Journal of Global Economic Analysis, Volume 2 (2017), No. 1, pp. 1-119.

The Standard GTAP Model, Version 7

By Erwin L. Corong[a], Thomas W. Hertel[a], Robert A. McDougall[a],
Marinos E. Tsigas[b] and Dominique van der Mensbrugghe[a]

Abstract: This paper provides complete documentation for version 7 of the 'standard' Global Trade Analysis Project (GTAP) model. This is the first comprehensive documentation of the model since the 1997 'GTAP book' and this updated version includes some important new features. On a substantive level, commodities and activities are separated, allowing for multi-product sectors, as well as multiple sectors producing the same commodity. Additional flexibility is provided for modeling of production and consumption behavior, and the valuation and naming conventions have been modified. In addition, this paper folds in important advances since the 1997 publication, including the revised treatment of non-homotheticity in final demand, the welfare decomposition and multi-modal international transportation. The paper opens with an overview which puts this widely used model in broader context. The model exposition is comprehensive and includes a bridging table linking the original, 'classic' model with the current version. This is followed by a section discussing the major extensions of the standard model and how they are being used. The paper closes with an overall assessment and a discussion of future research directions.

JEL codes: C68, D58, D60, F1

Keywords: Applied general equilibrium analysis; international trade policy; energy policy; climate change; global economic analysis

1. Introduction and overview

The purpose of this article is to document the latest version of the 'standard' Global Trade Analysis Project (GTAP) model. GTAP is a comparative static, global general equilibrium model, suitable for entry-level use—albeit suitably complex to be used for a wide variety of informative policy analyses—and as a basis for specialized extensions as detailed in Section 4. Another critical role for the model is to lend analytical structure to the GTAP Data Base—a product which is widely used in the global economic modeling community (Aguiar et al., 2016). Developing such a database in the absence of a model risks delivering a product with serious

---

[a] Center for Global Trade Analysis, Department of Agricultural Economics, Purdue University, 403 West State Street, West Lafayette, IN 47907, USA ((corresponding author: ecorong@purdue.edu), (hertel@purdue.edu), (rmcdougall@purdue.edu), (vandermd@purdue.edu)).

[b] United States International Trade Commission, 500 E Street, SW Washington DC, 20436, USA (marinos.tsigas@usitc.gov).

gaps. By specifying a standard model which can be run with the GTAP Data Base and parameter files, we ensure that this can be used directly, or with selective augmentation, by a large number of individuals. Indeed, there are currently more than 15,000 individuals in the GTAP network—most of whom draw on this database in some way, and many of whom use the standard model for their work in global economic analysis.

The original—and only complete—documentation of this model (outside of the code itself, which is maintained up-to-date on the GTAP web site) is contained in the second chapter of the 'GTAP book', published in 1997 by Cambridge University Press (Hertel, 1997). Since that time there have been many changes including: (a) the addition of an indispensable welfare decomposition module (Huff and Hertel, 2001), (b) a thorough reworking of the behavior of final demand in the model (McDougall, 2003), and (c) the disaggregation of international trade and transport activity, by mode of transport (Hertel et al., 2000). Taken on its own, a paper documenting the full model in its current form would be useful. However, in addition, two of the co-authors of this paper have recently incorporated a number of valuable new features into the standard model in response to the widening array of model applications. These offer the user greater flexibility. The most important of these features is the option for some sectors to produce multiple products (e.g., by-products from biofuel production as in Taheripour et al. (2010)), and the option for multiple sectors to produce the same or a closely substitutable product (e.g., electricity from multiple generation sources as in Peters (2016b)).

The GTAP model builds on a rich tradition of general equilibrium models—both single-region and global in scope. It was immediately preceded by the Sectoral Analysis of Liberalising Trade in the East Asian Region (SALTER) model (Jomini et al., 1994), developed at the Australian Productivity Commission in the late 1980's by a team which included one of us (McDougall). The SALTER model was, in turn, heavily influenced by the WALRAS model (Burniaux et al., 1990), developed at the OECD for analysis of agricultural trade policy in the industrialized economies of the world. WALRAS built on computable general equilibrium (CGE) modeling foundations established by John Whalley (Whalley, 1984), and John Shoven and John Whalley (Shoven and Whalley, 1992), as well as Victor Ginsburgh and Jean Waelbroeck (Ginsburgh and Waelbroeck, 1981). In addition, SALTER, and in turn, GTAP, were heavily influenced by the work of Peter Dixon and collaborators (Dixon et al., 1982). Dixon built on the work of Leif Johansen (Johansen, 1960) who developed the first computable general equilibrium model which involved totally differentiating the non-linear model and expressing the equations in terms of elasticities, cost shares and percentage change variables. This greatly facilitates analysis of the economic mechanisms at work in any given simulation—a point which will be further illustrated in this paper.

Of necessity, this is a long paper. It must develop and explain all the equations in the model. However, we have organized it in a way which will facilitate readers accessing only the parts of the paper which they need at a given point in time. Section 2 of the paper gives an overview of the GTAP model—its main design features, comparable to many other global CGE models, and some of its inevitable limitations. This is a key section for those who are unfamiliar with this type of global economic modeling. Section 3 develops the full model. However, it is broken into sections with different components. For those readers who are already familiar with the standard model, and want to learn what is new, Tables A.1 to A.4 in the Appendix provide a concise summary of old vs. new sets, parameters, input data flows and variables in the model. There are also code 'snippets' in the text which will provide those familiar with the General Equilibrium Modelling PACKage (GEMPACK) software direct access to these new features, as implemented in version 7 of the GTAP model (the *model code* along with aggregated versions of the GTAP Data Base are available as supplementary files published with this paper). Footnotes with signpost [*NEW*] are likewise added to flag the model's new features. For those who just want to understand the theory behind the model, easy-to-read, algebraic equations for each module of the model are provided, along with a discussion of their theoretical underpinnings and how each section of the model works. So, while Section 3 is at the heart of this paper, it need not be read in its entirety to be useful to the reader.

Section 4 summarizes key extensions of the model which have attracted widespread use and citations over the past two decades. These cover (a) model extensions which introduce new economic theory (e.g., imperfect competition, heterogeneous firms), (b) versions which seek to support particular types of policy analysis (e.g., climate mitigation, agricultural policy, migration, poverty reduction), and finally, (c) model versions which provide a bridge to biophysical data and the scientific modeling community (e.g., water, land and carbon). All of these build on the standard model. Where possible, the extensions are treated in a modular fashion, with the addition of equations at the bottom of the model file, accompanied by closure changes and perhaps additional parameters allowing for only modest alteration of the standard model. In this way, those who have mastered the standard model can gain ready access to a host of more advanced tools for analyses. This section will be of interest, both to newcomers who wish to learn all the different ways in which this framework is applied, as well as to veterans who may not be familiar with some of these new application domains.

The paper concludes with a section discussing strengths, limitations, and future directions for the GTAP modeling framework.

2. Design choices

General features

This section seeks to place the GTAP model in the context of the broader family of global economic models by enumerating key design choices which have gone into its formulation. As such it will help the reader to understand both its strengths and its limitations.

GTAP is *global*: It covers not a single country, or a select group of countries, but the whole world. There are no trade flows between countries represented in the model and some residual "rest of the world", since all countries are fully represented in the model. That is not to say that all countries are individually represented: some are incorporated in regional aggregates. Furthermore, the representation of each region follows a common template—albeit differentiated by differences in the base data and key behavioral elasticities. This stands in contrast to models in which each region has a different design as is the case with the Basic Linked System (BLS) model (Fischer et al., 1988). While intellectually appealing, allowing each region to follow a different structure makes teaching, interpretation and extension of the model challenging.

GTAP is a *general equilibrium* model. Unlike a partial equilibrium model, it does not confine itself to one sector, or a small group of sectors; unlike a macroeconomic model, it does not treat all production and consumption as being of a single good, or a very small number of stylized goods (exportable and importable, or tradable and non-tradable). Instead, it represents an economy of many goods, produced by many sectors.

GTAP is a *comparative static* model. A GTAP simulation presents not changes through time, but differences between different possible states of the global economy —a *base case* and a *policy case*—at a fixed point in time, or with respect to two points in time (base period vs. a future projections period). It can, nonetheless, readily be converted into a recursive dynamic model such as the GDyn (Ianchovichina and McDougall, 2000) extension of the GTAP model.

GTAP rests on an *input-output* accounting framework. The framework is complete, in that all sources and uses of each economic good are accounted for, as are all inputs into production.[1] Wherever a cost is incurred or a benefit obtained, it is accounted for as usage of specific products or primary factors. When we speak here of completeness, we mean of course completeness within the theory of the model, not necessarily in representation of the world.

GTAP has proven successful as a relatively generic, broad-based general equilibrium model. We use it in GTAP *short courses* as a vehicle for training economists with little or no background in general equilibrium modeling, with the intention of promoting their speedy engagement in practical policy analysis. To that end, it is appropriate to confine the features of the *standard model* to those required in a broad range of analyses. The standard model is also widely used in real-time policy analysis as well as serving as the basis for many specialized extensions (see Section 4).

---

[1] The GTAP Data Base can readily rest on a Social Accounting Matrix (SAM) framework that is detailed in Section C in the Appendix. The section also highlights the fundamental accounting relations between the GTAP model and a SAM.

Implementation

The standard GTAP model is implemented using the *GEMPACK* suite of economic modeling software (Harrison and Pearson, 1996). As usual in GEMPACK, the equations of the model are recorded not in levels but in *percentage change form*. So for instance in the GTAP code the production variable, *qo*, represents not the level Q of production, but the percentage differential, $dQ/Q \times 100$, taken along a path between two states of the world economy. However, despite its apparently linear representation, the model is non-linear due to the formulae and update equations, which result in changes in the underlying shares and price elasticities. Its solution requires non-linear methods (Dixon et al., 1982, chapter 5). Nonetheless, GEMPACK produces the identical solution obtained from a model implemented using the same underlying non-linear equations, as is customary in the General Algebraic Modeling Software (GAMS) software environment (Hertel et al., 1992; Horridge et al., 2013).

Use of the percentage change representation complicates simple equations and simplifies complicated behavioral equations. In the simple, adding up equations (e.g., market clearing), new *share coefficients* appear; in behavioral equations, complicated expressions involving intensity parameters are replaced again by simple share coefficients. These share coefficients are calculated from a database comprised by input-output and trade accounts, expressed in money values. A point of surprise to some new users is that the database does not include, and the model does not need, explicit data for quantities or prices; apart from a relatively small set of behavioral parameters (elasticities of substitution, and the like), the required coefficients can be obtained as ratios of money values.

As the intensity parameters disappear, so too does the need to calibrate them. And since the database already represents a set of world economic accounts, solution of the model does not entail creating a representation of a state of the world, but perturbing a representation of one state to obtain an alternative state.

The *closure*, or partition of the variables into endogenous and exogenous components, is not fixed in the theoretical structure, but set by the user for each simulation. Different closures may be used to represent different economic environments, or for different lengths of run. For a short-run simulation, for instance, one might fix the wage rate, while for a long-run simulation, the level of employment might be fixed.

General features of the theory

For a given length of run, the model represents a corresponding *equilibrium position* of the economy. For a run of five years, for instance, in simulating a policy shock applied in the year 2020, the initial database for the simulation might represent the medium-run equilibrium in 2025 under business as usual, and the final database of the equilibrium under the applied policy change. Usually, however, we do not think of a precise length of run, but more broadly of a short-, medium-, or long-run equilibrium. There is an inevitable tension between the theoretical impulse for an equilibrium database, and the practical requirement that the database represent a world recognizable by policy makers.[2]

Almost everywhere, the theoretical structure derives from *optimizing behavior* by agents such as firms and households. Households maximize utility, firms minimize costs, and all agents are *price takers*. We adopt the fiction of a *representative agent*: the household sector consists of infinitely many identical infinitesimal households, an industry of infinitely many identical infinitesimal firms, so that each sector has the budget shares or input-output ratios of its component agents.

All equations in the model display *price homogeneity*: for any given solution, an alternative solution may be found by scaling all price and money value variables by a common factor, while holding quantity variables fixed. Under standard closures, exactly one price variable is held fixed; all other prices are evaluated relative to this *numéraire*. The theoretical structure therefore displays the *neoclassical dichotomy* between real variables and relative prices on the one hand, and the price level on the other hand. Consistent with the assumption of *constant returns to* scale, the standard model likewise displays *quantity homogeneity*. The price and quantity homogeneity conditions provide useful checks on theoretical modifications of the model.

As the model has nothing useful to say about price levels, it does not incorporate multiple currencies. The GTAP Data Base is denominated in millions of base year US dollars. In simulations with a numéraire, price variables are in effect prices relative to that numéraire; in other simulations, interpretation is a task for the user.[3]

In the standard model, all taxes[4] are expressed in *ad valorem* form; given the indeterminacy of the price level, attempts to introduce specific rate taxes entail adding more information to the database.

Related to the need for an exogenously set price level is *Walras*' law, applicable to a large class of general equilibrium models, under which equilibrium in all but one of the markets in the model implies equilibrium in the last market. One market-clearing condition should therefore be omitted; in GTAP this is the market for investment funds. The model therefore does not explicitly impose equality between global saving and global investment expenditure, but merely records divergence between them; this should be endogenously zero. Computationally significant deviations from zero indicate errors in the theoretical structure or imbalances in the data.

---

[2] For instance, external imbalances that cannot be sustained indefinitely may yet be sustained over considerable periods of time. The same holds for levels of saving and investment, and for abnormal profits or losses in individual industries.

[3] Unlike some global models such as Globe (McDonald et al., 2007), GTAP does not carry nominal exchange rates—all prices, irrespective of region, are to be compared relative to the model numéraire or some other price, or price index.

[4] The GTAP model is written in log-linear form with all taxes implemented as the power of the tax, i.e., 1 + ad valorem tax rate.

Absent from the standard model are adjustment processes, money, financial instruments, imperfect competition and increasing returns to scale, multiple households within a region, emissions of carbon dioxide and other pollutants. Many of these features are addressed in extended versions of the standard model (Section 4).

Accounting framework

The input-output structure provides a framework within which to account for supply and use of the economic goods of the model: products (goods and services), distinguished by region of origin, and primary factors. These are called in GTAP parlance *tradable* and *endowment commodities*, consistent with the former only being internationally traded. Sources of supply are importation and production; uses are *current production activities* (use in particular industries) and *final demands*, the latter comprised of investment (fixed capital formation), private consumption, government consumption and exportation.

Some special restrictions are imposed. The most notable restriction on trade is *sourcing at the border*: for each product, *all domestic agents in an economy use the same mix of imports* from different countries, though each agent chooses its own combination of imports with the domestic product. This greatly reduces the size of the database and simulation run time, but rules out more elaborate supply-chain analyses such as that of Koopman et al. (2014); however, a supply-chain version based on an *MRIO* (multi-regional input-output) extension of the GTAP Data Base (Section 4) caters to that need.

Also fundamental is the absence of *domestic margins*, the transport, sales and other services incurred between point of production or importation and point of use. On the theoretical side, these are addressed in various model extensions (Peterson, 2006; Corong, 2017), and work is in train to provide matching data on an ongoing basis.

As noted, there is no *international trade in primary factors*; this might be a concern in, for instance, modeling mode 4 of the General Agreement on Trade in Services (GATS) which relates to the movement of people across national borders (see GMig (Walmsley et al., 2007) extension in Section 4).

We make no distinction between cross-border trade in goods and consumption abroad (travelers' expenditures), though import duties are liable to affect the two flows differently.

There are no *re-exports*; only domestic products are exported. Recognizing re-exports would, in principle, allow us better to represent re-export services, however difficult it might prove in practice to operationalize such improvements.

There is no *inventory investment*, a limitation requiring deviation from input-output statistics, but in accord with the natural limitations of our model theory. And correspondingly, *working capital* is not recognized as a factor of production.

The external accounts cover only trade in products and net capital inflow; there are no *foreign income receipts or payments*, no *remittances* and no *international aid flows* (see extensions GDyn and GMig in Section 4).

In the government accounts, we have an extensive treatment of taxes and subsidies, but no *transfer payments* or *property income receipts*. Accordingly, the database does not reflect any concept of a government budget deficit (see extension MyGTAP in Section 4 for a more complete treatment of government accounts). The gaps in the government accounts situate the model within the *trade-oriented* (as opposed to the *tax-oriented*) stream of general equilibrium modeling.

The theoretical structure cannot accommodate negative capital earnings or intermediate usage, though these are liable to occur in official input-output tables, through, for instance, *loss-making activities* or *sales by final buyers*.

