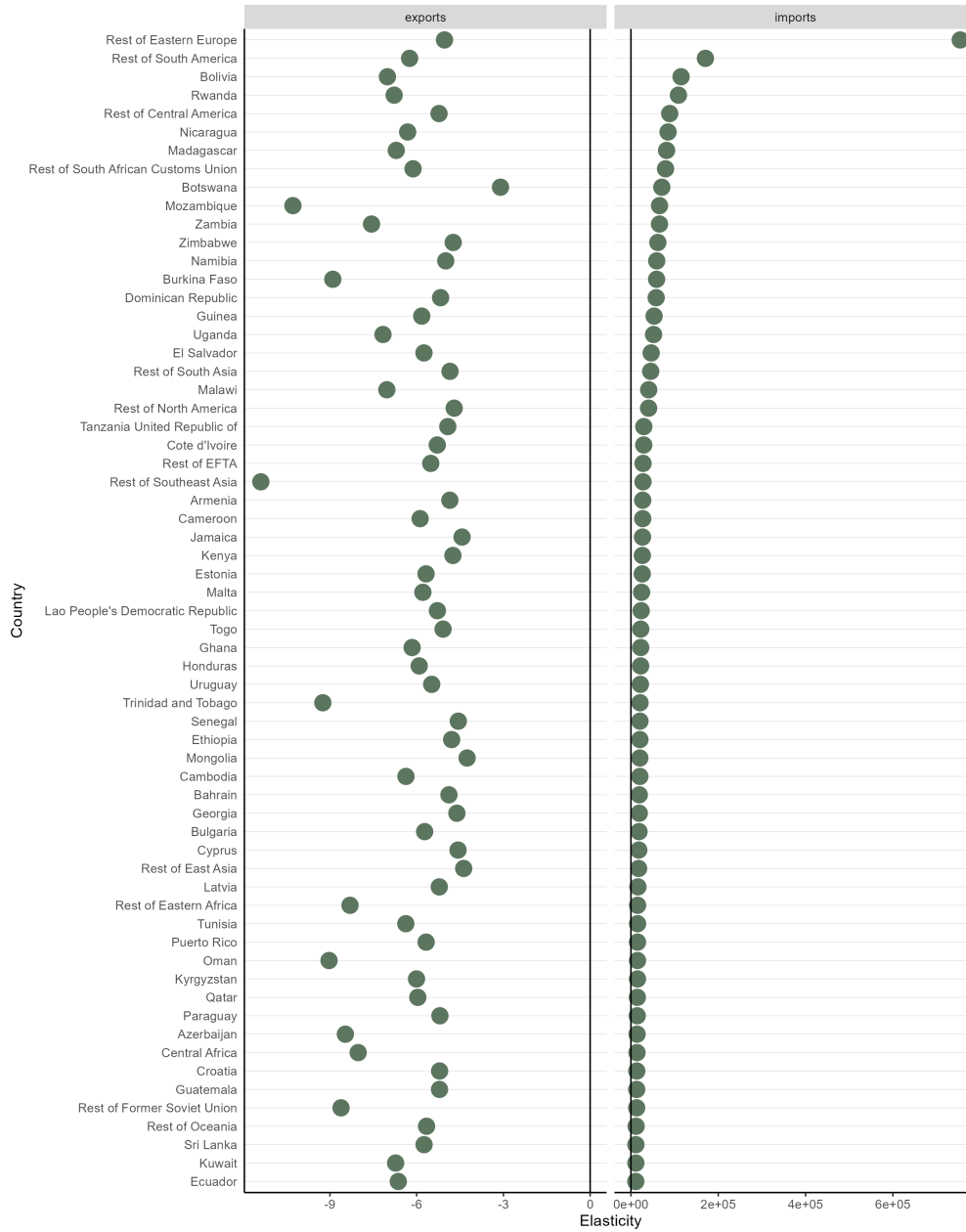
```
    tmperb(sec_i, '%country%') = 0;  
);
```

```
* -----  
* end  
* -----
```

---

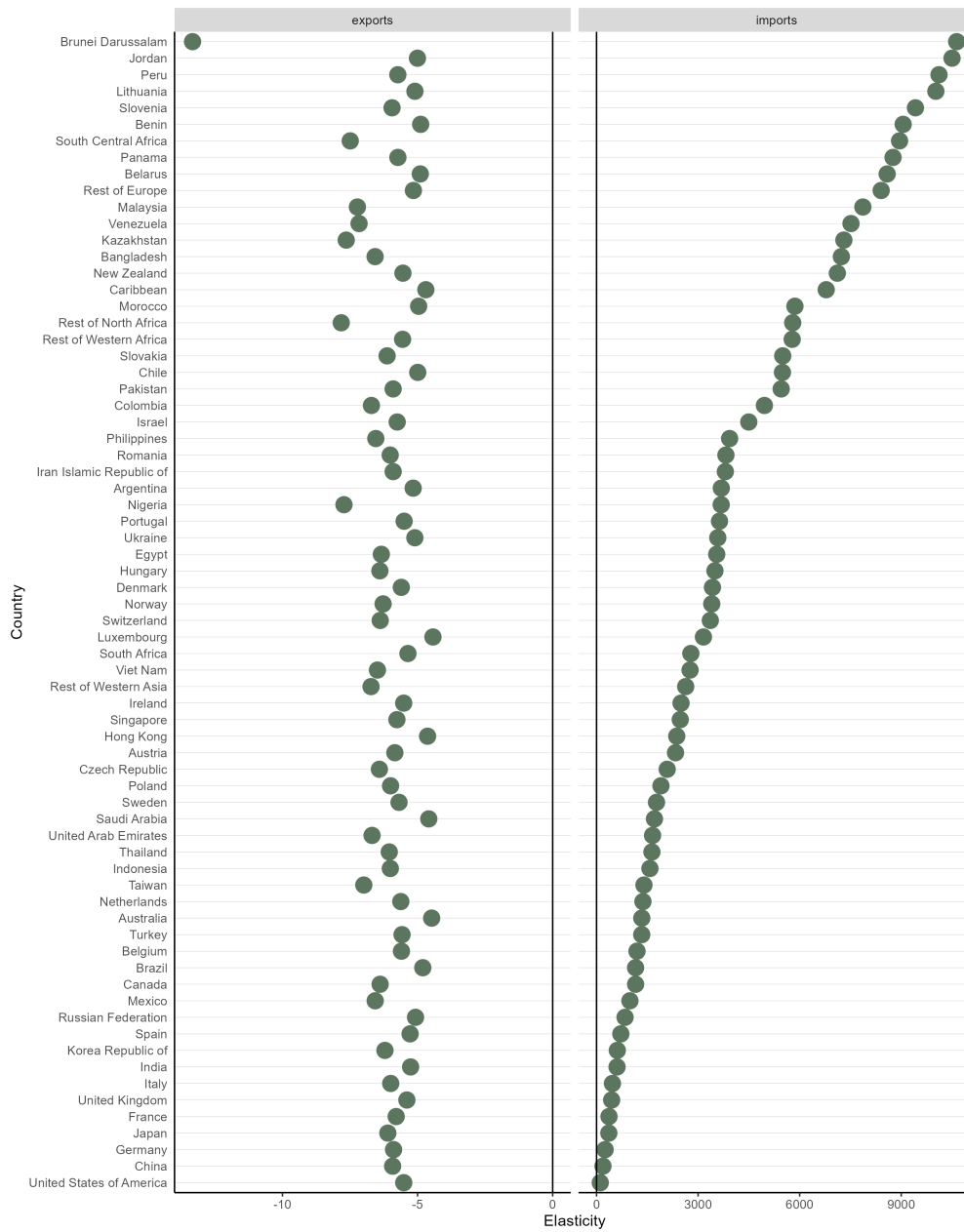
Because our aim is to characterize elasticities between a given country and an aggregation of the rest of the world (to avoid heterogeneity in behavioral responses by individual bilateral trade partners), we conduct this exercise by aggregating the GTAP model into two regions that include the given country and the rest of the world (or all other regions in the GTAP database aggregated together). As noted in the empirical literature, the behavioral response in trading partner countries is likely heterogeneous and would lead to a wide range of effects if left disaggregated; however, this is outside the scope of this paper.

(a) Top 50th Percentile in Import Supply Price Elasticities



Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

(b) Bottom 50th Percentile in Import Supply Price Elasticities [axis change]



**Figure 2.** Weighted Average Elasticities for GTAP Database

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

Figure 2 presents weighted average elasticities for each country in the database, ranked by the average import supply price elasticity. Weighted averages are com-

puted using the reference import and export values for a given country.<sup>9</sup> In general, larger elasticities in absolute value for both imports and exports suggest that the small open economy assumption likely does a reasonable job at approximating international trade linkages in an open-economy model. Figure 2a includes countries in the 50-100th percentiles whereas Figure 2b includes countries below the 50th percentile based on weighted import elasticities (note the x-axis change). Calculated elasticities are a function of the assumed underlying Armington elasticities in the GTAPinGAMS package and associated trade shares. As expected, larger countries with large levels of imports have smaller import supply elasticities as shifts in those economies are more likely to impact world prices than shifts in smaller economies that import a relatively smaller share of globally traded goods. Conversely, export demand price elasticities are smaller in absolute value and relatively more consistent across countries. On average, this suggests that while the small open economy assumption is likely reasonable in the import market, it may miss important price impacts in the export market for most countries in the GTAP database.

Figure 3 reports the average elasticities across countries for each commodity in the database ranked by average import supply price elasticities. In general, import supply elasticities are more varied than export demand elasticities. Furthermore, natural resource extraction-based commodities, or commodities primarily produced in certain parts of the world have smaller elasticities in absolute value. It is important to note that the results presented in this section summarize findings based on disaggregated elasticities. While reported weighted averages may provide general evidence for the LOE assumption over the SOE assumption or vice versa, underlying these averages are heterogeneous price elasticities (in total, approximately 15,000 elasticities). Modelers should take care in assessing these elasticities when considering a large open economy assumption.

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<sup>9</sup> Note that in some instances, the perturbation experiments produced outlier estimates, which we define as estimates with the wrong sign, due to numerical issues in modeling two countries where one country is relatively small. While more work is likely needed in some of these instances, we produced weighted averages excluding these outliers and advise caution when applying these methods in practice.



**Figure 3.** Weighted Average Commodity Elasticities for GTAP Database

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

### 3. Modeling Approach

We adopt an approach first introduced in Markusen (1995b)<sup>10</sup>, and later used by Yuan et al. (2019) in the U.S. Regional Energy Policy (USREP) model that intro-

<sup>10</sup> See Markusen (1995a) for the associated MPSGE code that implements the reduced form approach.



duces curvature into the flat portions of Figure 1 using a reduced form framework. The approach introduces curvature by requiring the use of a *fixed factors* for exporting or importing goods, where we assume a “rest of world” agent is endowed with the fixed factors and demands foreign exchange. The level of the fixed factors can be used to calibrate the model to exogenous export demand and import supply price elasticities. Fixed factors are used frequently in the CGE modeling literature to represent diminishing returns due to a fixed supply of natural resources (e.g., Hertel (2002); Marten, Schreiber, and Wolvertson (2021); Chen et al. (2022)). Here, the framework can be applied in a way that allows us to target specific supply and demand elasticities for imports and exports, respectively. Note that this specification is not meant to explicitly capture the physical process of trade, but rather approximate price and quantity impacts in the modeled country as if the rest of world were included in the model.