Time, investment and property income

The handling of future-oriented activities, saving and investment, in a single-period model is bound to involve some awkwardness, and GTAP is not exempt from this. In GTAP, we do not require that trade be balanced, but allow for trade deficits or surpluses, and matching capital inflows or outflows. At the same time, we do not recognize foreign income receipts or payments. So foreign assets are always being accumulated or run down, but never generate income flows. Greater theoretical coherence would be achieved by imposing trade balance on the database, but at the cost of drastic departure from the observed state of the world.

As noted below, in the regional household demand system, we treat saving as a good—a treatment with some advantages, but also some disadvantages. Some undesirable effects arise when net national saving is negative, as in a considerable number of countries the data indicate is the case. The treatment also involves defining a *price of saving*, a dubious concept, and one that leads to some inconvenience in the welfare decomposition (see Section 3).

It is usual in GTAP simulations to treat capital as mobile across industries in an economy, but fixed in aggregate within each region. This might be plausible for some classes of capital, for instance, office buildings, but it would be difficult to define a length of run in which it would be plausible overall.

Some of these issues find their natural resolution in GDyn (Section 4). An integrated treatment of inter-industry and international capital mobility, however, remains a research opportunity.

Special features

A *regional household* allocates regional income between private consumption, government consumption and saving so as to maximize the theoretical construct of regional utility. This unitary regional utility function, together with the reliance on optimizing behavior and the care taken numerically to balance the database, supports a rigorous *welfare decomposition*, distinguishing endowment, technological and allocative efficiency effects, divided into detailed subclasses (Section 3).

For the form of the private demand system, the most usual choice in applied general equilibrium modeling is perhaps the linear expenditure system (LES). For n goods this has n parameters—the expenditure elasticities—along with an average measure of substitutability, the so-called Frisch parameter. The most usual alternative approach is that of flexible functional forms, with approximately $n^2/2$ parameters. In GTAP we adopt Hanoch's intermediately placed, constant differences of elasticities (*CDE*) form (Hanoch, 1975), with $2n$ free parameters; this on the one hand is less demanding than the flexible functional forms, and on the other hand lets us calibrate to income elasticities and own-price elasticities independently, and importantly, is non-homothetic.

It is usual in general equilibrium modeling to treat some primary factors as fixed and some as mobile and across industries, with the division perhaps variable according to length of run, so that capital for instance is fixed in short-run simulations and mobile over the long-run. GTAP provides also the option of *sluggish* factors—mobile, but not perfectly mobile, between industries, according to some elasticity of transformation which can be calibrated to reproduce observed factor supply responses.

The two-level system of substitution between products from different sources—an import-domestic substitution nest above an import-import substitution nest—helps trade policy analysts display both limited import penetration from tariff reductions, by setting the import-domestic substitution elasticity relatively low, and modest terms of trade losses from unilateral tariff reforms, by setting the import-import substitution elasticity at a higher level.

In the database, rather than recording basic values and taxes, GTAP records tax-free and tax-paid values. This is a convenience in data validation, since it simplifies the sign conditions on the data: zero and strictly positive flows must match exactly between the tax-free and tax-paid arrays.

3. Model description

We now proceed to provide a detailed description of version 7 of the standard GTAP model.[5] We will describe the model using the circular flow logic of an economy outlined in Figure 1.[6] Here, production generates income accruing to endowments that is returned to the regional household and then spent on three sources of final demand: private expenditures, government spending and saving—which subsequently is translated into investment spending. Each source of spending, as well as purchases of intermediate goods comprise both domestic purchases and imported purchases, thereby generating both domestic and export sales by firms.

Before going into the details of the model specification, it is useful to describe the main price linkages for commodities—though the individual price equations

---

[5] We shall call version 7 of the GTAP model the *standard* GTAP model, i.e., it is the new standard. Version 6 will be referred to as GTAP 'classic'.

[6] The figure is an adaptation from Brockmeier (2001).

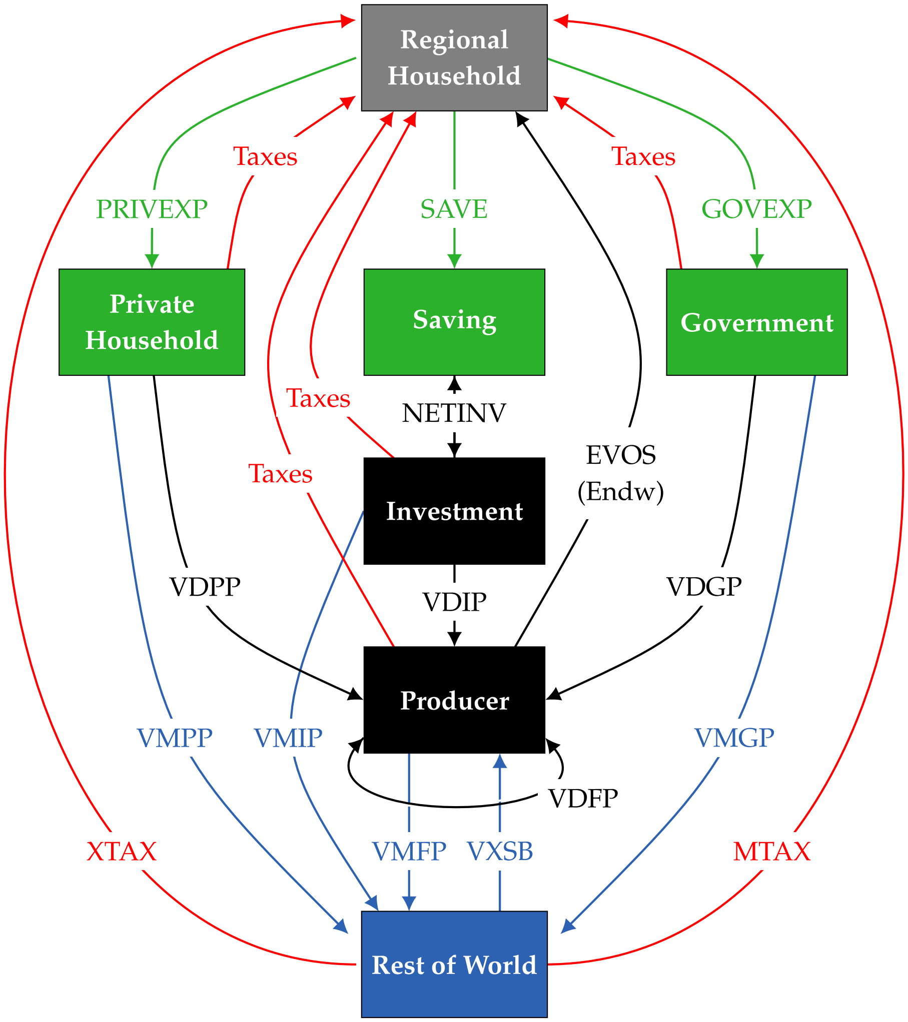

Figure 1. Circular flows in a regional economy

Notes: See Table A.3 for definition of each value flow

are introduced later. There is only one set of prices that truly determines all of the other prices. A natural way to think about these is as the prices which equilibrate supply and demand in this general equilibrium model. Thus we start with the market Price for Domestically Supplied commodity c in region r, $PDS_{c,r}$. For those familiar with the 'classic' GTAP model, a Bridge Table is provided at the end of Section 3, highlighting key differences between the current standard model and the 'classic' version. The top panel of Figure 2 depicts the various linkages. Working backwards to the suppliers of this commodity, we have the basic commodity- and activity-specific price, $PCA_{c,a,r}$. In the 'classic' GTAP model, where each activity produces a unique commodity, then $PCA_{c,a,r} = PDS_{c,r}$ for the appropriate correspondence between a and c. This is also the case when multiple activities produce a common commodity which is perfectly substitutable. However, in the new GTAP model there is scope for imperfect substitution at this stage, in which case these commodity prices are differentiated by activity, with the degree of differentiation governed by a substitution elasticity (discussed below).

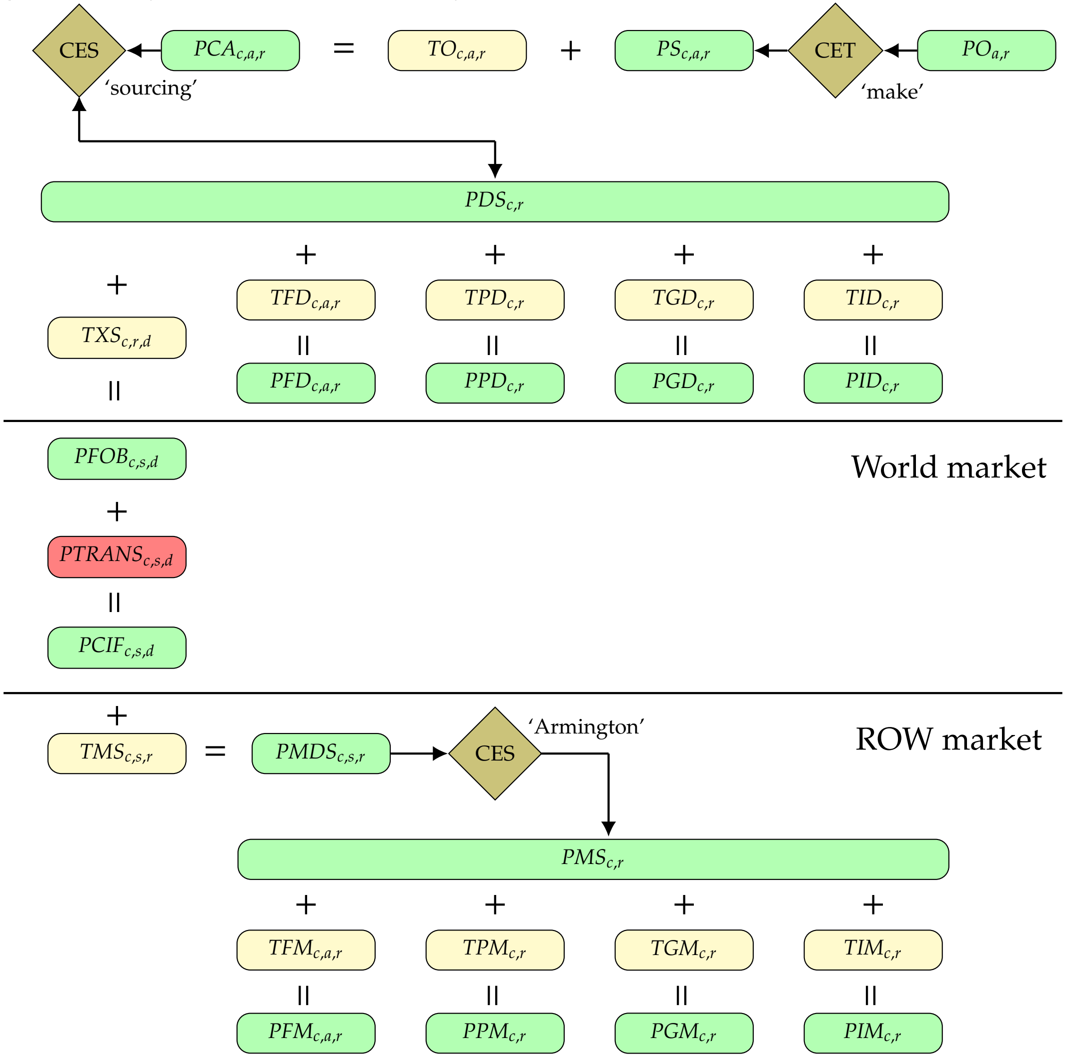

Figure 2. Price linkages in the model

Notes: See Table A.4 for a summary of price variables in the model

The basic commodity- and activity-specific price, $PCA_{c,a,r}$, is equal to the supplier's price ($PS_{c,a,r}$) plus a commodity- and activity-specific tax/subsidy ($TO_{c,a,r}$)—

in GTAP, all taxes are implemented as the power of the tax, i.e., 1 + tax rate[7] (this has the advantage of allowing for additive price linkage equations when the model is totally differentiated). In order to allow for multi-product activities, a 'make' matrix is introduced into this version of the GTAP model.[8] Therefore the supply prices must be aggregated before obtaining unit revenue associated with that activity. Due to zero profits, unit revenue must equal unit cost. Henceforth, we will refer to $PO_{a,r}$ as the *unit revenue of activity a in region r*. It is also useful to define an index for *unit revenue at basic prices* associated with sales by activity a in region r: $PB_{a,r}$. When employing the standard GTAP Data Base, the model will feature a diagonal 'make' matrix with a one-to-one correspondence between activity a and commodity c so that $PDS_{c,r} = PB_{a,r}$ for the appropriate correspondence between a and c.

From Figure 2, we see that domestic supplies are allocated across destination regions—the domestic market and all external destinations, i.e., bilateral exports. All of these sales of domestically supplied goods are priced at $PDS_{c,r}$. Export prices are obtained by multiplying $PDS_{c,r} \cdot TXS_{c,s,d}$ (where $TXS_{c,s,d}$ = 1 + export tax rate) and this converts the domestic supply price to the price of exports, $PFOB_{c,s,d}$, denoting the price before freight and insurance are added. Given the presence of a (potentially) bilaterally varying export tax, this price is now destination-specific. In the figure, the indices for the export price reflect the demand side, and not the supply side. The first regional index, in this case s, reflects the source region, and the second regional index, in this case d, reflects the destination region. The top of the figure has the source region as s, so from the supply side, the FOB price should be written as $PFOB_{c,s,d}$, where d is the destination region. The FOB price undergoes two further transformations en route to its final destination. A transportation margin ($PTRANS_{c,s,d}$) is added to the FOB price to generate the CIF price of imports, $PCIF_{c,s,d}$. Then a bilateral tariff ($TMS_{c,s,d}$) is added to the latter to generate the P̲rice of iM̲ports in the D̲omestic market by S̲ource, $PMDS_{c,s,r}$. A 'national' importer aggregates bilateral imports from all sources to 'produce' an aggregate import bundle with a P̲rice of iM̲ported S̲upplies, $PMS_{c,r}$.[9] Each agent in

---

[7] [NEW] The producer tax is now commodity- and activity-specific. It is introduced as a positive wedge between the producer cost and the basic price. In previous versions of the model, a producer tax was negative and a subsidy was positive.

[8] [NEW] Previous versions explicitly assumed a diagonal 'make' matrix with a one-to-one mapping between activities and commodities.

[9] In theory, it would be preferable to allow variation in the sourcing from individual exporters. However, there are two reasons for avoiding this. Firstly, the data are not there to support bilateral sourcing by agent. In practice, we are lucky if we can get the split of domestic and imported goods by sector/agent from database contributors (Aguiar et al., 2016). Secondly, if we were to introduce this bilateral sourcing by agent, we would have a large number of four-dimensioned arrays in the model (commodity × source × destination × sector) and this would create problems of model size for many users. For example, there are 140 regions and 57 sectors in the GTAP 9 Data Base. Full sourcing by agent would result in intermediate input arrays with more than 60 million elements! In light of the fact that the data to support such a model are not presently available, this seems like a poor choice.

the economy—firms, households, government and investment—access this common import bundle market at the common price, $PMS_{c,r}$, and which competes with domestically supplied goods which are priced at $PDS_{c,r}$. However, there are also agent-specific sales taxes which must be applied before reaching the prices actually paid by firms, private households, government and investors for the imported goods: $PFM_{c,a,r}$, $PPM_{c,r}$, $PGM_{c,r}$ and $PIM_{c,r}$, as well as for the domestic goods: $PFD_{c,a,r}$, $PPD_{c,r}$, $PGD_{c,r}$ and $PID_{c,r}$. The agents' price of the composite commodity obtained after aggregating the domestic and imported goods is represented by $PFA_{c,a,r}$, $PPA_{c,r}$, $PGA_{c,r}$ and $PIA_{c,r}$ respectively for firms, households, government and investment.

Appendix C has additional figures to illustrate the linkages across quantities in the model (Figure C.1) and the value linkages (Figure C.2).

3.1 Firm behavior

Each producing activity, indexed by a, combines a set of intermediate goods and factors to produce output. Similar to many CGE models, the production structure is based on a sequence of nested Constant Elasticity of Substitution (CES) functions that aims to re-produce the substitution possibilities across the full set of inputs. The nested structure, or "technology tree" is depicted in Figure 3. The top level nest is composed of two aggregate composite bundles—intermediate demand and value added.[10] The second level nests decompose each of the two aggregate nests into their components—on the one hand demand for individual intermediate goods (at the Armington level) and demand for individual factors.[11] A final nest decomposes demand for the composite good into domestic and imported components.

3.1.1 Top production nest

The composite index of output from activity a, represented by $QO_{a,r}$ in levels, or $qo_{a,r}$ in percentage change form, is a combination of an intermediate demand bundle, $qint_{a,r}$, with the value added bundle, $qva_{a,r}$. Equations (1) and (2) define, respectively, the demand for the two top level bundles where the key substitution elasticity is $ESUBT_{a,r}$ (typically assumed to be zero). Equation (3) (written more conveniently as a levels equation) represents the zero-profit condition for activity a, i.e., the total revenue of this activity is equal to the sum of all the input costs, which can be totally differentiated and simplified using the envelope condition to give (3′).