To formalize the reduced form approach, assume that we have a single country model (dropping the  $c$  subscript). Let  $pm_i$  denote the import price of commodity  $i$ ,  $pe_i$  denote the export price, and  $pfx$  denote the price of foreign exchange. Under the SOE assumption,  $pm_i = pe_i = pfx$ . To relax this assumption, we can introduce another set of prices,  $pfim_i$  and  $pfix_i$ , representing prices associated with the fixed factors for imports and exports, respectively, that are owned by a “rest of world” agent in the model. Assuming a Cobb-Douglas functional form, we can construct the following cost functions for the supply of world imports into a given country’s commodity market and the supply of all exports into the world market:

$$pm_i = pfx^{(1-\theta_i^m)} pfim_i^{\theta_i^m} \quad (3)$$

$$pfx = pe_i^{(1-\theta_i^x)} pfix_i^{\theta_i^x} \quad (4)$$

where  $\theta_i^m$  and  $\theta_i^x$  denote cost shares associated with the fixed factors for imports and exports, respectively. Note that when  $\theta_i^m \rightarrow 0$  and  $\theta_i^x \rightarrow 0$ , then the reduced form approach converges to the SOE assumption. In this setting and using conventional data for calibrating a CGE model, fixed factors are unknown and need to be calibrated. We therefore seek to calibrate the values of  $\theta_i^m$  and  $\theta_i^x$  to target exogenous price elasticities.

**Proposition 1.** *We can calibrate the value of  $\theta_i^m$  such that the model targets an exogenously specified price elasticity of supply,  $\eta_i$ , by setting:  $\theta_i^m = 1/(1 + \eta_i)$ .*

*Proof.* See Rutherford (2002) on benchmarking supply functions, setting  $\sigma = 1$ .<sup>11</sup>

□

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<sup>11</sup> This is the standard way that fixed factors are typically introduced into a CGE model relating the elasticity of substitution ( $\sigma$ ) with an exogenous supply elasticity ( $\eta$ ) and cost shares ( $\theta$ ):  $\sigma = \eta\theta/(1 - \theta)$ . Because we have *two* unknowns in this setting ( $\sigma, \theta$ ), we can assume a Cobb-Douglas functional form without loss of generality of the approach. Assuming a substitution elasticity different than  $\sigma = 1$  would simply adjust the calibrated

**Proposition 2.** We can calibrate the value of  $\theta_i^x$  such that the model targets an exogenously specified price elasticity of demand,  $\epsilon_i$ , by setting:  $\theta_i^x = 1 / -\epsilon_i$ .

*Proof.* Assume we know  $\epsilon_i$ . In Equation 4, our zero profit condition can be characterized by aggregating exports from the modeled country with exports from un-modeled countries into the world market. This latter component is the fixed factor. The zero profit condition for this aggregation relates the price of foreign exchange ( $pfx$ ) with the export price from the modeled country ( $pe_i$ ) and the price of the fixed factor ( $pfix_i$ ). Assume that the total supply of exports from the rest of the world (outside of the modeled country) is  $\overline{fix}_i$ . Using Shepard's Lemma, we can represent the market clearing condition for the fixed factor as:

$$xd_i \frac{\partial pfx}{\partial pfix_i} = \overline{fix}_i \quad (5)$$

where  $xd_i$  denotes total ROW demand for exported commodity  $i$  (including exports from the modeled country and ROW). Solving for  $pfix_i$ :

$$pfix_i = pfx \left( \frac{xd_i \theta_i^x}{\overline{fix}_i} \right) \quad (6)$$

Substituting this into the cost function in (4), we can characterize the following two relationships:

$$pfx = pe_i \left( \frac{xd_i \theta_i^x}{\overline{fix}_i} \right)^{\theta_i^x / (1 - \theta_i^x)} \quad (7)$$

and,

$$xd_i = \left( \frac{\overline{fix}_i}{\theta_i^x} \right) \left( \frac{pe_i}{pfx} \right)^{(\theta_i^x - 1) / \theta_i^x} \quad (8)$$

We can then solve for ROW export demand from the modeled country's market:

$$xd_i^{row} = xd_i \frac{\partial pfx}{\partial pe_i} = \left( \frac{\overline{fix}_i}{\theta_i^x} \right) \left( \frac{pe_i}{pfx} \right)^{1 - \theta_i^x} \quad (9)$$

Using the expression for the price elasticity (using relative prices) of export demand from the modeled country, we can solve for  $\theta_i^x$ :

$$\epsilon_i^x = \frac{\partial xd_i^{row}}{\partial (pe_i / pfx)} \frac{(pe_i / pfx)}{xd_i^{row}} = \frac{1}{-\theta_i^x} \quad (10)$$

or  $\theta_i^x = 1 / -\epsilon_i$ . □

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values for  $\theta_i^m$  and  $\theta_i^x$ . In other settings where the value of  $\theta$  may be known (for instance using land rents to characterize land as a fixed factor), the substitution elasticity can be backed out based on choices of  $\eta$  and  $\theta$ .

Using these calibrated cost shares for the fixed factors, the reference levels of the fixed factors for exports ( $\overline{fix}_i$ ) and imports ( $\overline{fim}_i$ ) are then defined as,

$$\overline{fix}_i = \frac{\theta_i^x}{(1 - \theta_i^x)} \overline{x}_i \quad (11)$$

and

$$\overline{fim}_i = \theta_i^m \overline{m}_i \quad (12)$$

where  $\overline{x}_i$  and  $\overline{m}_i$  are reference levels of exports and imports taken from the benchmark dataset used to calibrate the model. Given the calibration procedure, as  $|\epsilon_i| \rightarrow \infty$  and  $\eta_i \rightarrow \infty$ , then  $\theta_i^x \rightarrow 0$  and  $\theta_i^m \rightarrow 0$ .

This technique is attractive for several reasons. First, it does not require that the modeler carry along a world economy model for the purposes of capturing trade effects when the interest is in the impacts within a single country. Second, we can straightforwardly incorporate the above pricing equations into any CGE structure without relying on additional data other than estimates for supply and demand elasticities. This differs from a previous effort that incorporated compensated demand functions directly into a single country CGE model using the GTAP database (Rutherford and Tarr, 2003). Finally, this framework provides a simple transition between the small and large open economy assumption allowing the analyst to evaluate its importance in a policy simulation. However, the empirical issue of what values should be chosen for needed elasticities remains. While this section develops proofs of the approach, this paper's second purpose is to explore mechanisms for parameterizing the reduced form approach and explore its effectiveness in capturing international trade linkages.

#### 4. Simulations

We employ two modeling frameworks to explore the efficacy of the reduced form approach for approximating international trade linkages. We use an aggregate version of the canonical GTAPinGAMS model to directly test the performance of the reduced form approach relative to an internally consistent multi-regional model of the global economy and an equivalent small open economy representation. The default static GTAPinGAMS framework is discussed at length in Lanz and Rutherford (2016) and is a translation of the GTAP model into the GAMS (General Algebraic Modeling System) programming language.<sup>12</sup> We aggregate the GTAPinGAMS model to two regions (United States and ROW) and 22 sectors that

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<sup>12</sup> For the purposes of this paper, we rely on the default "out of the box" model from Lanz and Rutherford (2016) with one exception in adding an additional constraint on government purchases. We close the government budget for each country by endogenizing a lump sum transfer to hold government consumption fixed to align with the closure assumptions in the SAGE model.

**Table 1.** Sector Aggregation and Aggregate U.S. Import Supply and Export Demand Elasticity Estimates

<i>Sector Index</i>	<i>Description</i>	<i>Import Supply Elasticity</i>	<i>Export Demand Elasticity</i>
agf	Agriculture, forestry, fishing and hunting	72.8	-4.0
cru	Crude oil	19.5	-10.2
col	Coal mining	494.7	-5.0
min	Metal ore and nonmetallic mineral mining	161.6	-1.4
<b>ele</b>	<b>Electric generation, transmission and distribution</b>	<b>155.4</b>	<b>-5.6</b>
gas	Natural gas	118.9	-28.2
wsu	Water, sewage, and other utilities	88.4	-5.6
con	Construction	225.7	-3.8
fbm	Food and beverage manufacturing	99.5	-4.6
wpm	Wood product manufacturing	90.7	-5.6
<b>ref</b>	<b>Petroleum refineries</b>	<b>90.4</b>	<b>-3.8</b>
<b>chm</b>	<b>Chemical manufacturing</b>	<b>149.5</b>	<b>-5.8</b>
cem	Cement manufacturing	100.2	-5.2
pmm	Primary metal manufacturing	171.2	-6.8
fmm	Fabricated metal product manufacturing	133.6	-7.0
cpu	Electronics and technology manufacturing	145.0	-8.1
tem	Transportation equipment manufacturing	116.3	-5.7
<b>bom</b>	<b>Balance of manufacturing</b>	<b>138.8</b>	<b>-7.3</b>
trn	Transportation	263.9	-3.6
ttn	Truck transportation	70.2	-3.8
srv	Services	54.5	-3.8
hlt	Healthcare services	27.1	-3.7

*Notes:* Bolded rows indicate sectors that are shocked in the suite of scenarios.

*Source:* Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

mimic the sectoring scheme used in the U.S EPA's intertemporal CGE model of the U.S. economy called SAGE (Marten, Schreiber, and Wolverton, 2021).<sup>13</sup> Table 1 reports both the sectoral aggregation scheme as well as corresponding import supply and export demand elasticities for the United States that are simulated from the GTAPinGAMS framework. Because the GTAPinGAMS model is static, we then use the SAGE model to illustrate the importance of temporal assumptions in the calibration procedure.

The comparative exercise between models is conducted by considering policy simulations that mimic several illustrative environmental regulatory shocks to the United States economy. Following previous work, we model an environmental

<sup>13</sup> The SAGE model provides a greater degree of disaggregation in the manufacturing and energy sectors of the economy that are more regularly impacted by environmental regulation. Mapping files are available upon request. Mapping to SAGE sectors is imperfect as GTAP9 does not have plastic and rubber manufacturing separately disaggregated in its default sectoring scheme.

regulation as a productivity shock that mandates additional inputs to produce the same amount of output (e.g. the installation of pollution control technologies) (Marten, Garbaccio, and Wolverton, 2019; Pizer et al., 2006). Here, we consider a suite of sector-specific shocks in sectors that have historically been regulated by the U.S. EPA under the Clean Air Act between 1990-2020 (EPA, 2011). We limit the exercise by focusing on four sectors that have either born a significant share of regulatory costs and/or represent a significant share of total U.S. exports. The chosen sectors are bolded in Table 1: electricity, refined petroleum, chemical manufacturing, and the balance of manufacturing (representing a composite of all other manufacturing sectors). In this analysis, we assume that the shocked sector is required to purchase \$1 billion of additional inputs (not inclusive of taxes) to comply with the hypothetical regulation in proportion to the reference input-output production structure (including capital and labor).<sup>14</sup>

#### 4.1 GTAPinGAMS

We consider three model variants based on the default, static GTAPinGAMS modeling framework.

- A two-country version of the GTAPinGAMS model aggregated to the dimensions in Table 1, designated "2 country."
- A single region model with the reduced form large open economy framework and aggregated sectoring scheme in Table 1, designated "loe."
- A single region model with the small open economy assumption and aggregated sectoring scheme in Table 1, designated "soe."

The single region model is a parsimonious representation of the small open economy option outlined in Lanz and Rutherford (2016) and produces identical results when assuming a small open economy in the canonical multi-regional framework. See Appendix A for the single region model MPSGE code.<sup>15</sup>

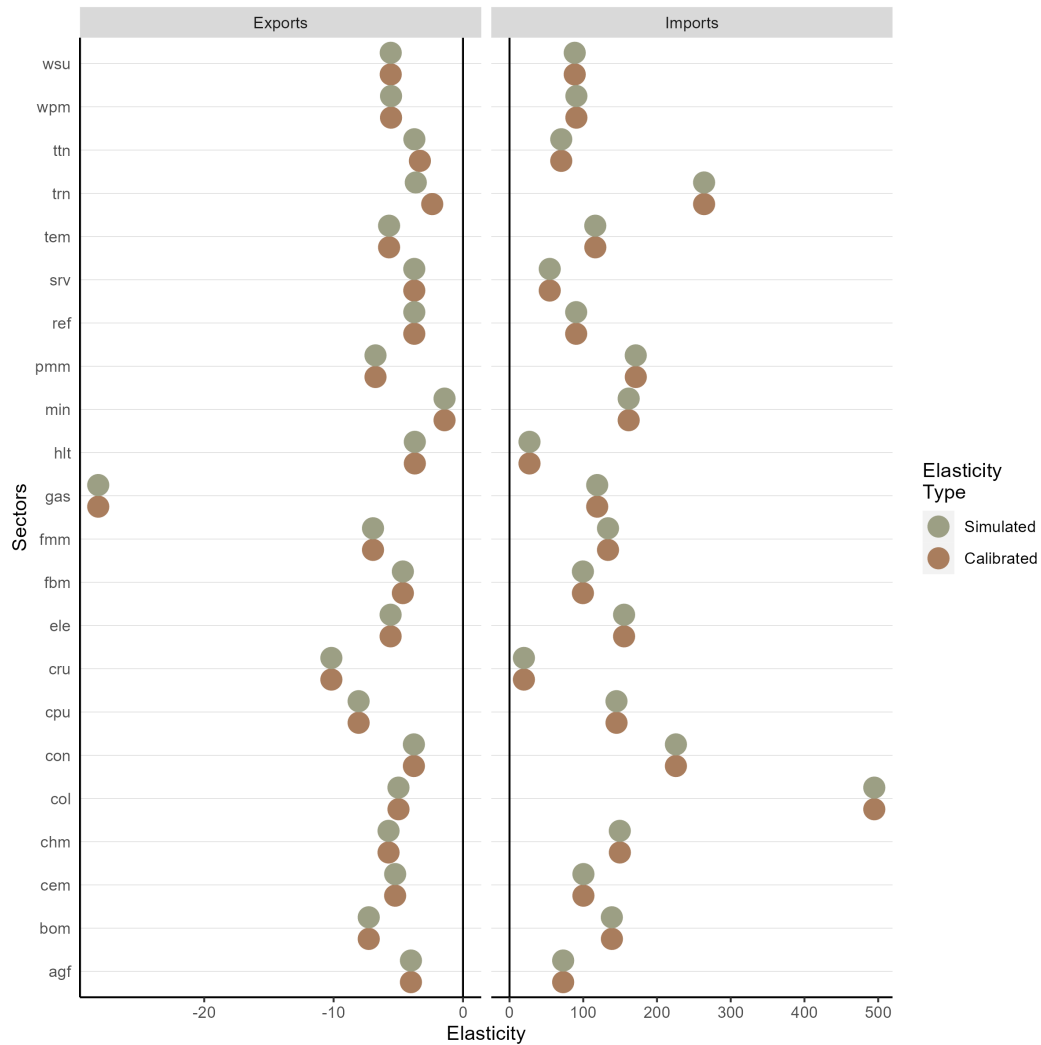
##### 4.1.1 Verifying the Calibration in a Single Country Model

Before conducting the illustrative policy experiment, we first verify the calibration of the reduced form large open economy specification. We compare the simulated values generated from the multi-regional model (reported in Table 1) with the

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<sup>14</sup> A proportional increase in inputs based on the reference input-output data is also called a Hicks Neutral productivity shock. Other research has found that the input composition of a productivity shock is important for assessing overall costs (Marten, Garbaccio, and Wolverton, 2019). We abstract away from this finding in this comparative exercise. For a more detailed description of modeling an environmental regulation, see Marten, Schreiber, and Wolverton (2021).

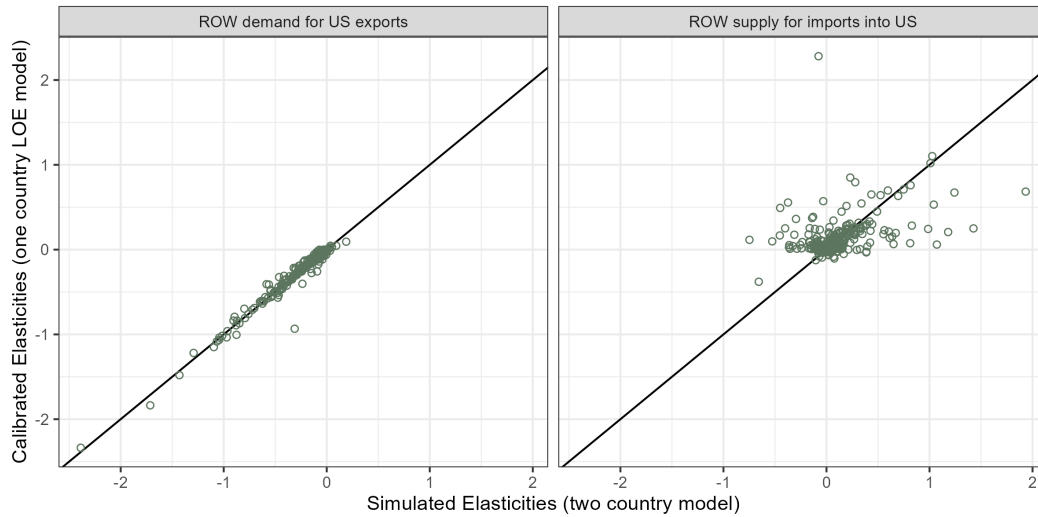
<sup>15</sup> The same model code can be run with the SOE or LOE assumptions by changing the fixed factor cost shares associated with the reduced form framework. That is, setting  $\theta_i^m$  and  $\theta_i^x$  to zero in Equations 3 and 4.



**Figure 4.** Own Price Export Demand and Import Supply Elasticities in the United States

*Source:* Authors' calculations using GTAP9 and [Lanz and Rutherford \(2016\)](#).

internally calibrated supply and demand elasticities from the single-region large open economy model. The latter values are computed by perturbing the price of imports or exports in the single region LOE model and solving for the percent change in export demand and import supply quantities relative to the small change in price. The calibrated values are reported in Figure 4. The simulated and calibrated values are almost identical verifying that the calibration procedure



**Figure 5.** Cross Price Export Demand and Import Supply Elasticities in the United States

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

is robust.<sup>16</sup>

The calibration of the reduced form framework only requires information on own-price elasticities, but estimates of cross-price effects can be used as an additional point of comparison on assessing how well the reduced form approach may capture trade impacts from the two region model. We calculate the cross-price elasticities as the percent change in export demand or import supply of good  $j$  relative to small changes in the export price or import price, respectively, of good  $i$ . Figure 5 characterizes this comparison by plotting the simulated (2 region model) cross-price elasticities on the x-axis and the calibrated (single region LOE model) cross-price elasticities on the y-axis.<sup>17</sup> The scatter plots are largely distributed along the 45° line, illustrating that the calibrated and simulated cross-price elasticities are relatively similar. The magnitude of import elasticities relative to export price elasticities (in absolute value) introduce some imprecision in the case of imports relative to exports.

#### 4.1.2 Model Comparison Results

Fully characterizing the impacts of the illustrative policy simulations is beyond the scope of the current paper. While understanding the role that trade may play in the costs and/or benefits of an environmental regulation is important, for this

<sup>16</sup> We did not include elasticities for transportation sectors in this figure. Simulated import elasticities are very large and obscure plotting. However, the consistency in the calibration holds in the transportation sector as well.

<sup>17</sup> Similar to Figure 4, we do not include transportation cross-prices elasticities in these plots.