---

[10] [NEW] The aggregate intermediate demand bundle is new to this version of the GTAP model. In the previous version, intermediate demand was linked directly to output with an explicit Leontief assumption. The revised specification provides users with added flexibility in specifying production technologies.

[11] Labor inputs are normally classified into skilled and unskilled categories; it is also possible to distinguish up to five labor types as per GTAP 9 Data Base.

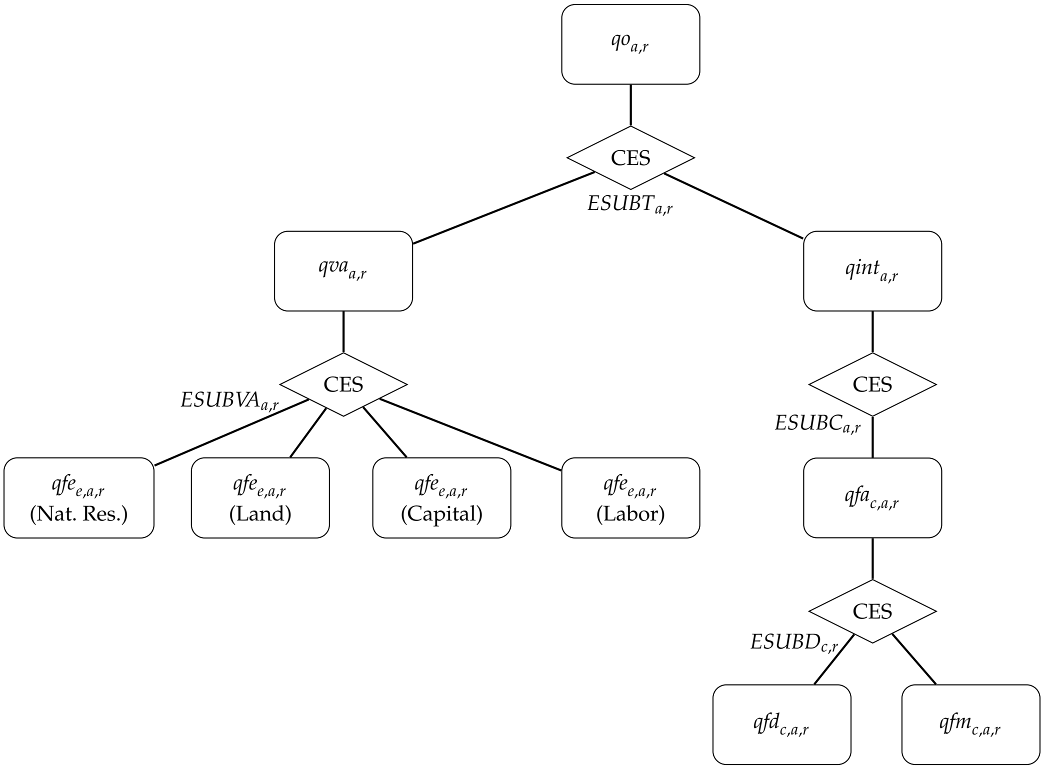

Figure 3. Production structure

In this representation of production, we allow for technological change. All technical change variables are given the first letter a in place of the relevant quantity upon which they operate. Thus, for example, Hicks-neutral technical change is described by changes in $ao_{a,r}$ whereas factor-augmenting technical change would work through the variable $afe_{e,a,r}$. These technological change variables operate in three ways: (1) they reduce the input requirement for the augmented factor, (2) they modify the effective price of the input, and (3) they alter the unit cost of production, and hence, through the zero profits condition, output price. Henceforth, *we will list behavioral equations in percentage change form* and *accounting equations in levels form*. This eases the theoretical exposition. However, implementation in GEMPACK, as well as the code snippets provided in the text, will be solely in linearized form.

$$qint_{a,r} = qo_{a,r} - ao_{a,r} - aint_{a,r} - ESUBT_{a,r} \left(pint_{a,r} - aint_{a,r} - po_{a,r} - ao_{a,r} \right) \quad (1)$$

$$qva_{a,r} = qo_{a,r} - ao_{a,r} - ava_{a,r} - ESUBT_{a,r} \left(pva_{a,r} - ava_{a,r} - po_{a,r} - ao_{a,r} \right) \quad (2)$$

$$PO_{a,r}QO_{a,r} = PINT_{a,r}QINT_{a,r} + PVA_{a,r}QVA_{a,r} \tag{3}$$

$$\begin{aligned} po_{a,r} &= \sum_c STC_{c,a,r}\left(pfa_{c,a,r} - af_{c,a,r} - aint_{a,r}\right) \\ &+ \sum_e STC_{e,a,r}\left(pfe_{e,a,r} - afe_{e,a,r} - ava_{e,r}\right) \\ &- ao_{a,r} \end{aligned} \tag{3'}$$

This unit describes the variables $qint_{a,r}$, $qva_{a,r}$ and $po_{a,r}$, using respectively equations `E_qint`, `E_qva` and `E_qo`.

Listing 1. GEMPACK equations for top level production nest

```
Equation E_qint
# sector demands for composite intermediate commodity inputs by act. a in r #
(all,a,ACTS)(all,r,REG)
   qint(a,r)
      = - aint(a,r) + qo(a,r) - ao(a,r)
      - ESUBT(a,r) * [pint(a,r) - aint(a,r) - po(a,r) - ao(a,r)];

Equation E_qva
# sector demands for primary factor composite #
(all,a,ACTS)(all,r,REG)
   qva(a,r)
      = -ava(a,r) + qo(a,r) - ao(a,r)
      - ESUBT(a,r) * [pva(a,r) - ava(a,r) - po(a,r) - ao(a,r)];

Equation E_qo
# industry zero pure profits condition #
(all,a,ACTS)(all,r,REG)
   po(a,r) + ao(a,r)
      = sum{e,ENDW, STC(e,a,r) * [pfe(e,a,r) - afe(e,a,r) - ava(a,r)]}
      + sum{c,COMM, STC(c,a,r) * [pfa(c,a,r) - afa(c,a,r) - aint(a,r)]}
      + profitslack(a,r);
```

3.1.2 Second level nests

The two top level bundles in Figure 3, $qint_{a,r}$ and $qva_{a,r}$, are disaggregated into their components using additional CES nests. The intermediate demand bundle, $qint_{a,r}$, is a CES aggregation over commodities of $qfa_{c,a,r}$, which represent the intermediate demand for composite commodity c by activity a, see equation (4). The key substitution elasticity is $ESUBC_{a,r}$, whose default value is 0.[12] There is a subsequent decomposition of composite intermediate input demand into demand for goods by source region described below. The price of the aggregate intermediate

---

[12] [NEW] $ESUBC_{a,r}$ is new to this version of GTAP. The old version did not allow for an elasticity of substitution amongst intermediate inputs which differed from the elasticity of substitution between intermediate inputs and value-added. This added nest offers users additional flexibility in specifying derived demand elasticities for intermediate inputs, such as the demand for electricity or fertilizer.

demand bundle, $pint_{a,r}$, is determined by the zero profit condition for this CES bundle, where $pfa_{c,a,r}$ represents the price of the intermediate components.

$$qfa_{c,a,r} = qint_{a,r} - afa_{c,a,r} - ESUBC_{a,r}\left(pfa_{c,a,r} - afa_{c,a,r} - pint_{a,r}\right) \tag{4}$$

$$PINT_{a,r}QINT_{a,r} = \sum_{c} PFA_{c,a,r}QFA_{c,a,r} \tag{5}$$

$$pint_{a,r} = \sum_{c} INTSHR_{c,a,r}\left(pfa_{c,a,r} - afa_{c,a,r}\right) \tag{5'}$$

In a similar fashion, the value added bundle, $qva_{a,r}$, is a CES aggregation of $qfe_{e,a,r}$, which represents demand for endowment (or primary factor) e by activity a, as given in equation (6).[13] The key substitution elasticity is $ESUBVA_{a,r}$, which is differentiated by activity and region (although the default is that this is region-generic). The price of the value added bundle is given by equation (7), where $PFE_{e,a,r}$ is the sector and factor-specific price of endowment e.

$$qfe_{e,a,r} = qva_{a,r} - afe_{e,a,r} - ESUBVA_{a,r}\left(pfe_{e,a,r} - afe_{e,a,r} - pva_{a,r}\right) \tag{6}$$

$$PVA_{a,r}QVA_{a,r} = \sum_{e} PFE_{e,a,r}QFE_{e,a,r} \tag{7}$$

$$pva_{a,r} = \sum_{e} SVA_{e,a,r}\left(pfe_{e,a,r} - afe_{e,a,r}\right) \tag{7'}$$

This unit describes the variables $qfa_{c,a,r}$, $pint_{a,r}$, $qfe_{e,a,r}$ and $pva_{a,r}$, using respectively equations `E_qfa`, `E_pint`, `E_qfe` and `E_pva`.

Listing 2. GEMPACK equations for second level production nests

```
Equation E_qfa
# industry demands for intermediate inputs c by act. a in region r #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
   qfa(c,a,r)
      = - afa(c,a,r) + qint(a,r)
      - ESUBC(a,r) * [pfa(c,a,r) - afa(c,a,r) - pint(a,r)];

Equation E_pint
# price of composite intermediate commodity inputs by act. a in r #
```

---

[13] [NEW] The endowment set has been split from the former set called `NSAV_COMM` and the endowment variables now have their own nomenclature, for example QE/PE instead of QO/PM. In addition, PFE and PEB are the relevant factor-use price for all endowments under all factor market specifications, i.e., the factor-use equations are no longer differentiated by the supply specification for endowments.

```
(all,a,ACTS)(all,r,REG)
   pint(a,r) = sum{c,COMM, INTSHR(c,a,r) * [pfa(c,a,r) - afa(c,a,r)]};

Equation E_qfe
# demands for endowment commodities #
(all,e,ENDW)(all,a,ACTS)(all,r,REG)
   qfe(e,a,r)
      = - afe(e,a,r) + qva(a,r)
      - ESUBVA(a,r) * [pfe(e,a,r) - afe(e,a,r) - pva(a,r)];

Equation E_pva
# effective price of primary factor composite in each sector/region #
(all,a,ACTS)(all,r,REG)
   pva(a,r) = sum{e,ENDW, VASHR(e,a,r) * [pfe(e,a,r) - afe(e,a,r)]};
```

3.1.3 Sourcing of commodities by firms

The final nest in production describes the composition of the commodity bundle, $qfa_{c,a,r}$, by source—domestic vs. imported. At this level, the demand for imports represents the demand for a composite bundle, i.e., a bundle of imports from all (external) source regions. Equations (8) and (9) determine, respectively, firms' demand for domestically produced goods ($qfd_{c,a,r}$) and the composite import good ($qfm_{c,a,r}$). The key substitution elasticity is $ESUBD_{c,r}$, the so-called (top-level) Armington[14] elasticity that determines the degree of substitutability between domestic and imported goods.[15] Equation (10) defines the price of the composite (Armington) bundle and (10′) gives the percentage change form of $pfa_{c,a,r}$.

$$qfd_{c,a,r} = qfa_{c,a,r} - ESUBD_{c,r}\left(pfd_{c,a,r} - pfa_{c,a,r}\right) \tag{8}$$

$$qfm_{c,a,r} = qfa_{c,a,r} - ESUBD_{c,r}\left(pfm_{c,a,r} - pfa_{c,a,r}\right) \tag{9}$$

$$PFA_{c,a,r}QFA_{c,a,r} = PFD_{c,a,r}QFD_{c,a,r} + PFM_{c,a,r}QFM_{c,a,r} \tag{10}$$

$$pfa_{c,a,r} = \left(1 - FMSHR_{c,a,r}\right) pfd_{c,a,r} + FMSHR_{c,a,r}pfm_{c,a,r} \tag{10′}$$

This unit describes $qfd_{c,a,r}$, $qfm_{c,a,r}$ and $pfa_{c,a,r}$, respectively equations `E_qfd`, `E_qfm` and `E_pfa`.

---

[14] Armington (1969) in a seminal paper described import demand using a differentiated goods model.

[15] [NEW] The top-level Armington elasticity is region-specific as well as commodity-specific. Previous versions of the model only differentiated the Armington elasticity across commodities. A further elaboration would be to distinguish the Armington elasticity across domestic agents—for example across different production activities. However, in practice, the empirical literature does not support this degree of differentiation.

Listing 3. GEMPACK equations for sourcing of commodities by firms

```
Equation E_qfd
# act. a demands for domestic good c #
(all,c,COMM)(all,a,ACTS)(all,s,REG)
  qfd(c,a,s) = qfa(c,a,s) - ESUBD(c,s) * [pfd(c,a,s) - pfa(c,a,s)];

Equation E_qfm
# act. a demands for composite import c #
(all,c,COMM)(all,a,ACTS)(all,s,REG)
  qfm(c,a,s) = qfa(c,a,s) - ESUBD(c,s) * [pfm(c,a,s) - pfa(c,a,s)];

Equation E_pfa
# industry price for composite commodities #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
  pfa(c,a,r) = [1 - FMSHR(c,a,r)] * pfd(c,a,r) + FMSHR(c,a,r) * pfm(c,a,r);
```

3.2 Commodity supply

The new standard GTAP model introduces one major innovation, which is the possibility of a non-diagonal ‘make’ matrix. In GTAP ‘classic’, each production activity was associated with one, and only one commodity. The new version allows for activities to produce more than one good, for example a biofuels sector that produces both ethanol and distiller’s dried grains with solubles (DDGS). This also allows for the supply of a single commodity (e.g., electricity) to be composed of output from multiple activities, for example nuclear and coal-fired generation. The ‘make’ matrix is an extension which many users have had to introduce themselves over the past two decades. Including this as an option in the standard model will enhance its utility as a basis for new extensions and applications.

On the supply side, activities with multiple outputs are given a Constant Elasticity of Transformation (CET) specification wherein they maximize their total revenue stream subject to being on the constant elasticity of transformation frontier. On the demand side, buyers of a commodity produced by multiple activities wish to minimize the total cost of supply subject to a CES preference function. However, the latter is written in such a way as to permit users to eliminate this feature, rendering the goods perfect substitutes, as might be the case, for example with irrigated and rain-fed wheat.

Equation (11) describes changes in the supply of commodity c produced by activity a, $qca_{c,a,r}$. This depends on the overall level of activity in the sector, $qo_{a,r}$, as well as any shift in the mix of commodities supplied by that sector. The latter will depend on changes in the price received by the firm for this commodity, $ps_{c,a,r}$, relative to the firm’s unit revenue of activity, $po_{a,r}$, as discussed previously. The key transformation elasticity is given by $ETRAQ_{a,r} < 0$. In the case of a diagonal ‘make’ matrix, this equation is harmless and simply transforms the output of activity a into the supply of commodity c.[16] Equation (12) computes unit revenue under the

---

[16] In the case of a diagonal matrix, $ps_{c,a,r} = po_{a,r}$, and the elasticity becomes redundant.

zero-profit assumption. Equation (13) links the basic price of commodity by activity, $pca_{c,a,r}$, to the commodity- and activity-specific unit cost, $ps_{c,a,r}$, via the power of the output tax, $to_{c,a,r}$.[17] Equation (14) calculates the average basic (tax-inclusive) price of activity, $pb_{a,r}$, as a share-weighted sum of basic commodity- and activity-specific prices, $pca_{c,a,r}$. Note that if the 'make' matrix is diagonal, then, $pb_{a,r} = pds_{c,r}$ for the appropriate correspondence between a and c.

$$qca_{c,a,r} = qo_{a,r} - ETRAQ_{a,r}\left(ps_{c,a,r} - po_{a,r}\right) \tag{11}$$

$$PO_{a,r}QO_{a,r} = \sum_c PS_{c,a,r}QCA_{c,a,r} \tag{12}$$

$$po_{a,r} = \sum_c MAKESACTSHR_{c,a,r}ps_{c,a,r} \tag{12'}$$

$$pca_{c,a,r} = ps_{c,a,r} + to_{c,a,r} \tag{13}$$

$$PB_{a,r}QO_{a,r} = \sum_c PCA_{c,a,r}QCA_{c,a,r} \tag{14}$$

$$pb_{a,r} = \sum_c MAKEBACTSHR_{c,a,r}pca_{c,a,r} \tag{14'}$$

Analogously, a 'national supplier' of composite commodity c purchases its inputs from all activities a producing c using a CES preference function (for example a national electricity supplier). Equation (15) reflects a CES price expression where the key parameter $ESUBQ_{c,r}$ represents the inverse of the CES substitution elasticity[18]—i.e., $ESUBQ = 1/\sigma$ with a default value of 0.[19] Thus Equation (15) simplifies to $pca_{c,a,r} = pds_{c,r}$ when $ESUBQ = 0$, suggesting that the law of one price holds and that the 'national supplier' can perfectly substitute among the same commodities produced by various activities. The variable $qca_{c,a,r}$ represents the desired demand for commodity c produced by activity a. Equation (16) represents the zero-profit

---

[17] [NEW] The power of output tax, in the new model version, is commodity- and activity-specific. It is applied to the unit cost of production (i.e., supply price), not the market price. A tax is now represented as a positive level and a subsidy is negative.

[18] Horridge (2014) notes that where high or infinite elasticity values are permissible, it is necessary to write the CES specification in GEMPACK as $p_i = p_{ave} - \tau(x_i - x_{tot})$ and $x_{tot} = \sum_i S_i x_i$ where $\tau = 1/\sigma$, instead of the primal form $x_i = x_{tot} - \sigma(p_i - p_{ave})$ and $p_{ave} = \sum_i S_i p_i$.

[19] [NEW] $ESUBQ_{c,r} \geq 0$ with a default value of 0 is hard coded in the model code. Users could change this default value by declaring $ESUBQ$ in the parameter file. A value of 0 implies perfect substitution, while higher (infinite) $ESUBQ$ values imply imperfect substitution.

condition and in essence determines the domestic supply of good c, $qc_{c,r}$.[20] These equations determine respectively $pca_{c,a,r}$ and $qc_{c,r}$. The variable qca from the CET-side is the supply, while the variable qca from the CES-side is the demand for these commodities and we could identify them separately and then include an equilibrium condition that determines pca. We skip this step and substitute out the equilibrium condition.

$$pca_{c,a,r} = pds_{c,r} - ESUBQ_{c,r}\left(qca_{c,a,r} - qc_{c,r}\right) \tag{15}$$

$$PDS_{c,r}QC_{c,r} = \sum_{a} PCA_{c,a,r}QCA_{c,a,r} \tag{16}$$

$$qc_{c,r} = \sum_{a} MAKEBCOMSHR_{c,a,r}qca_{c,a,r} \tag{16'}$$

This unit determines $qca_{c,a,r}$, $qo_{a,r}$, $ps_{c,a,r}$, $pb_{a,r}$, $pca_{c,a,r}$ and $qc_{c,r}$ using equations `E_qca`, `E_po`, `E_ps`, `E_pb`, `E_pca` and `E_qc`.

Listing 4. GEMPACK equations for commodity supply

```
Equation E_qca
# supply of commodities by act. a #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
  qca(c,a,r) = IF[MAKES(c,a,r) gt 0,
              qo(a,r) - ETRAQ(a,r) * [ps(c,a,r) - po(a,r)]];

Equation E_po
# average unit (tax-exclusive) cost of output of act. a #
(all,a,ACTS)(all,r,REG)
  po(a,r) = sum{c,COMM, MAKESACTSHR(c,a,r) * ps(c,a,r)};

Equation E_ps
# links basic and supply price of commodity c produced by activity a in r #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
  pca(c,a,r) = ps(c,a,r) + to(c,a,r);

Equation E_pb
# price index: basic (tax-inclusive) price of output of act. a #
(all,a,ACTS)(all,r,REG)
  pb(a,r) = sum{c,COMM, MAKEBACTSHR(c,a,r) * pca(c,a,r)};

Equation E_pca
# CES allocation of commodity output by activity (ESUBQ(c,r) is 1/CES elast.) #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
  pca(c,a,r) = IF[MAKEB(c,a,r) gt 0, pds(c,r)
          - ESUBQ(c,r) * [qca(c,a,r) - qc(a,r)]]; ! Inverse CES !

Equation E_qc
# market clearing condition for total commodity supply #
```

---

[20] Equation (16) could be replaced with the CES primal function, explicitly defining $QC_{c,r}$. The price $PDS_{c,r}$ is determined by the goods market equilibrium condition described below.

```
(all,c,COMM)(all,r,REG)
  qc(c,r) = sum{a,ACTS, MAKEBCOMSHR(c,a,r) * qca(c,a,r)}; ! Inverse CES !
```

3.3 Income distribution

In keeping with its primary role as a trade model, focusing on the international incidence of policies, the model has a single representative household for each region. The household receives all gross factor payments net of the capital depreciation allowance, plus the receipts from all indirect taxes. Equation (17) represents gross factor payments equal to total factor remuneration—summed across all activities and factors—evaluated at market prices, less the depreciation allowance.[21] Equation (18) represents total regional income—the sum of factor income and the fiscal revenues from all indirect taxes (sales tax on domestic and imported goods, taxes on factor use, output tax, and import and export taxes).[22] Section 3.9 will derive the indirect tax revenue flows and total tax revenues.[23]

$$FINCOME_r = \sum_a \sum_e PEB_{e,a,r} QES_{e,a,r} - \delta_r PINV_r KB_r \tag{17}$$

$$Y_r = FINCOME_r + INDTAX_r \tag{18}$$

This unit determines $FINCOME_r$ and Y_r using equations `E_fincome` and `E_y`.

Listing 5. GEMPACK equations for regional income equations

```
Equation E_fincome
# factor income at basic prices net of depreciation #
(all,r,REG)
FY(r) * fincome(r)
  = sum{e,ENDW, sum{a,ACTS, EVFB(e,a,r)*[peb(e,a,r) + qes(e,a,r)]}}
  - VDEP(r) * [pinv(r) + kb(r)];

Equation E_y
# regional income = sum of primary factor income and indirect tax receipts #
(all,r,REG)
INCOME(r) * y(r)
  = FY(r) * fincome(r)
  + 100.0 * INCOME(r) * del_indtaxr(r)
  + INDTAX(r) * y(r)
  + INCOME(r) * incomeslack(r);
```

---

[21] [NEW] Total factor remuneration is evaluated at the activity-level instead of at the economy-wide level. This reflects the fact that natural resources have been designated as sector-specific.

[22] There are also income taxes on total factor remuneration. These are incorporated in the *FINCOME* variable.

[23] The GTAP model evaluates tax revenues relative to regional income and this will be reflected in the TABLO implementation of the relevant equations, if not in the mathematical description. For the interested reader, Section B.2 in the Mathematical Appendix describes this with additional detail.

3.4 Allocation of income across expenditure categories

Regional income is distributed across three broad categories—private consumption, public expenditures and saving. The idea of treating saving as a commodity in a static utility function derives from Lluch (1973) and Howe (1975). The saving good proxies demand for future consumption in this comparative static model. Similarly, the demand for government spending is treated as a proxy for the welfare obtained from public goods provided by such spending. This idea is motivated by Keller (1980, chapter 8), who demonstrates that if: (1) preferences for public goods are separable from preferences for private goods, and (2) the utility function for public goods is identical across households within the regional economy, then we can derive a public utility function. The aggregation of this index with private utility in order to make inferences about regional welfare requires the further assumption that (3) the level of public goods provided in the initial equilibrium is optimal. Users who do not wish to invoke this assumption can employ an alternative closure, such as fixing the level of aggregate government utility while letting private consumption adjust to exhaust regional income on expenditures. However, doing so destroys the appealing welfare properties of the regional household utility function.

A top-level utility function, using a Cobb-Douglas specification, governs the allocation of aggregate expenditure across these three broad categories.[24] More specifically, regional households act so as to maximize utility:

$$U = A \sum_f U_f^{B_f}$$

subject to the budget constraint:

$$\sum_f E_f \left(U_f, P_f \right)$$

where U denotes overall regional utility, U_f is sub-utility from source f, $E_f \left(U_f, P_f \right)$ the expenditure required to achieve sub-utility U_f at price vector P_f. and B_f are the Cobb-Douglas distribution parameters; and the index f ranges over the three broad categories of private consumption, government consumption, and saving.

Saving is a unitary good, but for government and private consumption, aggregator functions relate overall sub-utility to consumption of individual commodities. For government consumption, the function is of the CES form (Cobb-Douglas by default); for private consumption, it is the Constant-Differences-of-Elasticities (CDE) system (Hanoch, 1975). This system affords no closed-form solution for expenditure, but that is not a problem in a numerical model where it can be numeri-

---

[24] This specification was introduced by McDougall (2003).

cally computed.

A key point is that the budget equation involves, in general, not prices of subutility but price vectors P_f. For saving, indeed, there is a single saving price P_s, and for government consumption, we can derive a constant marginal cost P_g. But for private consumption, CDE preferences being non-homothetic, we cannot determine the price of private utility independently of the level of private consumption (McDougall, 2003). With the marginal cost of utility from private consumption endogenous, the top-level expenditure shares, as it turns out, depend on the elasticity of expenditure with respect to utility, both in aggregate and for private consumption individually (for saving and government consumption, the elasticity is identically one). Accommodating this feature required a major theory extension which is fully developed in McDougall (2003), to which the technically-oriented reader is referred and an abbreviated version is developed in Section B.3 in the Mathematical Appendix.

Besides the variables U and U_f, the utility function involves parameters A and B_f. We treat these as variables in the model. Changes in the distribution parameters B_f represent changes in regional household preferences; users may endogenize these, in effect overruling the regional household demand system so as to meet targets represented by other, exogenized variables. Of course, as noted above, this destroys the welfare properties of the model.

"Expenditure" on the single saving good is the product of its price $PSAVE_r$ and quantity $QSAVE_r$, in percentage change form, $psave_r + qsave_r$. Its share in regional income is then $psave_r + qsave_r - y_r$. With fixed prices and preferences, this would be a constant—in percentage change form, zero—but in the more complex situation obtaining, we have

$$psave_r + qsave_r - y_r = uelas_r + dpsave_r \tag{19}$$

where $uelas_r$ denotes (percentage change in) the elasticity of expenditure with respect to utility and $dpsave_r$ the Cobb-Douglas distribution parameter B_S. Government consumption expenditure yg_r likewise is given by

$$yg_r - y_r = uelas_r + dpgov_r \tag{20}$$

where $dpgov_r$ is the Cobb-Douglas distribution parameter B_G. Private consumption expenditure is given by

$$yp_r - y_r = uelas_r - uepriv_r + dppriv_r \tag{21}$$

where $dppriv_r$ is the Cobb-Douglas distribution parameter B_P and $uepriv_r$ represents the elasticity of expenditure on private consumption with respect to utility therefrom. This elasticity, unlike its counterparts for government consumption and saving, is variable in levels and non-zero in percentage changes. The overall elasticity of expenditure $uelas_r$ is an index of the elasticities for the three broad categories, but, since two of the three are fixed, its percentage change equation involves only the remaining one:[25]

$$uelas_r = XSHRPRIV_r uepriv_r + dpav_r \tag{22}$$

where $XSHRPRIV_r$ represents the share of private consumption expenditure in regional income, and $dpav_r$ an index of the change in the distribution parameters:

$$dpav_r = XSHRPRIV_r dppriv_r + XSHRGOV_r dpgov_r + XSHRSAV_r dpsave_r \tag{23}$$

where $XSHRGOV_r$ and $XSHRSAV_r$ represent respectively the share of public expenditure and saving in regional income. This suffices to determine the top-level demands; we also calculate two descriptive variables, an overall price index for disposition of income:

$$p_r = XSHRPRIV_r priv_r + XSHRGOV_r pgov_r + XSHRSAV_r psave_r \tag{24}$$

and top-level utility,

$$\begin{aligned} u_r &= au_r + DPARPRIV_r \log\left(UTILPRIV_r\right) dppriv_r \\ &+ DPARGOV_r \log\left(UTILGOV_r\right) dpgov_r \\ &+ DPARSAV_r \log\left(UTILSAVE_r\right) dpsave_r \\ &+ UTILELAS_r^{-1}\left(y_r - pop_r - p_r\right) \end{aligned} \tag{25}$$

Here au_r represents the percentage change in the parameter A in the overall utility function, $DPARPRIV_r$ the levels value of the distributional parameter B_P for private consumption, $UTILPRIV_r$ the levels value of utility U_P from private consumption, and so on. In most simulations, the change in the distributional terms are zero (and even when we let the distributional parameters vary, we arrange things so their contributions are zero to first order), and the equation reduces to the simpler form

$$u_r = UTILELAS_r^{-1}\left(y_r - pop_r - p_r\right)$$

that is, utility u_r depends on real *per capita* income $y_r - pop_r - p_r$, with a sensitivity given by the inverse of the elasticity $UTILELAS_r$ of expenditure with respect to utility; this inverse being just the elasticity of utility with respect to income.

This unit determines $qsave_r$, yg_r, yp_r, $uelas_r$, $dpav_r$, p_r and u_r using equations `E_qsave`, `E_yg`, `E_yp`, `E_uelas`, `E_dpav`, `E_p` and `E_u`.

---

[25] This formula is developed in Section B.3 in the Mathematical Appendix.

Listing 6. GEMPACK equations for top level utility equations

```
Equation E_qsave
# saving #
(all,r,REG)
  psave(r) + qsave(r) - y(r) = uelas(r) + dpsave(r);

Equation E_yg
# government consumption expenditure #
(all,r,REG)
  yg(r) - y(r) = uelas(r) + dpgov(r);

Equation E_yp
# private consumption expenditure #
(all,r,REG)
  yp(r) - y(r) = -[uepriv(r) - uelas(r)] + dppriv(r);

Equation E_uelas
# elasticity of cost of utility wrt utility #
(all,r,REG)
  uelas(r) = XSHRPRIV(r) * uepriv(r) - dpav(r);

Equation E_dpav
# average distribution parameter shift #
(all,r,REG)
  dpav(r)
    = XSHRPRIV(r) * dppriv(r)
    + XSHRGOV(r) * dpgov(r)
    + XSHRSAVE(r) * dpsave(r);

Equation E_p
# price index for disposition of income by regional household #
(all,r,REG)
  p(r) = XSHRPRIV(r) * ppriv(r)
       + XSHRGOV(r) * pgov(r)
       + XSHRSAVE(r) * psave(r);

Equation E_u
# regional household utility #
(all,r,REG)
  u(r)
    = au(r)
    + DPARPRIV(r) * loge(UTILPRIV(r)) * dppriv(r)
    + DPARGOV(r) * loge(UTILGOV(r)) * dpgov(r)
    + DPARSAVE(r) * loge(UTILSAVE(r)) * dpsave(r)
    + [1.0 / UTILELAS(r)] * [y(r) - pop(r) - p(r)];
```

3.5 Domestic final demand

The top level distribution of regional income is disbursed for private and public expenditures. Domestic saving is used to purchase capital goods, i.e., investment. The supply of domestic saving may be adjusted by net foreign saving flow. A positive capital account leads to investment higher than domestic saving, and the reverse for a negative capital account balance. The allocation of global investment and capital account closure is described below.

3.5.1 Private expenditures

Equation (26) defines demand for the composite commodity c for private expenditures. In per capita terms, the percent change in demand for good c is the inner-product of the percent change in composite consumer prices with the appropriate row of the matrix of own- and cross-price elasticities ($EP_{c,r}$), plus the percent change in per capita income adjusted by the income elasticity ($EY_{c,r}$).

$$qpa_{c,r} - pop_r = \sum_k EP_{c,k,r} ppa_{k,r} + EY_{c,r} \left(yr_r - pop_r \right) \tag{26}$$

Equation (26) is just a generic Marshallian demand equation; what makes our private demand system an economically coherent demand system is the fact that these elasticities depend on prices and quantities (or budget shares) and are derived from an underlying sub-utility function for private expenditure. The particular functional form which is chosen here to represent preferences for private spending is based on the Constant Differences of Elasticities implicitly additive expenditure function (CDE) by Hanoch (1975). The CDE has been shown to be well-suited to CGE applications (Hertel et al., 1991), as it allows more flexibility than the CES or LES functional forms, since it has $2n$ behavioral parameters (where n is the number of commodities). Half of these parameters relate to the compensated price responsiveness and the remainder relate to the response of commodity demands to income. This contrasts with the LES, for example, where just one parameter governs the price responsiveness of all n demands. Another option, short of the parameter-hungry, fully flexible functional forms (e.g., translog) which may not be globally well-behaved, is the 'An Implicitly Directly Additive Demand System' (AIDADS) by Rimmer and Powell (1992). AIDADS is a generalization of the LES which allows for additional Engel flexibility by including two marginal budget shares for each commodity—one governing expenditure patterns at low income levels and one ruling the day at very high income levels. However, as with LES, the price responsiveness of AIDADS is still very limited, and, as income grows and subsistence quantities become relatively small, the uncompensated price elasticities of demand converge to one.[26] This is unattractive in comparative static simulations where income changes are small, relative to price changes.