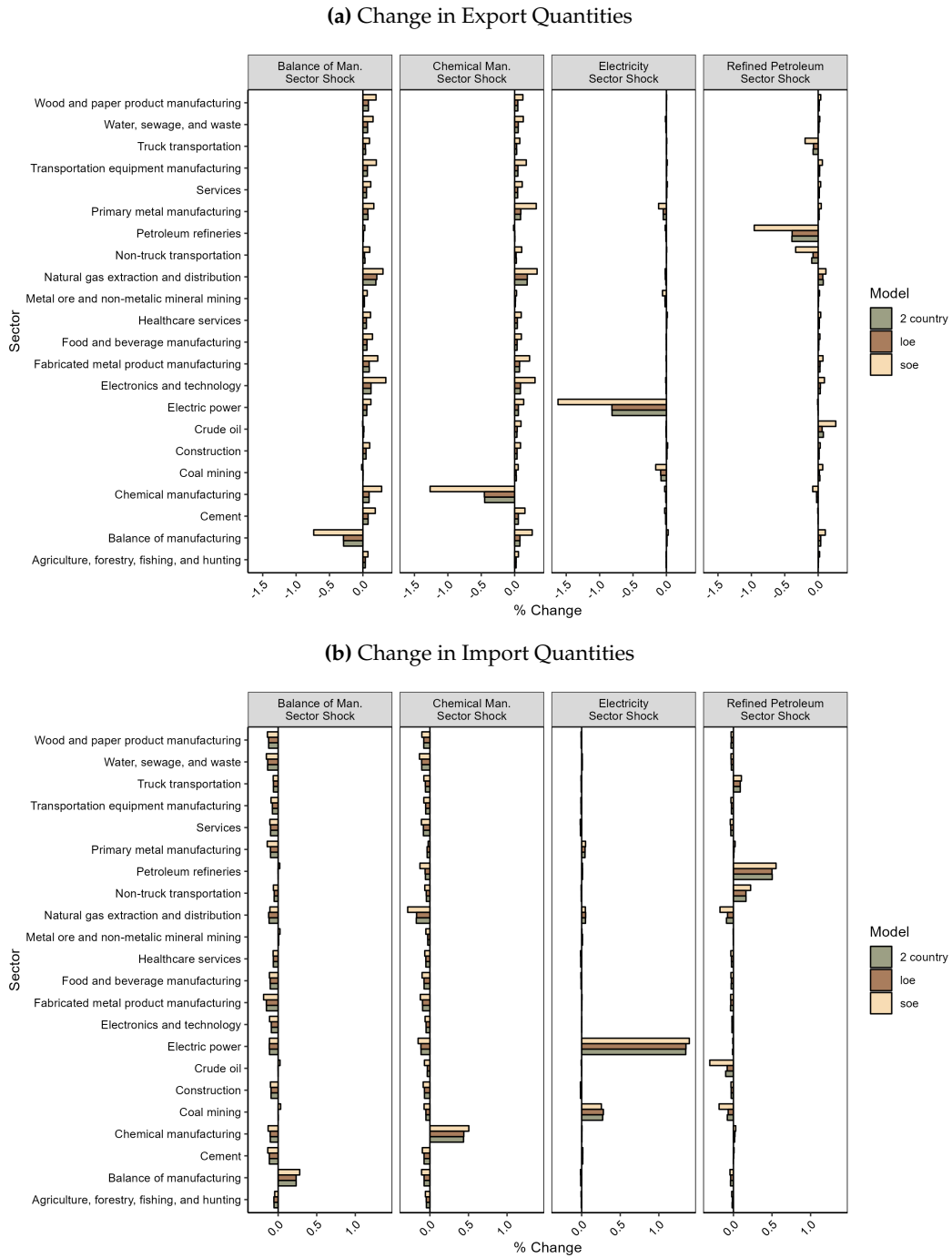


exercise we are mainly interested in exploring the efficacy of the reduced form LOE approach as compared to both the multi-regional and SOE frameworks. Thus, our comparative exercise is limited to a comparison of the changes in both quantities and prices for imports and exports across the model variants as well as the overall welfare implications.

Figure 6 reports the percent change in export and import quantities across the three model specifications and policy scenarios. In all modeled scenarios, the productivity shock increases the cost of production leading to both a decrease in output levels and an increase in the output price. Figures 6a and 6b illustrate that this produces a decrease in the level of exports and an increase in the level of import substitution for the regulated sector. The magnitude of the change, however, depends on the representation of trade. The SOE model produces larger quantity impacts relative to the two country and LOE model specifications. This is driven by the assumption of perfectly flat export demand and import supply curves since changes in the equilibrium outcomes will be born completely in quantity space. Introducing a downward sloping demand curve and upward sloping supply curve elicits a muted quantity impact due to changes in prices. Notably, because simulated and calibrated import elasticities are large, the small open economy trade representation produces very similar changes in import quantities relative to the other model variants. Finally, and most importantly, the quantity changes in the LOE and two country models are almost identical.

We report the percent change in export and import prices in Figure 7. For the reasons described above, the small open economy model does not allow for impacts to export and import prices. In Figure 7a, the export price increases in the regulated sector because the cost of production rises to comply with the regulation. Export price responses from the single country LOE model are close to, yet slightly smaller, than responses from the two country model. For imports, the price of the imported commodity that is substitutable with the regulated sector's output rises because of increased demand. While price responses are similar across the two country and LOE model variants, there are differences, and notably in the non-regulated sectors. This is due to the differences between the implicit cross-price elasticities in the calibrated LOE approach and the two country model. Since the import supply elasticities - either implicit or calibrated across the trade approaches - are large (e.g., import supply curves are *almost* flat), it can be challenging to precisely match the cross-price elasticities in the reduced form LOE approach, though the differences are numerically small.

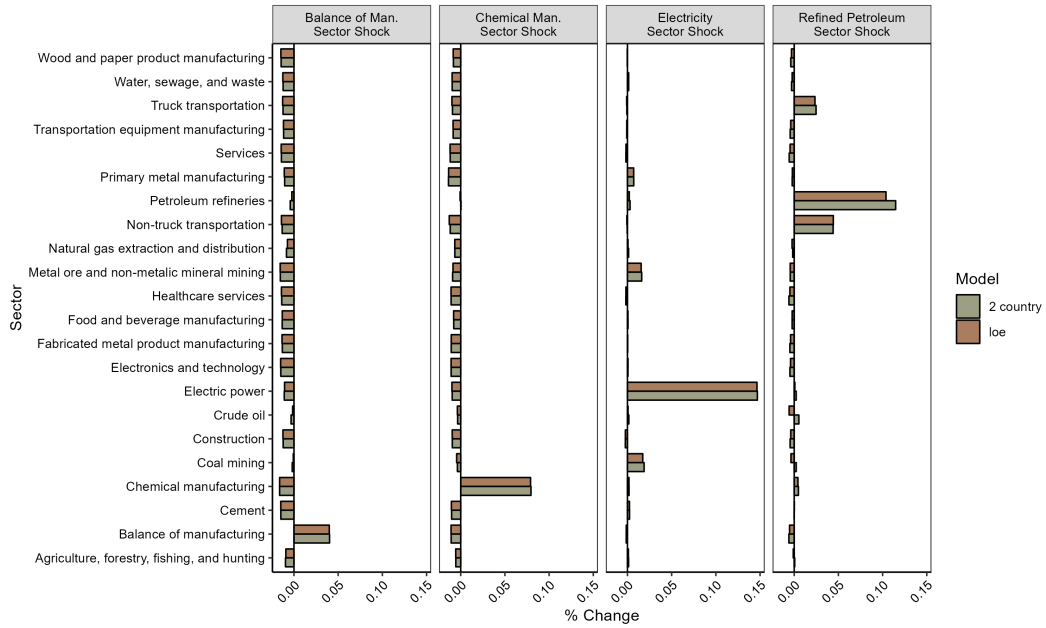
Finally, we report the welfare and terms of trade consequences across trade assumptions in Figure 8. We calculate welfare changes as equivalent variation as a percentage of total reference income. Across all model scenarios, the single region LOE model approximates the welfare effects of the multi-regional model closely. Both the multi-regional model and single region LOE model differ from the single region SOE model by capturing a terms of trade effect, or a change in the relative



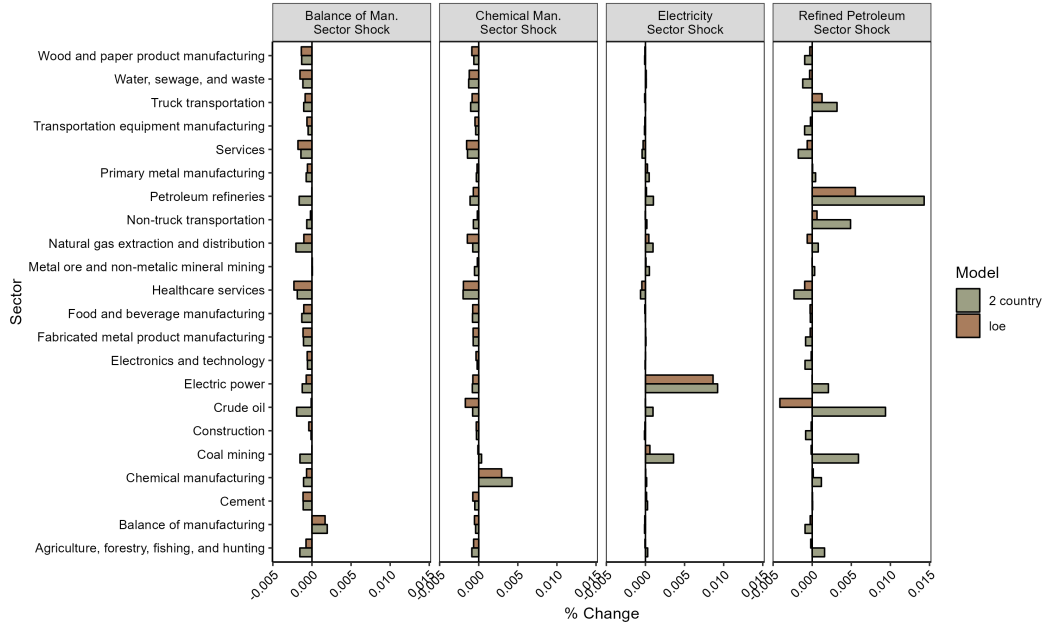
**Figure 6.** Percent Change in United States Export and Import Quantities