The CDE demand system has the following generic formulation:

$$\max U : \sum_c a_c U^{e_c b_c} \left(\frac{P_c}{Y} \right)^{b_c} \equiv 1 \quad \text{subject to } Y = \sum_c P_c X_c$$

---

[26] A further extension of AIDADS involves allowing subsistence quantities to vary as a function of per capita income. In this way, price responses at high income levels can be made more realistic. This Modified AIDADS (MAIDADS) demand system has been proposed by Preckel et al. (2010) and may hold promise for future CGE modeling—particularly when income growth plays an important role.

The parameter e is referred to as the expansion parameter and is linked to the income elasticity and b is the substitution parameter (respectively $INCPAR_{c,r}$ and $SUBPAR_{c,r}$ in the model). Using Roy's identity and the implicit function theorem, the budget shares are given by the following:

$$s_c = \frac{Z_c}{\sum_k Z_k} \quad \text{where } Z_c = a_c b_c U^{e_c b_c} \left(\frac{P_c}{Y}\right)^{b_c}$$

Note that with this definition for Z, the utility expression simplifies to:

$$\sum_c \frac{Z_c}{b_c} \equiv 1$$

The CDE system allows us to write out explicit formulae describing how the price and expenditure elasticities of demand, $EPC_{c,k,r}$ and $EY_{c,r}$, vary with changing budget shares and these are given by equations (27–29), that thus feed into equation (26).[27] Equation (27) simplifies the resulting expressions for the price elasticities. It defines the Allen partial elasticity for the CDE function. The parameter $ALPHA_c = 1 - SUBPAR_c$ and the parameter δ is the Kronecker δ that takes the value 1 when the indices are identical (i.e., for diagonal elements), otherwise the value 0. The coefficient $CONSHR$ represents the relevant budget share for commodity c. Equation (28) defines the income elasticities. Equation (29) defines the uncompensated price elasticities, which are a simple function of the Allen partial and income elasticities. Since the parameters of the CDE function are invariant, the only things that change in these elasticity expressions are the continuously updated budget shares.

$$\begin{aligned} APE_{c,k,r} &= ALPHA_{k,r} + ALPHA_{c,r}\left(1 - \frac{\delta_{c,k}}{CONSHR_{k,r}}\right) \\ &- \sum_{c'} CONSHR_{c',r} ALPHA_{c',r} \end{aligned} \tag{27}$$

$$\begin{aligned} EY_{c,r} &= \Big[INCPAR_{c,r}\left(1 - ALPHA_{c,r}\right) \\ &+ \qquad \textstyle\sum_k CONSHR_{k,r} INCPAR_{k,r} ALPHA_{k,r}\Big] \\ &/ \quad \left[\textstyle\sum_k CONSHR_{k,r} INCPAR_{k,r}\right] \\ &+ \quad \left[ALPHA_{c,r} - \textstyle\sum_k CONSHR_{k,r} INCPAR_{k,r}\right] \end{aligned} \tag{28}$$

$$EP_{c,k,r} = CONSHR_{c,r}\left(APE_{c,k,r} - EY_{c,r}\right) \tag{29}$$

For the top level of the demand system, we need the elasticity of private con-

---

[27] These expressions are derived in Section B.4 in the Mathematical Appendix.

sumption expenditure with respect to utility from private consumption; this in levels, as Hanoch (1975) shows, is the expenditure-share-weighted average of the CDE expansion parameters $INCPAR_{c,r}$, in percentage changes, the $INCPAR$-times-expenditure weighted average of the budget shares:

$$uepriv_r = \sum_c \frac{CONSHR_{c,r} INCPAR_{c,r}}{\sum_k CONSHR_{k,r} INCPAR_{k,r}} \left(ppa_{c,r} + qpa_{c,r} - yp_r \right) \tag{30}$$

We also calculate two descriptive variables. The private consumption price index $ppriv_r$ is just a weighted average of prices of the composite goods:

$$ppriv_r = \sum_c CONSHR_{c,r} ppa_{c,r} \tag{31}$$

utility from and per capita expenditure on private consumption are related by the percentage-change form of the expenditure function:

$$yp_r - pop_r = ppriv_r + UELASPRIV_r up_r \tag{32}$$

Private expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (33), (34) and (35) determine private demand for domestic goods ($qpd_{c,r}$), imported goods ($qpm_{c,r}$) and the consumer price of the composite good ($ppa_{c,r}$).

$$qpd_{c,r} = qpa_{c,r} - ESUBD_{c,r} \left(ppd_{c,r} - ppa_{c,r} \right) \tag{33}$$

$$qpm_{c,r} = qpa_{c,r} - ESUBD_{c,r} \left(ppm_{c,r} - ppa_{c,r} \right) \tag{34}$$

$$PPA_{c,r} QPA_{c,r} = PPD_{c,r} QPD_{c,r} + PPM_{c,r} QPM_{c,r} \tag{35}$$

$$ppa_{c,r} = (1 - PMSHR_{c,r})\, ppd_{c,r} + PMSHR_{c,r} ppm_{c,r} \tag{35'}$$

This unit determines $qpa_{c,r}$, $uepriv_r$, $ppriv_r$, up_r, $qpd_{c,r}$, $qpm_{c,r}$ and $ppa_{c,r}$ using equations `E_qpa`, `E_uepriv`, `E_ppriv`, `E_up`, `E_qpd`, `E_qpm` and `E_ppa`. In addition, consumer demand requires updating of the income and price elasticity expressions, $APE_{c,k,r}$, $EY_{c,r}$ and $EP_{c,k,r}$ using update formulas for $ALPHA_{c,r}$ and $CONSHR_{c,r}$.

Listing 7. GEMPACK equations for consumer demand equations

```
Formula (all,c,COMM)(all,r,REG)
  ALPHA(c,r) = 1 - SUBPAR(c,r);
```

```
Formula (all,c,COMM)(all,r,REG)
  CONSHR(c,r) = VPP(c,r) / PRIVEXP(r);

Formula (all,c,COMM)(all,k,COMM)(all,r,REG)
  APE(c,k,r)
    = ALPHA(c,r) + ALPHA(k,r) - sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)};

Formula (all,c,COMM)(all,r,REG)
  APE(c,c,r)
    = 2.0 * ALPHA(c,r)
    - sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)}
    - ALPHA(c,r) / CONSHR(c,r);

Formula (all,c,COMM)(all,r,REG)
  EY(c,r)
    = [1.0 / sum{n,COMM, CONSHR(n,r) * INCPAR(n,r)}]
    * [INCPAR(c,r) * [1.0 - ALPHA(c,r)]
    + sum{n,COMM, CONSHR(n,r) * INCPAR(n,r) * ALPHA(n,r)}]
    + [ALPHA(c,r) - sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)}];

Formula (all,c,COMM)(all,k,COMM)(all,r,REG)
  EP(c,k,r) = [APE(c,k,r) - EY(c,r)] * CONSHR(k,r);

Equation E_qpa
# private consumption demands for composite commodities #
(all,c,COMM)(all,r,REG)
  qpa(c,r) - pop(r)
    = sum{k,COMM, EP(c,k,r) * ppa(k,r)} + EY(c,r) * [yp(r) - pop(r)];

Equation E_uepriv
# elasticity of expenditure wrt utility from private consumption #
(all,r,REG)
  uepriv(r) = sum{c,COMM, XWCONSHR(c,r) * [ppa(c,r) + qpa(c,r) - yp(r)]};

Equation E_ppriv
# price index for private consumption expenditure #
(all,r,REG)
  ppriv(r) = sum{c,COMM, CONSHR(c,r) * ppa(c,r)};

Equation E_up
# computation of utility from private consumption in r #
(all,r,REG)
  UELASPRIV(r) * up(r) = yp(r) - ppriv(r) - pop(r) ;

Equation E_qpd
# private consumption demand for domestic goods #
(all,c,COMM)(all,r,REG)
  qpd(c,r) = qpa(c,r) - ESUBD(c,r) * [ppd(c,r) - ppa(c,r)];

Equation E_qpm
# private consumption demand for aggregate imports #
(all,c,COMM)(all,r,REG)
  qpm(c,r) = qpa(c,r) - ESUBD(c,r) * [ppm(c,r) - ppa(c,r)];

Equation E_ppa
# private consumption price for composite commodities #
(all,c,COMM)(all,r,REG)
  ppa(c,r) = [1 - PMSHR(c,r)] * ppd(c,r) + PMSHR(c,r) * ppm(c,r);
```

3.5.2 Public expenditures

The sub-utility function for public expenditure is based on a CES utility function.[28] Equation (36) determines composite commodity demand by the government for commodity c where $ESUBG_r$ is the substitution elasticity across commodities.[29] The government expenditure price index is provided in equation (37), where the ratio term expresses the budget shares.

$$qga_{c,r} = yg_r - pgov_r - ESUBG_r \left(pga_{c,r} - pgov_r \right) \tag{36}$$

$$pgov_r = \sum_c \frac{VGP_{c,r}}{GOVEXP_r} pga_{c,r} \tag{37}$$

Utility from and expenditure on government consumption are related by the percentage change form of a linearly homogeneous expenditure function:

$$yg_r - pop_r = pgov_r + ug_r \tag{38}$$

Public expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (39), (40) and (41) determine public demand for domestic goods ($qgd_{c,r}$), imported goods ($qgm_{c,r}$) and the government price of the composite good ($pga_{c,r}$).

$$qgd_{c,r} = qga_{c,r} - ESUBD_{c,r} \left(pgd_{c,r} - pga_{c,r} \right) \tag{39}$$

$$qgm_{c,r} = qga_{c,r} - ESUBD_{c,r} \left(pgm_{c,r} - pga_{c,r} \right) \tag{40}$$

$$PGA_{c,r} QGA_{c,r} = PGD_{c,r} QGD_{c,r} + PGM_{c,r} QGM_{c,r} \tag{41}$$

$$pga_{c,r} = \left(1 - GMSHR_{c,r}\right) pgd_{c,r} + GMSHR_{c,r} pgm_{c,r} \tag{41'}$$

This unit determines $qga_{c,r}$, $pgov_r$, ug_r, $qgd_{c,r}$, $qgm_{c,r}$ and $pga_{c,r}$ using equations `E_qga`, `E_pgov`, `E_ug`, `E_qgd`, `E_qgm` and `E_pga`.

---

[28] [NEW] The previous specification explicitly assumed a Cobb-Douglas utility function. Setting the substitution elasticity to 1 replicates the previous specification.

[29] There is no aggregate volume of government expenditure. It could be convenient to hold this fixed in some simulations and endogenize some instrument, such as the top-level utility share parameter for government expenditures. The equation $xg = yg - pgov$ would define the real volume of government expenditure (this is equivalent to ug).

Listing 8. GEMPACK equations for government demand

```
Equation E_qga
# government consumption demands for composite commodities #
(all,c,COMM)(all,r,REG)
  qga(c,r) = yg(r) - pgov(r) - ESUBG(r) * [pga(c,r) - pgov(r)];

Equation E_pgov
# price index for aggregate gov't purchases #
(all,r,REG)
  pgov(r) = sum{c,COMM, [VGP(c,r) / GOVEXP(r)] * pga(c,r)};

Equation E_ug
# utility from government consumption in r #
(all,r,REG)
  ug(r) = yg(r) - pgov(r) - pop(r);

Equation E_qgd
# government consumption demand for domestic goods #
(all,c,COMM)(all,r,REG)
  qgd(c,r) = qga(c,r) - ESUBD(c,r) * [pgd(c,r) - pga(c,r)];

Equation E_qgm
# government consumption demand for aggregate imports #
(all,c,COMM)(all,r,REG)
  qgm(c,r) = qga(c,r) - ESUBD(c,r) * [pgm(c,r) - pga(c,r)];

Equation E_pga
# government consumption price for composite commodities #
(all,c,COMM)(all,r,REG)
  pga(c,r) = [1 - GMSHR(c,r)] * pgd(c,r) + GMSHR(c,r) * pgm(c,r);
```

3.5.3 Investment expenditures

The sub-utility function for investment expenditure, i.e., gross investment, is based on a Leontief utility function.[30] Equation (42) determines composite commodity demand by the capital goods sector for commodity c. The aggregate volume of investment, $qinv_r$, will be defined below, but essentially it comes from the nominal investment equals saving identity, where saving is the sum of domestic saving and net capital inflows from foreign economies. The investment expenditure price index is provided in equation (43) and equation (43′) in percentage terms where the ratio term expresses the expenditure shares.

$$qia_{c,r} = qinv_r \tag{42}$$

---

[30] [NEW] Investment purchases were part of the 'production' activities in the previous version of the model, albeit with no value added, i.e., only purchases of goods and services. For this version of the model, the investment column has been extracted from the value of firms' purchases of domestic and imported intermediates at different price valuations (i.e., `VDFA`, `VDFM`, `VIFA` and `VIFM`) to have a more explicit formulation of domestic final demand.

$$PINV_r QINV_r = \sum_c QIA_{c,r} PIA_{c,r} \tag{43}$$

$$pinv_r = \sum_c \frac{VIP_{c,r}}{REGINV_r} pia_{c,r} \tag{43'}$$

Investment expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (44), (45) and (46) determine investement demand for domestic goods ($qid_{c,r}$), imported goods ($qim_{c,r}$) and the price of the composite investment good ($pia_{c,r}$).

$$qid_{c,r} = qia_{c,r} - ESUBD_{c,r} \left(pid_{c,r} - pia_{c,r} \right) \tag{44}$$

$$qim_{c,r} = qia_{c,r} - ESUBD_{c,r} \left(pim_{c,r} - pia_{c,r} \right) \tag{45}$$

$$PIA_{c,r} QIA_{c,r} = PID_{c,r} QID_{c,r} + PIM_{c,r} QIM_{c,r} \tag{46}$$

$$pia_{c,r} = (1 - IMSHR_{c,r})\, pid_{c,r} + IMSHR_{c,r} pim_{c,r} \tag{46'}$$

This unit determines $qia_{c,r}$, $pinv_r$, $qid_{c,r}$, $qim_{c,r}$ and $pia_{c,r}$ using equations `E_qia`, `E_pinv`, `E_qid`, `E_qim` and `E_pia`.

Listing 9. GEMPACK equations for investment demand

```
Equation E_qia
# Top level (Leontief) demand for investment goods #
(all,c,COMM)(all,r,REG)
  qia(c,r) = qinv(r);

Equation E_pinv
# defines the price of investment #
(all,r,REG)
  pinv(r) = sum{c,COMM, [VIP(c,r) / REGINV(r)] * pia(c,r)};

Equation E_qid
# demand for domestic investment commodity c #
(all,c,COMM)(all,r,REG)
  qid(c,r) = qia(c,r) - ESUBD(c,r) * [pid(c,r) - pia(c,r)];

Equation E_qim
# demand for imported investment commodity c #
(all,c,COMM)(all,r,REG)
  qim(c,r) = qia(c,r) - ESUBD(c,r) * [pim(c,r) - pia(c,r)];

Equation E_pia
# invesment price for composite commodities #
```

```
(all,c,COMM)(all,r,REG)
  pia(c,r) = [1 - IMSHR(c,r)] * pid(c,r) + IMSHR(c,r) * pim(c,r);
```

3.6 Trade, goods market equilibrium and prices

3.6.1 Sourcing of imports

At this juncture, all agents in the economy have a well-specified commodity-specific demand for domestic goods and composite imported goods.[31] The sourcing of imports by region of origin is done at the regional level in the destination country.[32] Equation (47) aggregates all of the agent-based demand for the import composite good (see also Figure C.1 in Appendix C that illustrates the quantity linkages). With a CES preference function for the sourcing of imports, the demand for each good by region of origin is given by equation (48), where $ESUBM_{c,d}$ is the substitution elasticity[33] for imports by destination region and the price $pmds_{c,s,d}$ is the basic price of commodity c produced in region s augmented by a bilateral export tax, the node-specific trade and transport margin and the relevant bilateral tariff.[34] (Recall the top panel of Figure 2.) The aggregate import price, $PMS_{c,d}$, is defined in equation (49).[35]

$$QMS_{c,r} = \sum_a QFM_{c,a,r} + QPM_{c,r} + QGM_{c,r} + QIM_{c,r} \tag{47}$$

$$qxs_{c,s,d} = qms_{c,d} - ESUBM_{c,d}\left(pmds_{c,s,d} - pms_{c,d}\right) \tag{48}$$

$$PMS_{c,d} QMS_{c,d} = \sum_s PMDS_{c,s,d} QXS_{c,s,d} \tag{49}$$

This unit determines $qms_{c,d}$, $qxs_{c,s,d}$ and $PMS_{c,d}$ using equations `E_qms`, `E_qxs` and `E_pms`.

---

[31] The domestic supplier of international trade and transport margins is explicitly assumed to only directly purchase domestic goods and services.