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

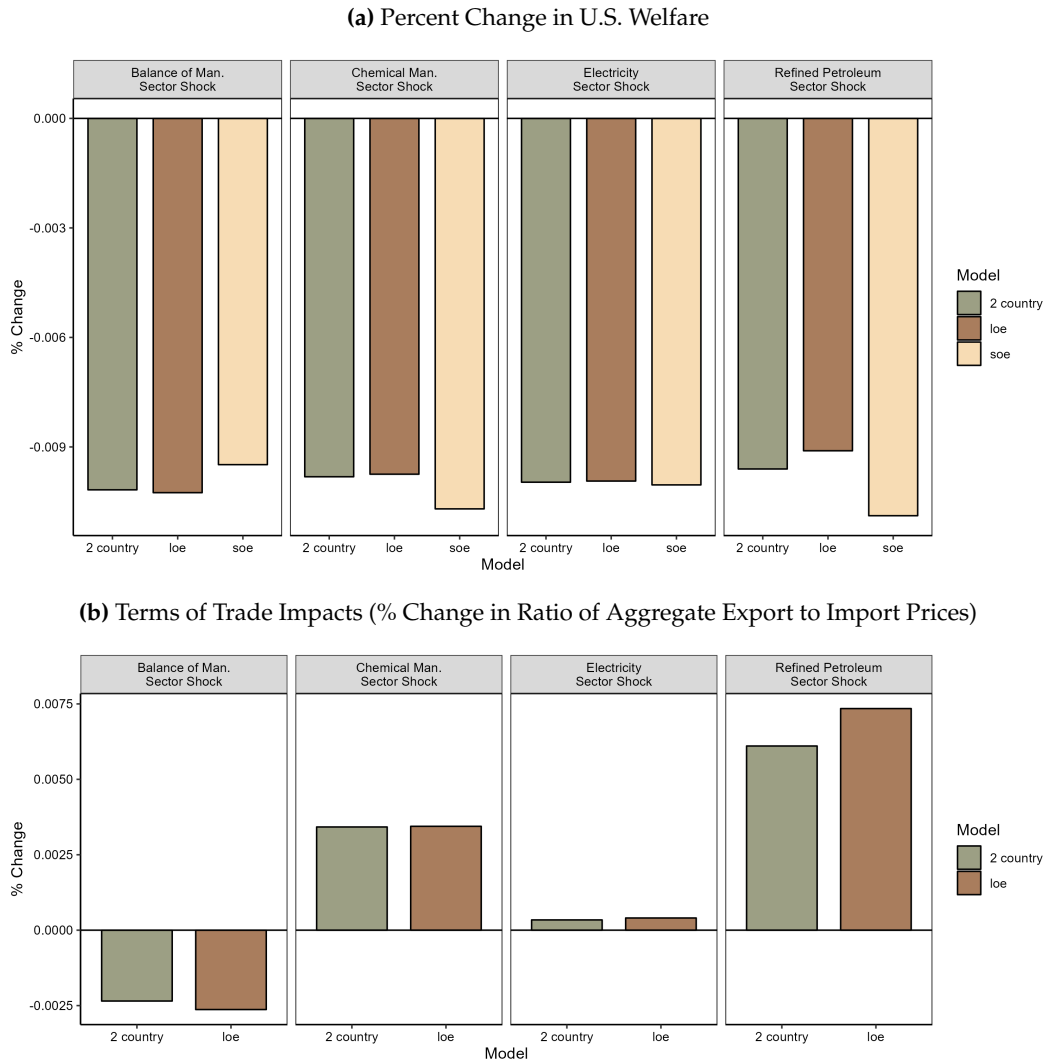
(a) Change in Export Prices



(b) Change in Import Prices



**Figure 7.** Percent Change in United States Export and Import Prices  
*Source:* Authors' calculations using GTAP9 and Lanz and Rutherford (2016).



**Figure 8.** Percent Change in U.S. Welfare and Terms of Trade

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).

value of exports to imports (here effectively comparing a price index for aggregate exports relative to a price index for aggregate imports). In general, the relative decrease (increase) in the value of exports relative to the value of imports means that less (more) of the imported variety can be bought for each exported good. This translates into a negative (positive) welfare consequence. Across our four illustrative shocks, the shock on the balance of manufacturing sector is unique in producing a reduction in the value of exports relative to imports and therefore both the multi-regional model and single region LOE model report smaller welfare costs relative to the SOE model. The opposite is true for the other scenarios. This

effect is modest for the electricity sector shock because only a small proportion of electricity is traded across international borders in the United States and therefore, all three model varieties produce very similar welfare impacts. To verify whether these findings are consistent for a range of shock sizes, we run the models with a range of illustrative shocks from \$1 billion to \$100 billion for each sector-specific shock. We find that these general results are insensitive to shock size (see Figure B.1 in Appendix B).

This section demonstrates that the reduced form approach for modeling export demand and import supply performs reasonably well compared to the internally consistent multi-regional international model. Its performance suggests that it is an attractive modeling framework for single country models with a large open economy representation where explicit linking to a multi-country model such as GTAP is impractical. Moreover, this section also highlights an additional terms of trade channel that can produce meaningful differences in welfare impacts from a regulatory policy shock that targets sectors that produce traded commodities relative to a small open economy model. In some cases, the LOE model can pass some of the welfare costs to the rest of the world through the form of relatively higher export prices, and in other cases, the opposite is true. In these illustrative simulations, the terms of trade effect caused an increase in welfare costs of roughly 8% in the balance of manufacturing shock and reductions in welfare costs between 1-16% in the remaining sector-specific shocks. The magnitude and direction of this effect is driven both by the level of trade exposure for a given commodity type (e.g., size of imports and exports) and the underlying parameterization of the export demand and import supply functions.

#### 4.2 SAGE

In this section, we use the simulated elasticities from the GTAP framework to parameterize the large open economy assumption in the U.S. EPA's SAGE model, v2.0.1 (Marten, Schreiber, and Wolverson, 2021). SAGE is a dynamic multi-sectoral, multi-regional computable general equilibrium model of the United States economy with perfect foresight. We illustrate how the calibration procedure described above can translate into a dynamic framework and compare the outcomes of several illustrative productivity shocks (mimicking the static shocks in the previous section) on export/import prices/quantities, welfare and terms of trade across alternative trade assumptions.

The SAGE model is generally structured similar to other CGE models designed to assess environmental and energy policies (e.g. Yuan et al. (2019)). The model features four sub-regions (defined by Census Region), 23 sectors (akin to Table 1 with an added plastic and rubber manufacturing sector), five household quintiles, five year time steps from 2016-2081, and is calibrated and parameterized using several data sources. Assumed substitution elasticities reflect the available estimates from the empirical literature (including Armington elasticities taken from

GTAP). The benchmark equilibrium is largely based on 2016 IMPLAN data (IMPLAN Group LLC., 2016). However, the baseline is augmented using information from Census, the Bureau of Economic Analysis, the Congressional Budget Office, the Energy Information Administration, and the Bureau of Labor Statistics. Baseline refinements capture marginal and average income tax rates, disaggregation of production between crude oil and natural gas, projections of energy intensities, government finances and income from the rest of the world and heterogeneous labor productivity growth by sector. Export supply and import demand is governed by the Armington assumption, where goods are differentiated by place of production (Armington, 1969). The import aggregation function distinguishes imported varieties from the regional (defined at the Census Region), national, and international markets through a nested CES function. Conversely, production is allocated to the regional, national, and international markets through a constant elasticity of transformation function. The model is written as a mixed complementarity problem in GAMS and solved using PATH. For more technical information on model structure, see Marten, Schreiber, and Wolvertson (2021).<sup>18</sup>

We calibrate the reduced form large open economy assumption in SAGE in a

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<sup>18</sup> Firms are assumed to be perfectly competitive and maximize profits subject to technological constraints. Production is generally characterized by nested CES functions that differentiate substitution possibilities between factors, energy inputs, and other material inputs. For sectors associated with natural resources, the model employs the use of fixed factors calibrated to empirical supply elasticities that adds an additional top-level nest in the production functions. For each modeled time step, factor markets are generally assumed to clear by region. The model distinguishes between new and extant capital such that new capital is malleable across sectors and subject to alternative production function assumptions relative to extant capital. Production with extant capital is assumed to operate with Leontief technologies. Though extant capital is fixed to a region, the model does afford very limited ability to shift the use of extant capital across sectors. While extant capital is based on base year capital stocks and depreciates into the future, the law of motion in the model adds new capital through endogenous investment decisions in each region. On the demand side of the SAGE model, consumers maximize their intertemporal per-capita welfare subject to a budget constraint composed of endowment income from capital returns, natural resources and time. Within each time step, each representative household has an intratemporal utility function that governs both the labor-leisure choice in the model as well as consumption of material goods. The within time step preference structure is characterized by a nested CES-LES (Linear Expenditure System) function with a CES aggregator in the top level nest between leisure and aggregate commodity consumption and an LES expenditure function for commodity consumption in the lower nest. We calibrate the leisure-composite material goods substitution elasticity to existing estimates for labor supply income and substitution elasticities. The LES sub-nest is calibrated to empirically estimated income elasticities by separating subsistence and discretionary spending. There is a single representative government representing all jurisdictions. The government raises revenue through ad valorem taxes on capital, labor, and production and balances its budget through lump sum transfers.

similar fashion to the methodology described in the static setting, however, with one key difference. Because SAGE is a dynamic model, we now must characterize the assumed temporal path of import supply and export demand elasticities. We modify Equation 10 to include a time subscript and, importantly, *within time step* relative prices:

$$\epsilon_{it} = \frac{\partial x d_{it}^{row}}{\partial (pe_{it}/pfx_t)} \frac{pe_{it}/pfx_t}{x d_{it}^{row}} = \frac{1}{-\theta_{it}^x} \quad (13)$$

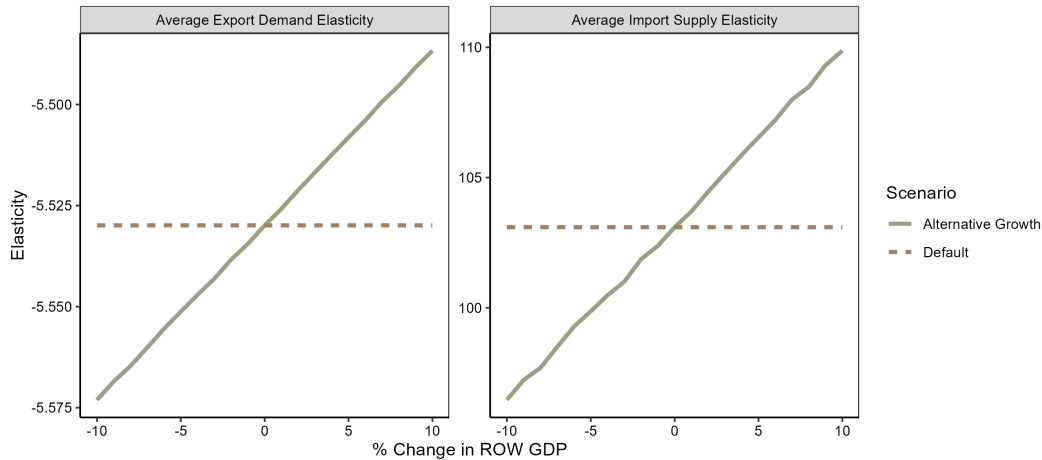
or  $\theta_{it}^x = 1 / -\epsilon_{it}$  for good  $i$  in time step  $t$ . Similarly for imports,  $\theta_{it}^m = 1 / (1 + \eta_{it})$ . We therefore must make an assumption regarding how our simulated elasticities in the base year propagate over time.

#### 4.2.1 Scenario Design: Implications of Economic Growth

Projecting export demand and import supply elasticities is dependent on assumptions made about the relative growth of the modeled country to the rest of the world. To inform our dynamic modeling simulations, we first assess the direction and magnitude of the effect of economic growth on trade elasticities using the GTAPinGAMS static model. Using the multi-regional model, we develop a routine that grows the ROW's GDP by a specified amount by scaling factor endowments to reflect differential growth relative to the modeled country. We then reassign benchmark parameters to the new "projected" equilibrium, and re-simulate export demand and import supply elasticities. Figure 9 reports the outcomes of these simulations, which illustrates the change in average elasticities when the ROW GDP grows at  $\pm 10\%$  leaving U.S. GDP constant. As the ROW grows relatively faster than the United States, both import supply and export demand elasticities get larger. This means that in the market for imports, the United States would tend toward a small open economy if the ROW continuously grows faster and on the export side, export demand elasticities would tend toward zero to reflect the ROW's growing market share. That is, the price of imports into the U.S. would tend toward the price of foreign exchange (less dependent on the fixed factor price) whereas in the market for exports, the U.S. export price would contribute less to the overall price of foreign exchange.

We use these results to consider four types of trade assumptions in the SAGE model: (1) *reference*, or when export demand and import supply elasticities remain constant over time, (2) *soe*, or the small open economy model where  $\theta_{it}^x = \theta_{it}^m = 0$  for all  $t$ , (3) *low\_growth*, which assumes that the ROW GDP grows by 5% leaving U.S. GDP unchanged (average export demand elasticities increase by 0.4% and average import supply elasticities increase by 3.4% each year), and (4) *high\_growth*, which assumes that the ROW GDP falls by 5% relative to an unchanged GDP in the United States (average export demand elasticities decrease by 0.4% and average import supply elasticities decrease by 3.1% each year). These growth assumptions are illustrative and designed to inform a sensitivity analy-





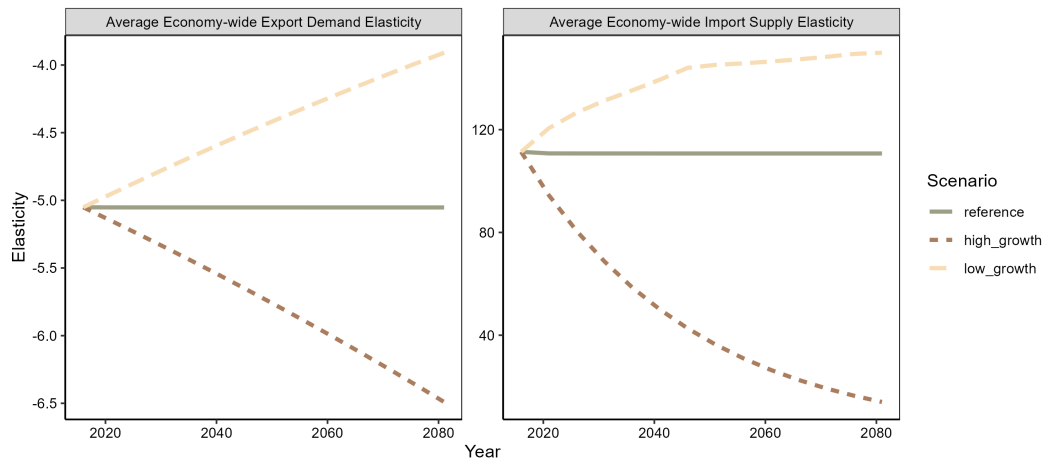
**Figure 9.** Trade Elasticities Under Differential Growth

Source: Authors' calculations using GTAP9 and [Lanz and Rutherford \(2016\)](#).

sis that is approximately representative of upper and lower bounds on possible growth projections.<sup>19</sup> Reflecting the illustrative nature of this sensitivity analysis, we use weighted average changes in export demand and import supply elasticities as opposed to sector differentiated elasticities growth rates, as the latter are sensitive to the method of projecting the economy forward. Furthermore, the static model does not explicitly model time. Therefore, the interpretation of *when* the impacts are expected to occur is inherently imprecise and can be roughly tied to assumptions on factor market closures and the elasticity structure in the model. The elasticities used to parameterize the GTAP model are long run measures, and therefore our interpretation of these growth rates as annual changes would be an overestimate. Figure 10 presents the calibrated (and simulated) weighted average elasticities for the three versions of the SAGE model implementing the reduced form LOE framework (elasticities are infinite in the SOE version).<sup>20</sup> In the large open economy specifications, we let the fixed factors associated with import supply and export demand grow at the steady state growth rate in the SAGE model.

<sup>19</sup> Furthermore, changes in elasticity estimates from asymmetric economic growth assumptions are likely case specific. These rates are specific to the United States economy. Modeling exercises focused on other countries would likely need to test whether these sets of results are externally valid.

<sup>20</sup> To circumvent numerical issues due to very small numbers introduced by calibrating the reduced form approach with large import supply elasticities, we set an upper bound on elasticities equal to 150, which we have found approximates the small open economy assumption well.



**Figure 10.** Assumptions for Elasticity Projections (Implicitly Calibrated)

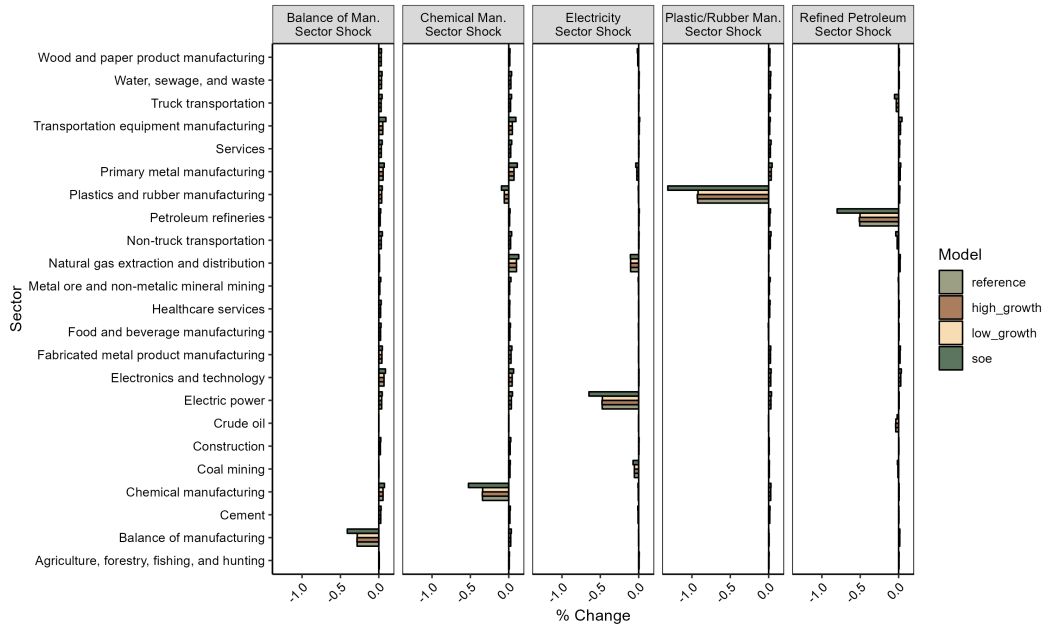
Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).

#### 4.2.2 SAGE Results

This section reports the results from modeling the same illustrative policy scenarios as used in the GTAPinGAMS simulations. With the exception that we also include a separate illustrative regulation in the plastics and rubber manufacturing sector. This sector is included in the chemical manufacturing sector in the GTAP database but is reflected as a separate sector in SAGE. Therefore, for completeness we consider this additional scenario in this section. For all cases, we consider a \$1 billion sector-specific Hicks-Neutral productivity shock that begins in 2021 and scales with output into perpetuity. We also assume that these shocks affect production with both new and extant capital.

Near term results are insensitive to economic growth projections. Figure 11 displays the percent change in export and import quantities between the policy equilibrium and the baseline in 2021. In this year, the SAGE model produces very similar impacts to the modeled shock in the GTAPinGAMS static framework. Across trade representations, the productivity shock increases the cost of production, raising the output price. This produces a decreased level of exports in the regulated sector. The model substitutes away from more expensive domestic and regional output toward more international imports. Like the static framework, the quantity impacts are partially muted in the reduced form LOE specifications relative to the SOE model because we introduce curvature in the import supply and export demand functions. Because the differential growth in export demand and import supply elasticities across scenarios is relatively small in 2021, impacts across the reference, high growth, and low growth scenarios are virtually identical. Figure 12 describes the price impacts in 2021. We normalize price impacts by the

(a) Change in Export Quantities



(b) Change in Import Quantities

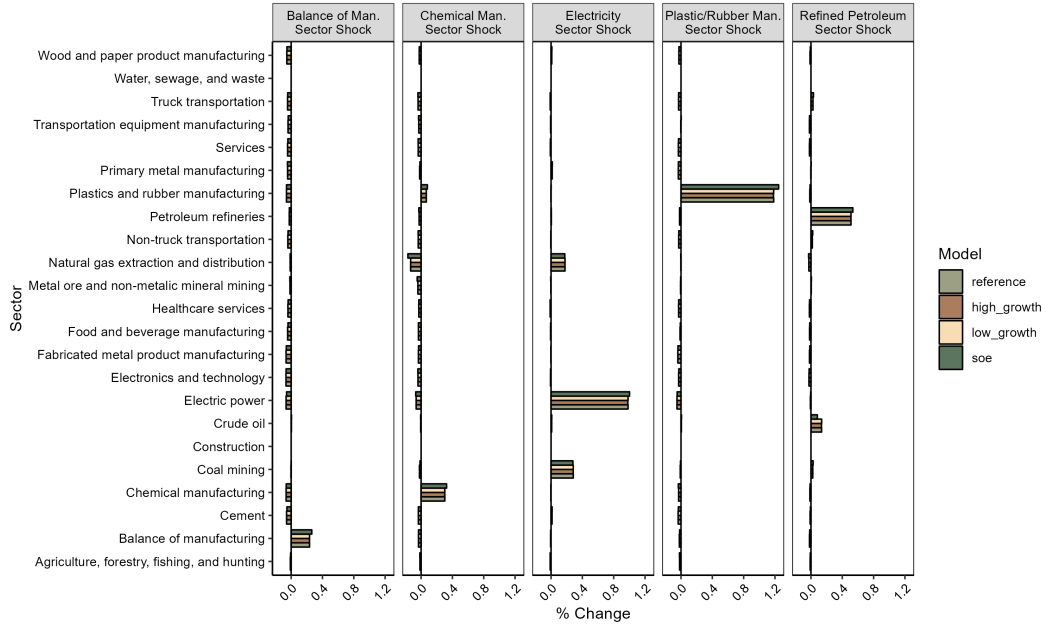
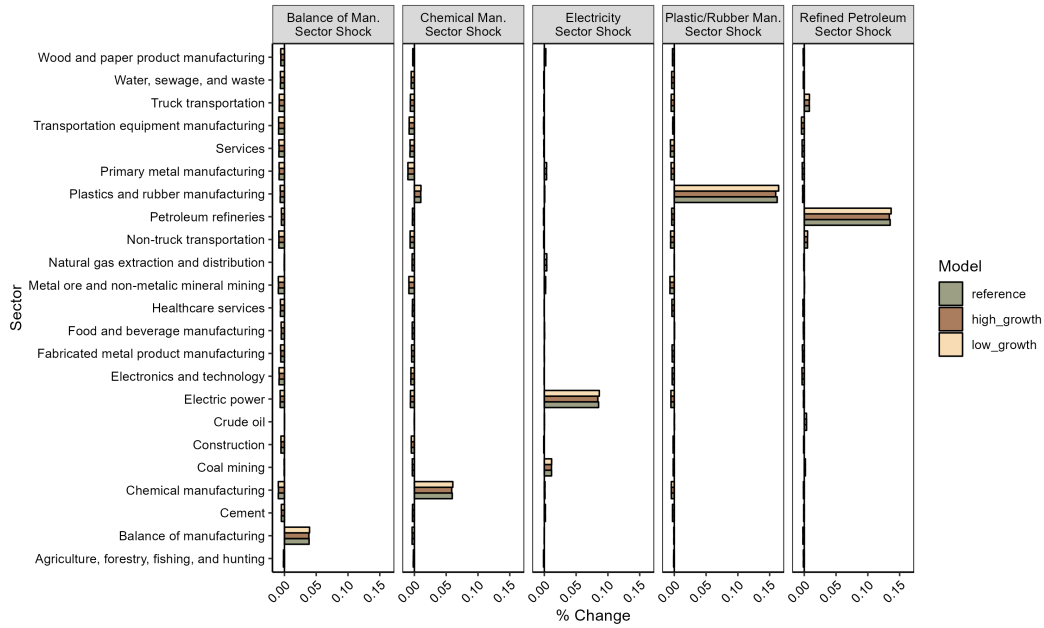


Figure 11. SAGE Percent Change in Export and Import Quantities in 2021

Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).