[32] This implies an assumption that the preferences for imports by source region are uniform across all domestic agents. A MRIO-based model assumes that sourcing by region is done at the agent level.

[33] [NEW] The second-level Armington elasticity is now region- and commodity-specific. In the previous version, it was only commodity-specific.

[34] All bilateral variables have two regional indices. The first is always the source region and the second is the destination region. Thus total exports from region s is written as follows where d is the importing (i.e., destination) region:

$$TEXP_{c,s} = \sum_d QXS_{c,s,d}$$

[35] The CES sourcing specification allows for a shift in preferences using the variable *ams*.

Listing 10. GEMPACK equations for sourcing of imports by region

```
Equation E_qms
# assures mkt clearing for imported goods entering each region #
(all,c,COMM)(all,r,REG)
  qms(c,r)
      = sum{a,ACTS, FMCSHR(c,a,r) * qfm(c,a,r)}
      + PMCSHR(c,r) * qpm(c,r)
      + GMCSHR(c,r) * qgm(c,r)
      + IMCSHR(c,r) * qim(c,r);

Equation E_qxs
# regional demand for disaggregated imported commodities by source #
(all,c,COMM)(all,s,REG)(all,d,REG)
  qxs(c,s,d)
    = -ams(c,s,d) + qms(c,d)
    - ESUBM(c,d) * [pmds(c,s,d) - ams(c,s,d) - pms(c,d)];

Equation E_pms
# price for aggregate imports #
(all,c,COMM)(all,d,REG)
  pms(c,d) = sum{s,REG, MSHRS(c,s,d) * [pmds(c,s,d) - ams(c,s,d)]};
```

3.6.2 International trade and transport margins

Trade flows from region s to region d generate demand for trade and transport services. Equation (50) describes the demand for trade and transport service m, to deliver good c from region s to region d. Demand is in fixed proportion to the quantity being delivered, with the possibility of improvements in transport efficiency as captured by the technical coefficient *atmfsd* giving the per unit efficiency of Transportation by Mode of Freight c from Source to Destination. Given the lack of bilateral supplies of shipping services, each mode of transport, m, is supplied at a uniform price PT_m across the world. This global transport price is a composite based on the price of national margin services exports, as given by equation (54). To compute the composite FOB-CIF margin, it is necessary to aggregate these modal-specific prices over all relevant modes of transport for that particular commodity. Any transport efficiency changes enter into this calculation as well, giving equation (51).

The global demand for margin service m is the sum of demand across all commodities and across all bilateral trade nodes, equation (52). There is a fictional 'global' transport sector which purchases the services m from each region. The global purchaser wishes to minimize the cost of purchasing the services across regions subject to a CES preference function. Optimal demand is given by equation (53), which determines $QST_{m,r}$, i.e., the regional supply of trade service m.

$$qtmfsd_{m,c,s,d} = qxs_{c,s,d} - atmfsd_{m,c,s,d} \tag{50}$$

$$ptrans_{c,s,d} = \sum_m VTFSD\_MSH_{m,c,s,d} \left(PT_m - atmfsd_{m,c,s,d}\right) \tag{51}$$

$$QTM_m = \sum_c \sum_s \sum_d QTMFSD_{m,c,s,d} \tag{52}$$

$$qst_{m,r} = qtm_m - ESUBS_m \left(pds_{m,r} - pt_m\right) \tag{53}$$

$$PT_m QTM_m = \sum_r PDS_{m,r} QST_{m,r} \tag{54}$$

This unit determines $qtmfsd_{m,c,s,d}$, $ptrans_{c,s,d}$, QTM_m, $qst_{m,r}$ and PT_m using equations `E_qtmfsd`, `E_ptrans`, `E_qtm`, `E_qst` and `E_pt`.

Listing 11. GEMPACK equations for international trade margins

```
Equation E_qtmfsd
# bilateral demand for transport services #
(all,m,MARG)(all,c,COMM)(all,s,REG)(all,d,REG)
  qtmfsd(m,c,s,d) = qxs(c,s,d) - atmfsd(m,c,s,d);

Equation E_ptrans
# generates flow-specific modal average cost of transport index (cf. HT7) #
(all,c,COMM)(all,s,REG)(all,d,REG)
  ptrans(c,s,d) = sum{m,MARG, VTFSD_MSH(m,c,s,d)
            * [pt(m) - atmfsd(m,c,s,d)]};

Equation E_qtm
# global demand for margin m #
(all,m,MARG)
  qtm(m) = sum{c,COMM, sum{s,REG, sum{d,REG,VTMUSESHR(m,c,s,d)
       * qtmfsd(m,c,s,d)}}};

Equation E_qst
# generate demand for regional supply of global transportation service #
(all,m,MARG)(all,r,REG)
  qst(m,r) = qtm(m) - ESUBS(m)*[pds(m,r) - pt(m)];

Equation E_pt
# generate price index for composite transportation services #
(all,m,MARG)
  pt(m) = sum{r,REG, VTSUPPSHR(m,r) * pds(m,r)};
```

3.6.3 Trade prices

Each bilateral trade flow is associated with four prices (recall Price Linkages in Figure 2). Domestic supplies are made available at the price $PDS_{c,r}$ which is the same across all regions of destination, i.e., from the supplier's perspective, there is no differentiation across destination markets (including the domestic market).[36] The uniform supply price is subject to a (potentially) bilateral export tax/subsidy between the supplier and the border. The border price, known as the *free on board price* or the FOB export price, is described in levels in equation (55). The variable $TXS_{c,s,d}$ represents the power of export tax/subsidy.[37] The first regional index is the region imposing the tax and the second regional index refers to the destination region of the exports.

Between the port of origin and the port of destination, an additional wedge is added to the price of exports that represents the international trade and transport margins, described above. This converts the FOB price to the import border price, also known as the *cost, insurance and freight*, or CIF, price. This is represented in levels in equation (56), where the parameter ζ represents an index of efficiency of margin services used per unit of export and $PTRANS_{c,s,d}$ is the average price of the margin services. The final transformation of the export price is the adjustment for bilateral import tariffs. This generates the price $PMDS_{c,s,d}$ in levels in equation (57) where $TMS_{c,s,d}$ represents the power of the tariff. It should be noted that, while the export and import taxes are multiplicative wedges, the transport margin is an additive wedge. The latter implies that if the FOB export price doubles, the CIF import price will not double unless the price of the margin services also doubles.

$$PFOB_{c,s,d} = PDS_{c,s} TXS_{c,s,d} \tag{55}$$

$$PCIF_{c,s,d} = PFOB_{c,s,d} + \zeta PTRANS_{c,s,d} \tag{56}$$

$$PMDS_{c,s,d} = PCIF_{c,s,d} TMS_{c,s,d} \tag{57}$$

This unit determines the variables $pfob_{c,s,d}$, $pcif_{c,s,d}$ and $pmds_{c,s,d}$ using equations `E_pfob`, `E_pcif` and `E_pmds`.

Listing 12. GEMPACK equations for trade prices

```
Equation E_pfob
# links basic and FOB exports prices #
(all,c,COMM)(all,s,REG)(all,d,REG)
  pfob(c,s,d) = pds(c,s) + tx(c,s) + txs(c,s,d);
```

---

[36] Some multi-regional trade models introduce imperfect transformation across destination markets—using for example a nested CET structure (see for example van der Mensbrugghe (2013)). This results in differentiated supply prices across destination markets.

[37] The FOB export price equation includes an additional, region-generic, tax/subsidy shifter that is only source specific and is a handy way to increase the tax wedge uniformly across all destination regions. [NEW] The base price of the export tax is the price *PDS*. In the previous version of the model, the base of the export tax was the border price, i.e., *PFOB*. With this change, $TXS_{c,s,d}$ is positive for a tax and negative for a subsidy.

```
Equation E_pcif
# links FOB and CIF prices for good c shipped from region s to d #
(all,c,COMM)(all,s,REG)(all,d,REG)
  pcif(c,s,d)
    = FOBSHR(c,s,d) * pfob(c,s,d)
    + TRNSHR(c,s,d) * ptrans(c,s,d);

Equation E_pmds
# links basic domestic import prices and CIF import prices #
(all,c,COMM)(all,s,REG)(all,d,REG)
  pmds(c,s,d) = pcif(c,s,d) + tm(c,d) + tms(c,s,d);
```

3.6.4 Goods market equilibrium

There is a single equilibrium condition for the goods market that determines the domestic market price $pds_{c,r}$. Domestic supply of good c must equal demand for good c, and demand is the sum of domestic demand plus the sum of exports to all export destinations. Equation (58) represents the sum of domestic demands for domestic goods across domestic agents—firms, private households, government and investment—excluding margin services exporters.[38] The equilibrium condition is represented by equation (59), which adds in merchandise exports as well as sales to the global trade and transport sector and this determines the market price of commodity c in region r, where $QC_{c,r}$ represents commodity supply.

$$QDS_{c,r} = \sum_a QFD_{c,a,r} + QPD_{c,r} + QGD_{c,r} + QID_{c,r} \tag{58}$$

$$QC_{c,r} = QDS_{c,r} + QST_{c,r} + \sum_d QXS_{c,r,d} \tag{59}$$

This unit determines $qds_{c,r}$ and $pds_{c,r}$ using equations `E_qds` and `E_pds`.

Listing 13. GEMPACK equations for goods market equilibrium

```
Equation E_qds
# assures market clearing for domestic sales #
(all,c,COMM)(all,r,REG)
   qds(c,r) = sum{a,ACTS, FDCSHR(c,a,r) * qfd(c,a,r)}
            + PDCSHR(c,r) * qpd(c,r) + GDCSHR(c,r) * qgd(c,r)
            + IDCSHR(c,r) * qid(c,r);

Equation E_pds
# assures market clearing for commodities #
(all,c,COMM)(all,r,REG)
   qc(c,r) = DSSHR(c,r) * qds(c,r) + sum(d,REG, XSSHR(c,r,d) * qxs(c,r,d))
           + IF[c in MARG, STSHR(c,r) * qst(c,r)]
           + tradslack(c,r);
```

---

[38] It is the domestic goods counterpart to equation (47).

3.6.5 Agents' prices for goods

Each domestic agent faces two market prices—$PDS_{c,r}$ and $PMS_{c,r}$—which are uniform across all agents. $PDS_{c,r}$ represents the price of domestic goods sold to the domestic market and $PMS_{c,r}$ is the CES aggregate price of imports, the latter based on the price $PDS_{c,r}$ of these goods in the source regions, but is also inclusive of the aforementioned bilateral trade wedges. The price actually paid depends on agent-specific sales taxes, which are also differentiated between domestic and import goods. The following set of equations determines the agents' price of demand for respectively domestic goods and the import bundle.

$$PFD_{c,a,r} = PDS_{c,r} TFD_{c,a,r} \tag{60}$$

$$PFM_{c,a,r} = PMS_{c,r} TFM_{c,a,r} \tag{61}$$

$$PPD_{c,r} = PDS_{c,r} TPD_{c,r} \tag{62}$$

$$PPM_{c,r} = PMS_{c,r} TPM_{c,r} \tag{63}$$

$$PGD_{c,r} = PDS_{c,r} TGD_{c,r} \tag{64}$$

$$PGM_{c,r} = PMS_{c,r} TGM_{c,r} \tag{65}$$

$$PID_{c,r} = PDS_{c,r} TID_{c,r} \tag{66}$$

$$PIM_{c,r} = PMS_{c,r} TIM_{c,r} \tag{67}$$

This unit determines $pfd_{c,a,r}$, $pfm_{c,a,r}$, $ppd_{c,r}$, $ppm_{c,r}$, $pgd_{c,r}$, $pgm_{c,r}$r, $pid_{c,r}$ and $pim_{c,r}$ using equations `E_pfd`, `E_pfm`, `E_ppd`, `E_ppm`, `E_pgd`, `E_pgm`, `E_pid` and `E_pim`.

Listing 14. GEMPACK equations for agent's prices of goods

```
Equation E_pfd
# links domestic basic and firm prices #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
  pfd(c,a,r) = pds(c,r) + tfd(c,a,r);

Equation E_pfm
# links domestic basic and firm prices #
(all,c,COMM)(all,a,ACTS)(all,r,REG)
```

```
  pfm(c,a,r) = pms(c,r) + tfm(c,a,r);

Equation E_ppd
# links basic and private consumption prices for domestic c #
(all,c,COMM)(all,r,REG)
  ppd(c,r) = pds(c,r) + tpd(c,r);

Equation E_ppm
# links domestic basic and private consumption prices #
(all,c,COMM)(all,r,REG)
  ppm(c,r) = pms(c,r) + tpm(c,r);

Equation E_pgd
# links domestic basic and government consumption prices #
(all,c,COMM)(all,r,REG)
  pgd(c,r) = pds(c,r) + tgd(c,r);

Equation E_pgm
# links imported basic and government consumption prices #
(all,c,COMM)(all,r,REG)
  pgm(c,r) = pms(c,r) + tgm(c,r);

Equation E_pid
# links basic and investment prices for domestic commodity c #
(all,c,COMM)(all,r,REG)
  pid(c,r) = pds(c,r) + tid(c,r);

Equation E_pim
# links basic and investment prices for imported commodity c #
(all,c,COMM)(all,r,REG)
  pim(c,r) = pms(c,r) + tim(c,r);
```

3.7 Factor market equilibrium

Factors of production, or endowments, are of three types: perfectly mobile (e.g., labor and capital), partially mobile or sluggish (e.g., land) and sector-specific factors (natural resources).[39] It should be noted that the model user has full flexibility in designating the degree of mobility for all factors—this is determined in the GTAP aggregation facilities (i.e., GTAPAgg2 or FlexAgg).

3.7.1 Mobile endowments

Perfect mobility implies that prices should be equated across all uses. Therefore, the percent change in endowment returns is uniform across activities and market equilibrium is determined by setting aggregate demand equal to (exogenous) supply. Equation (68) represents the equilibrium condition for mobile endowments where $QE_{e,r}$ represents the (fixed) aggregate endowment and $QFE_{e,a,r}$ is demand for endowment e by activity a.[40] Equation (68) thus determines $QE_{e,r}$, which is the

---

[39] [NEW] The three categories for sectoral mobility in the model replace the two categories—mobile and sluggish—used previously. In GTAP 'classic' the sector-specific factors were assumed to be sluggish, but with a very small transformation elasticity, essentially making them sector-specific.

[40] The endowment subset *ENDWM* covers mobile endowments.

economy-wide return to perfectly mobile factors. To simplify specification of activities' unit cost definitions and accounting identities, the model includes equation (69) that produces the activity-specific factor cost, which is uniform across activities.

$$QE_{e,r} = \sum_{e} QFE_{e,a,r} \qquad \text{for } e \in \{ENDWM\} \tag{68}$$

$$pes_{e,a,r} = pe_{e,r} \qquad \text{for } e \in \{ENDWM\} \tag{69}$$

3.7.2 Sluggish endowments

For each sluggish endowment, there is an aggregate quantity, typically in fixed supply, for example total agricultural land. The supply of the aggregate factor to individual activities is less than perfectly elastic, as there is a transformation frontier that moderates the movement of the factor across activities. A CET specification is used as the transformation frontier. Equation (70) determines the supply of the sluggish factor for use in activity a, $qes_{e,a,r}$, where the key transformation elasticity is $ETRAE_{e,r}$ and $pes_{e,a,r}$ represents the (after tax) activity-specific return to the sluggish factor.[41] Equation (71) defines the aggregate return to the sluggish factor, $pe_{e,r}$.[42] Equation (72) represents the equilibrium condition that determines the after tax-market equilibrium price, $pes_{e,a,r}$, for the use of the sluggish factor in activity a.

$$qes_{e,a,r} = qe_{e,r} - ETRAE_{e,r}\left(pes_{e,a,r} - pe_{e,r}\right) \qquad \text{for } e \in \{ENDWS\} \tag{70}$$

$$PE_{e,r}QE_{e,r} = \sum_{a} PES_{e,a,r}QES_{e,a,r} \qquad \text{for } e \in \{ENDWS\} \tag{71}$$

$$QES_{e,a,r} = QFE_{e,a,r} \qquad \text{for } e \in \{ENDWS\} \tag{72}$$

3.7.3 Sector-specific endowments

The final endowment category is the set of sector-specific factors, typically these are natural resources—such as mineral deposits, fossil fuel reserves, forestry stocks, etc. The sector-specific supply is exogenous, i.e., $QES_{e,a,r}$ is fixed. Only one equation is thus needed, which is the equilibrium condition[43], identical to that for sluggish

---

[41] The endowment subset *ENDWS* covers sluggish endowments.