(a) Change in Export Prices



(b) Change in Import Prices

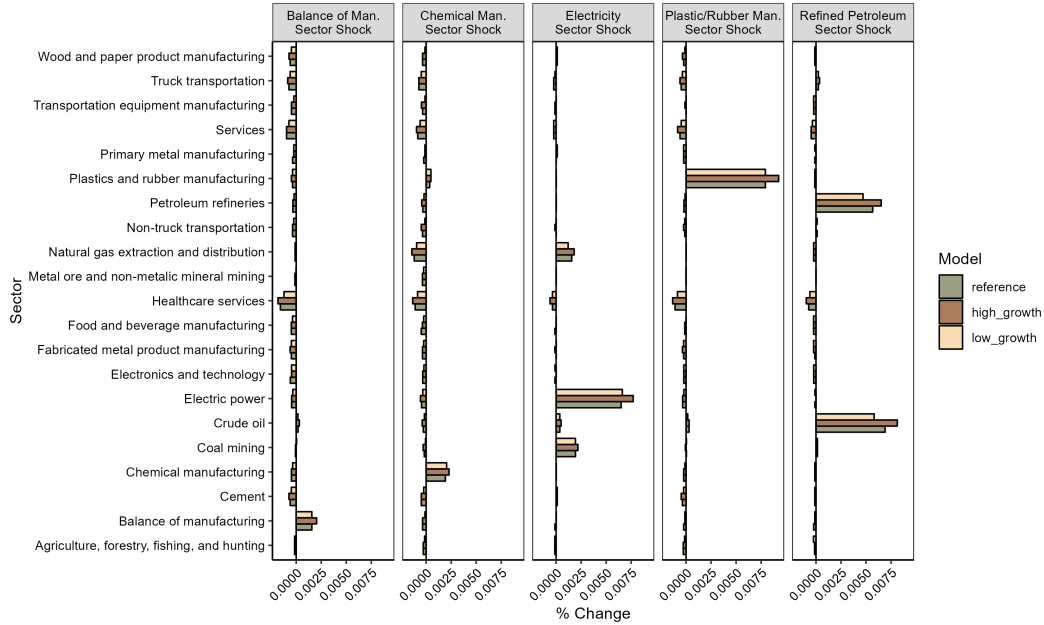
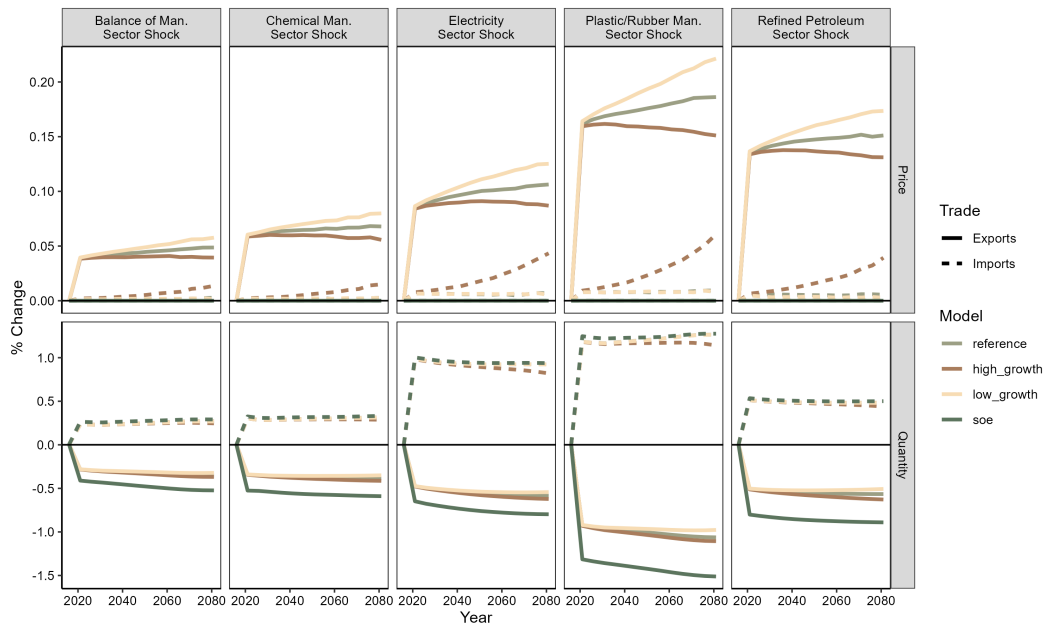


Figure 12. SAGE Percent Change in Export and Import Prices in 2021

Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).

within-period price of foreign exchange.<sup>21</sup> As with the quantity impacts, the percent change in prices are similar to what is produced in the GTAPinGAMS model. Across the model specifications, the low growth scenario produces relatively larger export price impacts (smaller export demand elasticities) and relatively smaller import price impacts (larger export demand elasticities), though, in 2021, differences are relatively small.



**Figure 13.** SAGE Percent Change in Quantities/Prices Over Time

Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).

In contrast, longer term export and import price and quantity impacts are more sensitive to projections of trade elasticities. Figure 13 presents the price and quantity impacts in the regulated sectors over the course of the model's time horizon. The percent change in price is reported above the percent change in quantities. In the reference representation of the LOE framework, the price and quantity of exports and imports change in the year that the shock is imposed and remain relatively constant for the remainder of the model horizon. Adjusting the elasticity projections to account for differential economic growth assumptions can lead to differences in export and import markets impacts farther out in the model's time horizon. The effects of the elasticity projections are more prominent for exports as

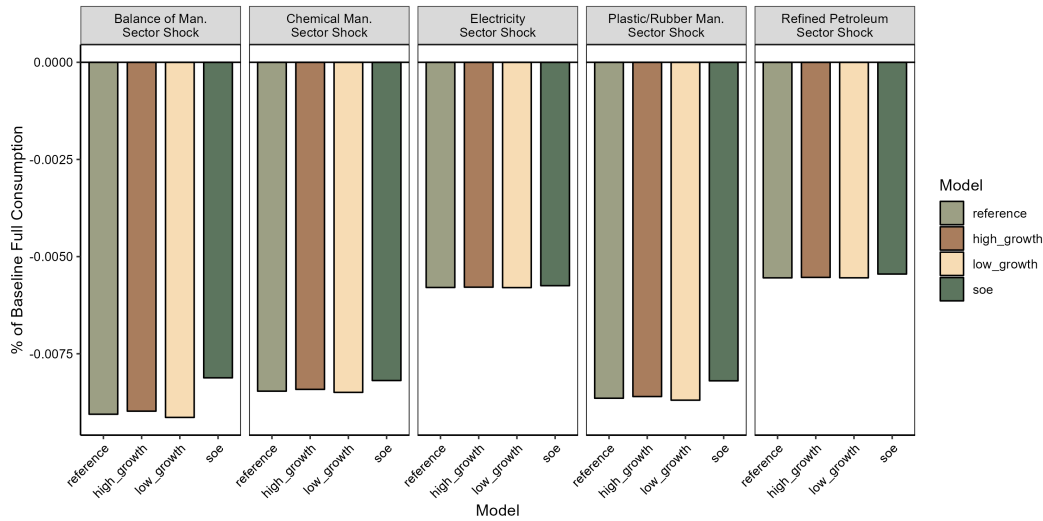
<sup>21</sup> In the SAGE model, we assign the numeraire as the price of foreign exchange in the base period. If we do not normalize price changes, the small open economy framework produces a percent change in prices due to changes in the price of foreign exchange calculated by the model relative to the base year (which is equivalent across sectors and regions).

the import supply elasticities are relatively large across all specifications, such that differences have less impact. Though the smaller import supply elasticities under the high growth scenario do shift supply price changes upward, with a more muted but opposite effect in the low growth scenario. In the high growth scenario, smaller export demand elasticities shift export price impacts downward, with the opposite effect in the low growth scenario. Taken together, these effects lead to lower levels of trade (exports and imports) in the regulated sector's commodity under the high growth scenario and higher levels of trade under the low growth scenario, relative to the reference specification.

Differential growth assumptions do not, however, translate into large differences in welfare costs in the United States, which are reported in Figure 14. In the SAGE model, we calculate welfare by computing equivalent variation for each of the 20 representative households within the model approximated for an infinite horizon (assuming prices and quantities follow their steady state growth paths in the post-terminal period) and then aggregating across households. We represent aggregate welfare as a percentage of an equivalent metric for the present value of full consumption (commodity consumption plus leisure). The magnitude of the welfare impacts are similar to those produced by the GTAPinGAMS model but can differ across sectors due to underlying differences in model structure and parameterization. The economy-wide terms of trade impacts in these simulations tend to be negative in trade affected sectors (see Figure B.2) causing welfare cost increases in the LOE specifications relative to the SOE specifications. The reference LOE specification produces a 1% to 12% welfare difference from the SOE specification. The high growth and low growth scenarios only produce a -1% to 1% difference from the reference LOE specification. This suggests that though the different growth scenarios can produce differences in export and import markets for the scenarios explored in this paper, the way in which elasticities are calibrated over time does not necessarily have significant impacts on estimates of welfare changes in the model.<sup>22</sup>

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<sup>22</sup> Notably, this result applies to the United States economy and may not be externally valid in models of other countries.



**Figure 14.** SAGE Welfare (% of Baseline Full Consumption)

Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).

## 5. Conclusion

In this paper, we assess numerical evidence on the potential importance of capturing terms of trade effects when modeling domestic policies and the potential for specifying a LOE within a single country CGE model using a reduced form approach. We find that, particularly in the market for exported goods, introducing a downward sloping export demand curve can pick up on important price effects otherwise missed by the SOE assumption often employed in single country models. We introduce a novel methodology for parameterizing the reduced form LOE representation of international trade from Yuan et al. (2019) in a single-country CGE model setting and test the efficacy of this framework against an internally consistent multi-regional international static framework. When calibrated correctly, we find that the reduced form framework is able to replicate impacts on imports and exports compared to the multi-regional international framework. This methodology is then implemented in SAGE, U.S. EPA's dynamic, large-scale CGE model of the United States, which we use to explore the implications of economic growth on trade elasticity projections. We find that a large open economy specification produces meaningful terms of trade impacts relative to a small open economy assumption leading to welfare estimates that are on the order of 1% to 12% different across a series of illustrative regulatory scenarios. However, these results are not very sensitive to capturing how differential economic growth between the U.S. and the ROW may impact international trade relationships.

This exercise highlights the general importance of considering a LOE specification when modeling a policy change in a trade exposed sector. Across both model-



ing applications, we find that the small open economy assumption can miss potentially important terms of trade impacts that can result in changes to household welfare. This represents a relatively understudied feature of non-trade related general equilibrium modeling applications. For instance, [Marten, Garbaccio, and Wolvertson \(2019\)](#) explore similar productivity shocks when evaluating the social costs of environment regulation and find that the composition of inputs used to comply with an environmental regulation was important for comparing the direct costs of compliance with the welfare consequences calculated by the model (where, depending on specific circumstances, welfare costs exceeded direct compliance costs). In this paper, we find that, for trade exposed sectors like manufacturing, the terms of trade impact leads to additional welfare consequences not considered in [Marten, Garbaccio, and Wolvertson \(2019\)](#) as they assumed a small open economy. This is likely true in other policy settings and could be the subject of future research. However, we also note that the relative value added of this additional welfare channel is dependent on context and it could be that other model features may potentially crowd out the LOE specification to limit model dimensionality concerns.

The reduced form approach outlined in this paper has several advantages and some caveats. Rather than build in a more complicated representation of the rest of world through linking with a multi-region model such as GTAP or explicitly building in the international structure, we are able to reasonably reproduce export and import price and quantity impacts simply through calibrating to a set of elasticities. This methodology translates a SOE specification into a LOE specification through the addition of at most  $2 \times n$  (where  $n = \#$  of commodities in the model) parameters. The total number of needed parameters may be even less depending on the magnitude of a given country's set of export demand and import supply elasticities (one could argue that a SOE assumption on imports may be a reasonable approximation in several cases). However, the reduced form framework does induce slightly different terms of trade impacts, but this difference is relatively small. Furthermore, export demand and import supply elasticities produced in this paper are entirely dependent on the GTAP model and are influenced by assumed Armington elasticities (and potentially other behavioral parameters) in the international economy framework. The magnitude of the simulated trade elasticities should be considered in the context of uncertainty in the underlying GTAP estimates.

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## Appendix A. Single Region GTAPinGAMS Model

### Single Region Model (single\_region\_model.gms)

```

$ontext
$model:gtap9

$sectors:
Y(g)$vom_(g)          ! Supply
MT(i)$vim_(i)        ! Imports and margin aggregate
YT(j)$vtw_(j)        ! Transportation services
FT(f)$$(sf(f)*evom_(f)) ! Specific factor transformation
LOE_M(i)$vxmd_(i)    ! Large open economy imports
LOE_X(i)$vxm_(i)     ! Large open economy exports

$commodities:
P(g)$$(vom_(g)-vxm_(g)) ! Domestic output price
PE(g)$vxm_(g)          ! Export market price
PM(i)$vxmd_(i)        ! Import market price
PMT(j)$vim_(j)        ! Import with margin composite
PT(j)$vtw_(j)        ! Transportation services
PF(f)$evom_(f)        ! Primary factors rent
PS(f,g)$$(sf(f)*vfm_(f,g)) ! Sector-specific primary factors
PFX                    ! Real exchange rate (SOE model)
PFIM(i)$fim0(i)       ! Fixed factor for imports
PFI(i)$fix0(i)        ! Fixed factor for exports

$consumers:
RA                    ! Representative agent
ROW                   ! Rest of world
GOV                   ! Government

$auxiliary:
TRANS                ! Budget balance rationing variable

$prod:Y(g)$vom_(g) s:0 t:etadx(g) m:esub_(g) va:esubva(g) i.tl(m):esubdm(i)
o:P(g)               q:(vom_(g)-vxm_(g)) a:GOV t:rto_(g) p:(1-rto_(g))
o:PE(g)              q:vxm_(g) a:GOV t:rto_(g) p:(1-rto_(g))
i:P(i)               q:(prod_shr(g,i,'domestic')*vdfm_(i,g)) p:((1+rtfd0_(i,g))/
prod_shr(g,i,'domestic')) i.tl: a:GOV t:rtfd_(i,g)
i:PMT(i)             q:(prod_shr(g,i,'imports')*vifm_(i,g)) p:((1+rtfi0_(i,g))/
prod_shr(g,i,'imports')) i.tl: a:GOV t:rtfi_(i,g)
i:PS(sf,g)          q:(prod_shr(g,sf,'factors')*vfm_(sf,g)) p:((1+rtf0_(sf,g))/
prod_shr(g,sf,'factors')) va: a:GOV t:rtf_(sf,g)
i:PF(mf)            q:(prod_shr(g,mf,'factors')*vfm_(mf,g)) p:((1+rtf0_(mf,g))/
prod_shr(g,mf,'factors')) va: a:GOV t:rtf_(mf,g)

$report:
v:XS(g) o:PE(g) prod:Y(g)

$prod:MT(i)$vim_(i) s:esubm(i)
o:PMT(i)           q:vim_(i)
i:PM(i)            q:vxmd_(i) p:pvxmd_(i) a:ROW t:(-rtxs_(i)) a:GOV t:(rtms_(i)
*(1-rtxs_(i)))
i:PT(j)           q:vtwr_(j,i) p:pvtwr_(i) a:GOV t:rtms_(i)

$report:

```

```
v:MD(i) i:PM(i) prod:MT(i)

$prod:YT(j)$vtw(j) s:1
o:PT(j) q:vtw_(j)
i:PE(j)$vxm_(j) q:vst_(j)
i:P(j)$ (not vxm_(j)) q:vst_(j)
i:PFX q:vst(j,"row")

$prod:LOE_X(i) s:1
o:PFX q:(vxm_(i)-vst_(i)+fix0(i))
i:PE(i) q:(vxm_(i)-vst_(i))
i:PFIX(i) q:fix0(i)

$report:
v:XD(i) i:PE(i) prod:LOE_X(i)

$prod:LOE_M(i) s:1
o:PM(i) q:vxmd_(i)
i:PFX q:(vxmd_(i)-fim0(i))
i:PFIM(i) q:fim0(i)

$prod:FT(sf)$evom_(sf) t:etrae(sf)
o:PS(sf,j) q:vfm_(sf,j)
i:PF(sf) q:evom_(sf)

$demand:RA s:0
d:P(cd) q:vom_(cd)
e:P("i") q:(-vom_("i"))
e:PF(f) q:evom_(f)
e:PFX q:incadj0 r:TRANS

$demand:GOV
d:P("g") q:vom_("g")
e:PFX q:govdef0
e:PFX q:(-incadj0) r:TRANS
e:PFX q:vb("usa")

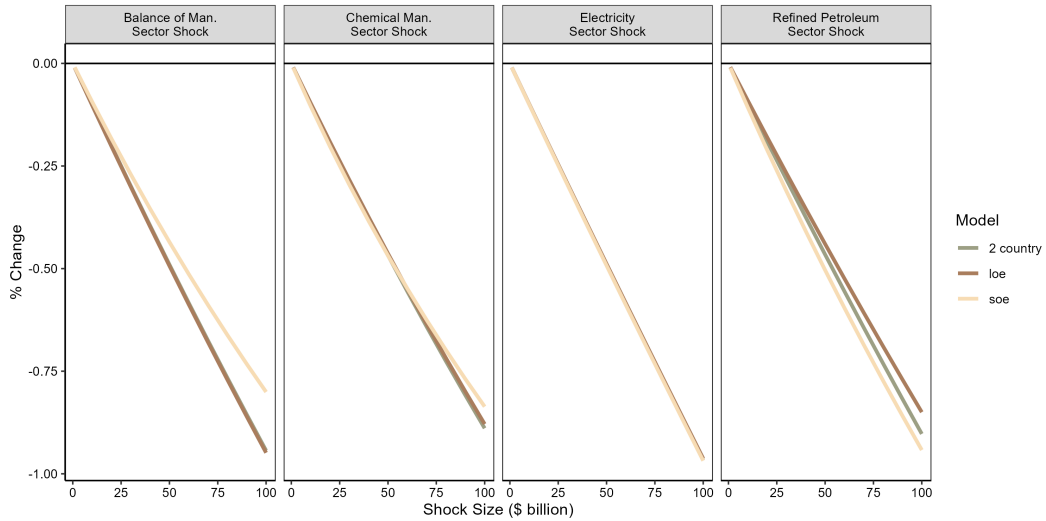
$demand:ROW
d:PFX
e:PFX q:rowpfx
e:PFIX(i) q:fix0(i)
e:PFIM(i) q:fim0(i)
e:PT(j) q:(-sum((i,s), vtwr(j,i,s,"row")))

$constraint:TRANS
GOV =e= P("g")*vom_("g");

$offtext
```

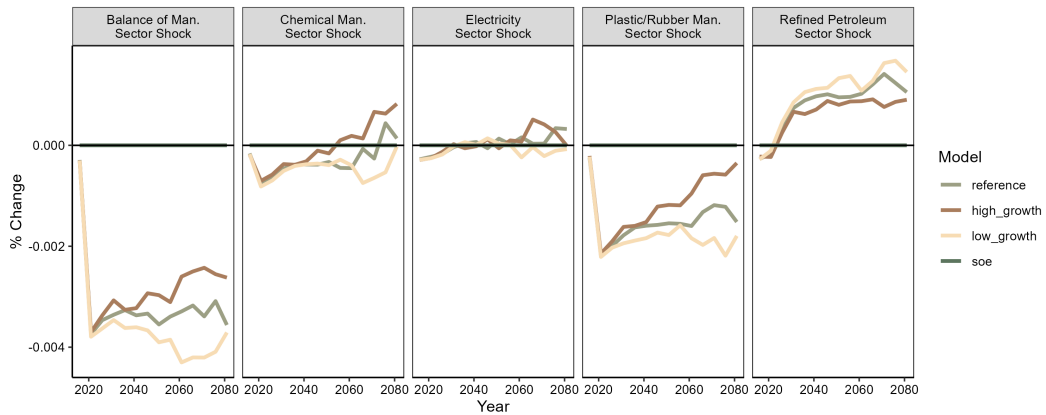
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**Appendix B. Additional Figures and Tables**



**Figure B.1.** GTAPinGAMS: Percent Change in U.S. Welfare Across Shock Sizes

Source: Authors' calculations using GTAP9 and Lanz and Rutherford (2016).



**Figure B.2.** SAGE: Terms of Trade Impacts (% Change in Ratio of Aggregate Export to Import Prices)

Source: Authors' calculations using v2.0.1 of the SAGE model calibrated to elasticity estimates from GTAP9 and Lanz and Rutherford (2016).