[42] There is a subtle difference in the interpretation of $PE_{e,r}$ between mobile and sluggish factors. In the case of mobile factors, $PE_{e,r}$, is the economy-wide pre-(income) tax return to the endowment. In the case of sluggish factors, $PE_{e,r}$ represents the CET aggregate price of the post-(income) tax returns to the endowment.

[43] The endowment subset *ENDWF* covers sector-specific endowments.

endowments, thus equation (73) replicates equation (72).[44]

$$QES_{e,a,r} = QFE_{e,a,r} \qquad \text{for } e \in \{ENDWF\} \tag{73}$$

Equation (74) links the equilibrium market price of endowments to the producer price that includes an endowment and activity-specific tax, where the power of the tax is identified with $TFE_{e,a,r}$. Equation (75) links the pre- and post- (income) tax price of endowments (at the activity level—though in most cases the income tax is independent of which activity the endowment is used for). In summary, each factor is associated with three prices and two wedges. $PFE_{e,a,r}$ represents the activities' price of the endowment use, inclusive of the tax $TFE_{e,a,r}$ that is imposed on the price $PEB_{e,a,r}$, which represents the equilibrium price-derived—through economy-wide arbitration, or activity-specific arbitration. $PES_{e,a,r}$ represents the take-home, or after tax remuneration, where the $TINC_{e,a,r}$ represents the (power of the) tax on income derived from endowment use.[45]

$$PFE_{e,a,r} = PEB_{e,a,r} TFE_{e,a,r} \tag{74}$$

$$PEB_{e,a,r} = PES_{e,a,r} TINC_{e,a,r} \tag{75}$$

In summary, the aggregate endowment, $QE_{e,r}$, is fixed for mobile and sluggish endowments, whereas the sector-specific endowment, $QES_{e,a,r}$, is fixed at the activity level.[46] Activity-level equilibrium conditions determine activity-specific returns for sluggish and fixed endowments. For accounting purposes and model simplification, $PEB_{e,a,r}$ for mobile endowments is included and is equal to the economy-wide return, $PE_{e,r}$, determined by market equilibrium.

This unit determines $PE_{e,r}$ for mobile and sluggish endowments, $PEB_{e,a,r}$ and $PES_{e,a,r}$ for all endowments, and $QES_{e,a,r}$ for sluggish endowments. Equation `E_pe2` implements equation (68), `E_qes1` implements equation (69), `E_qes2` implements equation (70), `E_pe1` implements equation (71), `E_peb` implements equations (72) and (73) which also holds for mobile endowments and simply sets supply equal to demand. Equation `E_pfe` implements equation (74) and equation `E_pes` implements equation (75).

---

[44] In the *TABLO* code, equations (72) and (73) are merged in `E_peb` and the equilibrium equation is conditioned on the set *ENDW*. For mobile endowments, this trivially sets activity supply to activity demand.

[45] In the *TABLO* code, the power of income tax is $TINC_{e,a,r} = EVFB_{e,a,r}/EVOS_{e,a,r}$.

[46] [NEW] The model does not include a definition for $QE_{e,r}$ for the sector-specific endowments (nor for the variable $PE_{e,r}$) as the interpretation of the aggregate is not very intuitive. This requires changes to the income identities—factor income (FY_r) and revenues generated by taxes on factor use (see below).

Listing 15. GEMPACK equations for equilibrium conditions for factor markets

```
Equation E_pe1
# mkt clearing for perfectly mobile endowments in each r #
(all,e,ENDWM)(all,r,REG)
  qe(e,r) = sum{a,ACTS, ENDWMSHR(e,a,r) * qfe(e,a,r)} + endwslack(e,r);

Equation E_qes1
# basic price of mobile endowments in a in r #
(all,e,ENDWM)(all,a,ACTS)(all,r,REG)
  pes(e,a,r) = pe(e,r);

Equation E_qes2
# allocation of sluggish endowments across sectors #
(all,e,ENDWS)(all,a,ACTS)(all,r,REG)
  qes(e,a,r) = qe(e,r) - ETRAE(e,r) * [pes(e,a,r) - pe(e,r)]
          - endwslack(e,r);

Equation E_pe2
# composite price for sluggish endowments #
(all,e,ENDWS)(all,r,REG)
  pe(e,r) = sum{a,ACTS, REVSHR(e,a,r) * pes(e,a,r)};

Equation E_peb
# mkt clearing for endowments in each r #
(all,e,ENDW)(all,a,ACTS)(all,r,REG)
  qfe(e,a,r) = qes(e,a,r);

Equation E_pfe
# links basic and firm demand prices for mobile endowments #
(all,e,ENDW)(all,a,ACTS)(all,r,REG)
  pfe(e,a,r) = peb(e,a,r) + tfe(e,a,r);

Equation E_pes
# links supply (pre-) and basic (post-tax) endowment prices #
(all,e,ENDW)(all,a,ACTS)(all,r,REG)
  peb(e,a,r) = pes(e,a,r) + tinc(e,a,r);
```

3.8 Allocation of global savings

Savings of each regional household are aggregated across all regions and given to a global investor, also known as the 'global bank'. The disbursement of global saving across regions to finance investment is specified using one of two different mechanisms. Under one mechanism the 'global bank' maximizes its returns by an investment allocation which achieves *equiproportionate* changes in the expected regional rates of return to investment.[47] The second mechanism ignores any expectations about region specific capital markets and allocates investment across regions according to the initial regional investment shares.[48]

---

[47] This approach in theory is a simple application of the theory of investment allocation across industries in the single-country ORANI model of the Australian economy (Dixon et al., 1982).

[48] A third mechanism, which may be implemented in the standard GTAP model after changing the closure, is to fix net foreign capital flows, which is the same as fixing the trade balance in the relatively simple representation of the balance of payments. In this case the global bank ignores changes in the

3.8.1 Investment preliminaries

Given that GTAP is a static model, the beginning of period capital stock, KB_r, is fixed and not normally impacted by a simulation shock. Changes to the investment level only affect the composition of demand, but have no impact on the beginning of period capital stock. The end of period capital stock, KE_r, is equal to the initial capital stock, less depreciation, plus new investment. Equation (76) determines the end of period capital stock, where $QINV_r$, is the volume of investment.

$$KE_r = (1 - \delta_r)\, KB_r + QINV_r \tag{76}$$

The rate of return on capital, $RENTAL$, is defined as the after-tax return to capital.[49] In levels, it is given by equation (77). $PES_{endwc,a,r}$ is the after tax return to capital in sector a.

$$RENTAL_r = \sum_a PES_{endwc,a,r} QES_{endwc,a,r} \Big/ KB_r \tag{77}$$

Equation (78) defines the net current rate of return, the rental rate adjusted for the price of replacing capital, less the depreciation rate. Therefore, a tariff reduction, that would lower the price of investment goods, would reduce the replacement price of capital and increase the rate of return (all else equal).[50]

$$RORC_r = \frac{RENTAL_r}{PINV_r} - \delta_r \tag{78}$$

3.8.2 Rate-of-return sensitive investment allocation

The first specification for allocating investment is that the global investor is sensitive to changes in relative rates of return. Equation (79) defines the expected rate of return—it represents an adjustment in expectations relative to the net rate of return, where the adjustment depends on the rate of growth of the capital stock. A large increase in the capital stock, all else equal, will tend to dampen the expected rate of return relative to the current net rate of return, where the elasticity $RORFLEX_r$ determines the level of the dampening. Obviously, at one extreme, a

---

relative rates of return, as in the case of a fixed investment allocation, and it is the net saving flow that is fixed, not the regional investment shares.

[49] The rental equation in GTAP.TAB is based on the ratio of $VES_{endwc,r}$ to $GROSSCAP_r$ (i.e., gross returns on capital) where *endwc* is a single element set containing capital endowment. This ratio is defined on an after-income taxes basis.

[50] When log-differentiated, the equation takes the form:

$$\dot{rorc}_r RORC_r = (RENTAL_r / PINV_r)\left(\dot{rental}_r - \dot{pinv}_r\right)$$

Replacing the ratio of $(RENTAL_r / PINV_r)$ with $RORC + \delta_r$, leads to the version in the TABLO code, where $GRNETRATIO$ is equal to $(RORC + \delta_r)/RORC$.

value of 0 means that there is no change in expectations. High values of the elasticity reflect high changes in the expected rate of return.

$$RORE_r = RORC_r \left(\frac{KE_r}{KB_r}\right)^{-RORFLEX_r} \tag{79}$$

The equilibrating mechanism is provided in equation (80) that equates the expected rate of return across countries. In effect, equation (80) determines $QINV_r$, i.e., global investment will be allocated across regions so as to insure that the expected rate of return is equalized across regions.

$$RORE_r = RORG \tag{80}$$

The equation that determines *RORG* is the global saving equals global investment identity. However, Walras' Law is invoked and that equation is dropped. It is checked after the fact to verify that the model has been properly implemented.

One can also show that the following accounting identity must hold:

$$PINV_r QINV_r = SAVE_r + FSAVE_r + \delta_r PINV_r KB_r \tag{81}$$

where $FSAVE_r$ is the region's net foreign capital flow. In this case, equation (80) could be seen as determining a region's net foreign capital flow and the equation above determines domestic investment. Summing the equation above across all regions, we have gross investment on the left-hand side and saving plus depreciation on the right-hand side. The sum of $FSAVE_r$ across all regions will be zero. An alternative, therefore could be to keep the equation above, but define it only for $(R-1)$ regions.

3.8.3 Investment allocation based on initial capital shares

The second specification for allocating investment assumes that the regional composition of capital stocks is invariant and does not respond to changes in expected relative rates of return. Equation (82) defines global net investment, i.e., the sum across all regions of regional net investment.

$$GLOBALCGDS = \sum_r QINV_r - \delta_r KB_r \tag{82}$$

Regional investment is assumed to be a constant share of global investment, where the share is calculated using base investment shares, equation (83).

$$QINV_r - \delta_r KB_r = \chi_r^I GLOBALCGDS \tag{83}$$

Note that this equation only needs to be defined for $(R-1)$ regions as the global saving/investment balance will determine the residual regional investment level. In the case of this ‘insensitive’ allocation rule, the model still produces the aggregate rate of return variable, $RORG$, which becomes the weighted sum of the regional expected rate of return variable, equation (84). The weights are provided by the regional share of global net investment.

$$RORG = \sum_r \varphi_r RORE_r \qquad \text{where } \varphi_r = \frac{PINV_r\left(QINV_r - \delta_r KB_r\right)}{\sum_s PINV_s\left(QINV_s - \delta_s KB_s\right)} \tag{84}$$

This unit determines $GLOBALCGDS$, KE_r, $RENTAL_r$, $RORC_r$, $RORE_r$, $QINV_r$ and $RORG$ using equations `E_ke`, `E_rental`, `E_rorc`, `E_rore`, `E_qinv` that implements equations (78) and (83), `E_globalcgds` that implements equations (82) and (84).[51]

Listing 16. GEMPACK equations for investment allocation

```
Equation E_ke
# ending capital stock equals beginning stock plus net investment. #
(all,r,REG)
  ke(r) = INVKERATIO(r) * qinv(r) + [1.0 - INVKERATIO(r)] * kb(r);

Equation E_rental
# defines a variable for capital rental rate #
(all,r,REG)
  rental(r) = sum{e,ENDWC, [VES(e,r) / GROSSCAP(r)] * pe(e,r)};

Equation E_rorc
# current rate of return on capital in region r #
(all,r,REG)
  rorc(r) = GRNETRATIO(r) * [rental(r) - pinv(r)];

Equation E_rore
# expected rate of return depends on the current return and investment #
(all,r,REG)
  rore(r) = rorc(r) - RORFLEX(r) * [ke(r) - kb(r)];

Equation E_qinv
# either gross investment or expected rate of return in region r #
(all,r,REG)
  RORDELTA * rore(r)
    + [1 - RORDELTA]
    * [[REGINV(r) / NETINV(r)] * qinv(r) - [VDEP(r) / NETINV(r)] * kb(r)]
    = RORDELTA * rorg + [1 - RORDELTA] * globalcgds + cgdslack(r);

Equation E_globalcgds
# either expected global rate of return or global net investment #
  RORDELTA * globalcgds + [1 - RORDELTA] * rorg
    = RORDELTA
    * sum{r,REG,
      [REGINV(r) / GLOBINV] * qinv(r) - [VDEP(r) / GLOBINV] * kb(r)}
```

---

[51] The parameter *RORDELTA* determines global investment closure. A value of 1 uses the rate-of-return sensitive closure. A value of 0 uses the fixed investment allocation closure.

```
    + [1 - RORDELTA] * sum{r,REG, [NETINV(r) / GLOBINV] * rore(r)};
```

3.8.4 Price of saving

This final section of the investment module defines the price of saving, $PSAVE_r$, needed among other things to complete the utility module, i.e., to define the utility of saving. In the absence of cross-border saving flows, the price of saving would be equal to the price of capital goods, i.e., $PINV_r$. In the presence of capital flows, regional investment is assumed to occur at the cost of purchasing capital goods regionally ($PINV_r$), but the counterbalancing saving flow is a mix of both domestic and international saving. We have adopted a somewhat *ad hoc* rule but that has the advantage of holding exactly at the global level, i.e., the global price of saving is equal to the global price of investment. Equation (85) defines the regional price of saving and it is set to $PINV_r$ plus an adjustment factor that is common across all regions. The adjustment factor is a weighted share of the regional investment price indices. The share parameter is equal to the regional share of the difference between net investment and saving in global net investment. Adding these up globally gives us the desired outcome that at the global level the price of saving is equal to the price of investment as the adjustment factor sums to 0 when aggregated across regions.

$$psave_r = pinv_r + \sum_s \varphi_s pinv_s \qquad \text{where } \varphi_s = \frac{NETINV_s - SAVE_s}{GLOBINV} \tag{85}$$

This unit determines $PSAVE_r$ using equation `E_psave`.

Listing 17. GEMPACK equations for price of saving

```
Equation E_psave
# price of saving #
(all,r,REG)
  psave(r)
    = pinv(r)
    + sum{s,REG, [[NETINV(s) - SAVE(s)] / GLOBINV] * pinv(s)}
    + psaveslack(r);
```

3.9 Tax revenue streams

The aggregate indirect tax revenue stream, $INDTAX_r$, was referenced above when describing income distribution. We have collected them in this section as now all of the relevant variables have been described that fully define these tax revenue streams.

3.9.1 Tax revenues generated in production

There are three separate taxes paid by firms: tax on output, taxes on factor use and taxes on purchases of domestic and imported goods. Equation (86) defines the tax revenues derived from the production tax, where $TO_{c,a,r} - 1$ represents the *ad valorem* tax rate[52], $PS_{c,a,r}$ is the pre-tax supply price (equal to unit cost in the case of perfectly competitive firms with constant returns to scale) and $QCA_{c,a,r}$ is output by activity a of commodity c.[53] The revenues from factor use are described in equation (87), where $TFA_{e,a,r} - 1$ represents the relevant tax rate. Equation (88) describes the sales tax revenues from intermediate consumption of goods and services, where $TFD_{c,a,r} - 1$ and $TFM_{c,a,r} - 1$ represent respectively the commodity- and activity-specific tax rates on domestic and imported goods.

$$TAXROUT_r = \sum_c \sum_a \left(TO_{c,a,r} - 1\right) PS_{c,a,r} QCA_{c,a,r} \tag{86}$$

$$TAXRFU_r = \sum_e \sum_a \left(TFE_{e,a,r} - 1\right) PEB_{e,a,r} QFE_{e,a,r} \tag{87}$$

$$\begin{aligned} TAXRIU_r \quad &= \quad \textstyle\sum_c \sum_a \left(TFD_{c,a,r} - 1\right) PDS_{c,r} QFD_{c,a,r} \\ &+ \quad \textstyle\sum_c \sum_a \left(TFM_{c,a,r} - 1\right) PMS_{c,r} QFM_{c,a,r} \end{aligned} \tag{88}$$

This unit determines $TAXROUT_r$, $TAXRFU_r$ and $TAXRIU_r$ using GEMPACK equations `E_del_taxrout`, `E_del_taxrfu` and `E_del_taxriu`.

Listing 18. GEMPACK equations for tax revenues from domestic production

```
Equation E_del_taxrout
# change in ratio of output tax payments to regional income #
   100.0 * INCOME(r) * del_taxrout(r) + TAXROUT(r) * y(r)
         = sum{c,COMM, sum{a,ACTS,
           MAKEB(c,a,r) * to(c,a,r) + PTAX(c,a,r) * [ps(c,a,r) + qca(c,a,r)]}};

Equation E_del_taxrfu
# change in ratio of tax payments on factor usage to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxrfu(r) + TAXRFU(r) * y(r)
    = sum{e,ENDW, sum{a,ACTS,
      VFP(e,a,r) * tfe(e,a,r) + ETAX(e,a,r) * [peb(e,a,r) + qfe(e,a,r)]}};

Equation E_del_taxriu
# change in ratio of tax payments on intermediate goods to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxriu(r) + TAXRIU(r) * y(r)
    = sum{c,COMM, sum{a,ACTS,
      VDFP(c,a,r) * tfd(c,a,r) + DFTAX(c,a,r) * [pds(c,r) + qfd(c,a,r)]}}
```

---

[52] [NEW] The previous implementation had the tax relative to PCA (i.e., the price PM in GTAP 'classic'), and thus a tax was negative and a subsidy was positive.

[53] Note that the implemented equations are somewhat more complex. First, they are defined as a percent of regional income, Y_r. And second, since 0 is a legitimate value, they are defined as an ordinary change and not a percent change. We are ignoring these complications in the write-up, but Section B.2 in the Mathematical Appendix provides additional description of the implementation of these equations.

```
      + sum{c,COMM, sum{a,ACTS,
        VMFP(c,a,r) * tfm(c,a,r) + MFTAX(c,a,r) * [pms(c,r) + qfm(c,a,r)]}};
```

3.9.2 Tax revenues generated in domestic final demand

Equations (89), (90) and (91) represent respectively the revenues from sales taxes on domestic and imported purchases by the private agent, the government and investment.[54] Note that the pre-tax supply prices are identical across all agents.

$$TAXRPC_r = \sum_c \left[(TPD_{c,r} - 1)\, PDS_{c,r} QPD_{c,r} + (TPM_{c,r} - 1)\, PMS_{c,r} QPM_{c,r} \right] \quad (89)$$

$$TAXRGC_r = \sum_c \left[(TGD_{c,r} - 1)\, PDS_{c,r} QGD_{c,r} + (TGM_{c,r} - 1)\, PMS_{c,r} QGM_{c,r} \right] \quad (90)$$

$$TAXRIC_r = \sum_c \left[(TID_{c,r} - 1)\, PDS_{c,r} QID_{c,r} + (TIM_{c,r} - 1)\, PMS_{c,r} QIM_{c,r} \right] \quad (91)$$

This unit determines $TAXRPC_r$, $TAXRGC_r$ and $TAXRIC_r$ using GEMPACK equations `E_del_taxrpc`, `E_del_taxrgc` and `E_del_taxric`.

Listing 19. GEMPACK equations for tax revenues from domestic final demand

```
Equation E_del_taxrpc
# change in ratio of private consumption tax payments to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxrpc(r) + TAXRPC(r) * y(r)
    = sum{c,COMM,
      VDPP(c,r) * tpd(c,r) + DPTAX(c,r) * [pds(c,r) + qpd(c,r)]}
    + sum{c,COMM,
      VMPP(c,r) * tpm(c,r) + MPTAX(c,r) * [pms(c,r) + qpm(c,r)]};

Equation E_del_taxrgc
# change in ratio of government consumption tax payments to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxrgc(r) + TAXRGC(r) * y(r)
    = sum{c,COMM,
      VDGP(c,r) * tgd(c,r) + DGTAX(c,r) * [pds(c,r) + qgd(c,r)]}
    + sum{c,COMM,
      VMGP(c,r) * tgm(c,r) + MGTAX(c,r) * [pms(c,r) + qgm(c,r)]};

Equation E_del_taxric
# change in ratio of investment tax payments to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxric(r) + TAXRIC(r) * y(r)
    = sum{c,COMM,
      VDIP(c,r) * tid(c,r) + DITAX(c,r) * [pds(c,r) + qid(c,r)]}
    + sum{c,COMM,
      VMIP(c,r) * tim(c,r) + MITAX(c,r) * [pms(c,r) + qim(c,r)]};
```

---

[54] [NEW] Investment taxes are part of firms' taxes in GTAP 'classic'.

3.9.3 Tax revenues generated in international trade

Equation (92) describes revenues generated by import tariffs where $TMS_{c,s,d} - 1$ represents the *ad valorem* tariff rate imposed in region d for exports sourced in region s for commodity c. The relevant base price is the border price of imports, i.e., $PCIFS_{c,s,d}$. Equation (93) represents the revenues generated by export taxes/subsidies, where $TXS_{c,s,d} - 1$ represents the export tax imposed by region s on exports towards region d for commodity c.[55] A careful look at the indices will show that import tariff revenues are collected by the destination country, d, and that export tax revenues are collected by the source country, s. These differences are also reflected in the different indices used for the regional sums.

$$TAXRIMP_d = \sum_s \sum_c \left(TMS_{c,s,d} - 1\right) PCIF_{c,s,d} QXS_{c,s,d} \tag{92}$$

$$TAXREXP_s = \sum_d \sum_c \left(TXS_{c,s,d} - 1\right) PDS_{c,s} QXS_{c,s,d} \tag{93}$$

This unit determines variables $TAXRIMP_d$ and $TAXREXP_s$ using GEMPACK equations `E_del_taxrimp` and `E_del_taxrexp`.

Listing 20. GEMPACK equations for tax revenues from international trade

```
Equation E_del_taxrimp
# change in ratio of import tax payments to regional income #
(all,d,REG)
  100.0 * INCOME(d) * del_taxrimp(d) + TAXRIMP(d) * y(d)
    = sum{c,COMM, sum{s,REG, VMSB(c,s,d) * [tm(c,d) + tms(c,s,d)]
    + MTAX(c,s,d) * [pcif(c,s,d) + qxs(c,s,d)]}};

Equation E_del_taxrexp
# change in ratio of export tax payments to regional income #
(all,s,REG)
  100.0 * INCOME(s) * del_taxrexp(s) + TAXREXP(s) * y(s)
    = sum{c,COMM, sum{d,REG, VFOB(c,s,d) * [tx(c,s) + txs(c,s,d)]
    + XTAXD(c,s,d) * [pds(c,s) + qxs(c,s,d)]}};
```

3.9.4 Income taxes and other tax identities

Equation (94) describes revenues generated by income taxes where $TINC_{e,a,r} - 1$ represents the income tax rate on endowment e used in activity a.[56] The base price of the tax is $PES_{e,a,r}$. Thus if the power of tax, $TINC_{e,a,r}$, is equal to 1.2, the tax rate

---

[55] [NEW] The export tax is now imposed on the post production tax price of production, $PDS_{c,r}$. In the GTAP 'classic', the export tax is imposed on the border price of exports, i.e., $PFOB_{c,s,d}$.

[56] [NEW] The income tax is specific to the source activity—this allows to differentiate, for example, taxes on profits across activities. Our source data does not currently differentiate income tax by source activity, but one could use *Altertax* (Malcolm, 1998), for example, to impose a differentiated tax structure.

is 20%. Equation (95) describes total revenues from all indirect taxes, $INDTAX_r$. This is part of household income. The other part of household income is total factor remuneration that is evaluated at the equilibrium price for factors and is thus inclusive of income taxes. Equation (96) defines total tax revenues—indirect and direct. This variable is purely definitional and does not appear anywhere else in the model (though presumably could be endogenized to target a total tax to income ratio level).

$$TAXRINC_r = \sum_e \sum_a (TINC_{e,a,r} - 1)\, PES_{e,a,r} QFE_{e,a,r} \tag{94}$$

$$\begin{aligned} INDTAX_r &= TAXROUT_r + TAXRFU_r \\ &+ TAXRIU_r + TAXRPC_r + TAXRGC_r + TAXRIC_r \\ &+ TAXRIMP_r + TAXREXP_r \end{aligned} \tag{95}$$

$$TTAXR_r = INDTAX_r + TAXRINC_r \tag{96}$$

This unit determines variables $TAXRINC_r$, $TAXRINC_r$ and $TTAXR_r$ using GEMPACK equations `E_del_taxrinc`, `E_del_indtaxr` and `E_del_ttaxr`.

Listing 21. GEMPACK equations for income tax revenues and tax identities

```
Equation E_del_taxrinc
# change in ratio of income tax payments to regional income #
(all,r,REG)
  100.0 * INCOME(r) * del_taxrinc(r) + TAXRINC(r) * y(r)
    = sum{e,ENDW, sum{a,ACTS, EVFB(e,a,r) * [tinc(e,a,r)]
    + INCTAX(e,a,r) * [pes(e,a,r) + qfe(e,a,r)]}};

Equation E_del_indtaxr
# change in ratio of indirect taxes to INCOME in r #
(all,r,REG)
  del_indtaxr(r)
    = del_taxrout(r) + del_taxrfu(r) + del_taxriu(r)
    + del_taxrpc(r) + del_taxrgc(r) + del_taxric(r)
    + del_taxrimp(r)+ del_taxrexp(r);

Equation E_del_ttaxr
# change in ratio of taxes to INCOME in r #
(all,r,REG)
  del_ttaxr(r) = del_indtaxr(r) + del_taxrinc(r);
```

3.10 Numéraire and closure

Any single price, or price index, could be chosen as the model numéraire, or price anchor. The default numéraire is a global price index of factor remuneration, *PFACTWLD*, which is aggregated over all endowments, activities and regions, i.e., it represents the average global return to endowments, equation (97), where the weights represent the base level endowment remuneration shares in global factor remuneration. The left-out equation, or Walras' Law was described in the investment section and represents the global saving=global investment identity.[57]

$$PFACTWLD = \sum_e \sum_a \sum_r \varphi_{e,a,r} PEB_{e,a,r} \tag{97}$$

Listing 22. GEMPACK equations for numéraire definition and Walras' Law

```
Equation E_pfactor
# computes % change in price index of primary factors, by region #
(all,r,REG)
  VENDWREG(r) * pfactor(r)
    = sum{e,ENDW, sum{a,ACTS, EVFB(e,a,r) * peb(e,a,r)}};

Equation E_rorg
# computes % change in global price index of primary factors #
  VENDWWLD * pfactwld = sum{r,REG, VENDWREG(r) * pfactor(r)};

Equation E_walras_sup
# extra equation: computes change in supply in the omitted market #
  walras_sup = pcgdswld + globalcgds;

Equation E_walras_dem
# extra equation: computes change in demand in the omitted market #
  GLOBINV * walras_dem = sum{r,REG, SAVE(r) * [psave(r) + qsave(r)]};

Equation E_walraslack
# Check Walras' Law. Value of "walraslack" should be zero #
  walras_sup = walras_dem + walraslack;
```

3.11 Measurement and decomposition of welfare[58]

Regional welfare in the standard GTAP model is reported as the percentage change in regional utility, or, alternatively, as the associated *Equivalent Variation* (EV). Most policy-oriented studies report the latter, as policy makers prefer to think about the value-based welfare change associated with a given policy. However, in a model with vastly different sized regional economies, expressing EV as a percentage of initial period expenditure, or equivalently, reporting the percentage change in utility, is preferred for inter-regional comparisons. Small percentage changes in welfare in large regional economies can dwarf proportionately more important changes in the welfare of smaller economies.

The most difficult aspect of general equilibrium policy analysis is that of explaining the results—in particular the welfare results. In the standard GTAP model, these are a function of terms of trade changes (inter-regional shifting of welfare)

---

[57] The equation in the TABLO code is labeled `E_rorg`. This is because *PFACTWLD* is exogenous as the model's price anchor and thus this equation 'explains' *RORG* which has no separate equation. Note as well that the equation is separated into two. The first defines a regional index for factor prices, and the second defines the global index.

[58] This section draws heavily on Hertel (2013)

and allocative efficiency changes (i.e., changes in production or consumption efficiency due to the presence of distortions). In many simulations, authors also vary technology, population, and possibly endowments as well. Some of these components may vary endogenously via closure changes (e.g., unemployment, technological spillovers, etc.), or they may be determined exogenously (they may simply assume that 'something good happens' due to a policy reform—e.g., improved productivity). Disentangling all of these factors affecting regional welfare is a very difficult task indeed. Fortunately, an analytical welfare decomposition has been developed which permits a break-down of the sources of welfare gain to be undertaken. The decomposition was originally developed by Huff and Hertel in the early 1990's and subsequently revised (Huff and Hertel, 2001) in light of the work of McDougall (2003), and involves a rather lengthy set of algebraic substitutions and simplifications, resulting in an expression for regional equivalent variation which, instead of being based on the regional household's expenditure function, is based instead on the various sources of efficiency changes as well as changes in endowments, technology and the region's terms of trade.[59]

Huff and Hertel (2001) begin this decomposition with the model equation which expresses the change in regional income as a function of payments to endowments (net of depreciation), plus tax revenue, less subsidies paid. Into this income change equation they substitute the linearized zero profit conditions for each sector, the linearized market clearing conditions for traded goods and endowments, and the price linkage equations. The change in income on the left hand side of this expression is next deflated by the change in the regional household price index, and this is also subtracted from the right hand side of the expression. Through a series of algebraic simplifications, an expression is obtained which gives the change in real per capita expenditure as a function of changes in endowments and taxes, interacting with quantity changes. Appropriate scaling converts the real income change into the regional equivalent variation. Due to the non-homotheticity in final demand, we must also factor in population in this welfare decomposition. Due to its complexity, we do not seek to translate this from the model code, but rather present the code in Listing 23, along with some edits to facilitate discussion and interpretation.

Listing 23. GEMPACK equations for welfare decomposition in the GTAP model

```
Equation E_EV_ALT
# decomposition of Equivalent Variation #
(all,r,REG)
    EV_ALT(r) =

    : Preference shifts
        -[0.01 * UTILELASEV(r) * INCOMEEV(r)]
        * [DPARPRIV(r) * loge(UTILPRIVEV(r) / UTILPRIV(r)) * dppriv(r)
        + DPARGOV(r) * loge(UTILGOVEV(r) / UTILGOV(r)) * dpgov(r)
```

---

[59] Baldwin and Venables (1995) independently developed a theoretical decomposition in the context of the gains from regional trade agreements which is similar in spirit.

```
        + DPARSAVE(r) * loge(UTILSAVEEV(r) / UTILSAVE(r)) * dpsave(r)]

    : Scaling factor
        + [0.01 * EVSCALFACT(r)]

    : Changes in allocative efficiency
        * [sum{c,COMM, sum{a,ACTS, PTAX(c,a,r) * [qca(c,a,r) - pop(r)]}}
        + sum{e,ENDW, sum{a,ACTS, INCTAX(e,a,r) * [qes(e,a,r) - pop(r)]}}
        + sum{e,ENDW, sum{a,ACTS, ETAX(e,a,r) * [qfe(e,a,r) - pop(r)]}}
        + sum{c,COMM, sum{a,ACTS, MFTAX(c,a,r) * [qfm(c,a,r) - pop(r)]}}
        + sum{c,COMM, sum{a,ACTS, DFTAX(c,a,r) * [qfd(c,a,r) - pop(r)]}}
        + sum{c,COMM, MPTAX(c,r) * [qpm(c,r) - pop(r)]}
        + sum{c,COMM, DPTAX(c,r) * [qpd(c,r) - pop(r)]}
        + sum{c,COMM, MGTAX(c,r) * [qgm(c,r) - pop(r)]}
        + sum{c,COMM, DGTAX(c,r) * [qgd(c,r) - pop(r)]}
        + sum{c,COMM, MITAX(c,r) * [qim(c,r) - pop(r)]}
        + sum{c,COMM, DITAX(c,r) * [qid(c,r) - pop(r)]}
        + sum{c,COMM, sum{d,REG, XTAXD(c,r,d) * [qxs(c,r,d) - pop(r)]}}
        + sum{c,COMM, sum{s,REG, MTAX(c,s,r) * [qxs(c,s,r) - pop(r)]}}

    : Changes in endowments
        + sum{e,ENDW, sum{a,ACTS, EVOS(e,a,r) * [qes(e,a,r) - pop(r)]}}

    : Depreciation
        - VDEP(r) * [kb(r) - pop(r)]

    : Changes in technology
        + sum{a,ACTS, VOS(a,r) * ao(a,r)}
        + sum{a,ACTS, VVA(a,r) * ava(a,r)}
        + sum{c,COMM, sum{a,ACTS, VFP(c,a,r) * aint(a,r)}}
        + sum{a,ACTS, sum{e,ENDW, VFP(e,a,r) * afe(e,a,r)}}
        + sum{a,ACTS, sum{c,COMM, VFP(c,a,r) * afa(c,a,r)}}
        + sum{m,MARG, sum{c,COMM, sum{s,REG,
            VTMFSD(m,c,s,r) * atmfsd(m,c,s,r)}}}
        + sum{c,COMM, sum{s,REG, VMSB(c,s,r) * ams(c,s,r)}}

    : Changes in terms of trade
        + sum{c,COMM, sum{s,REG, VFOB(c,r,s) * pfob(c,r,s)}}
        + sum{m,MARG, VST(m,r) * pds(m,r)}
        + NETINV(r) * pinv(r)
        - sum{c,COMM, sum{s,REG, VFOB(c,s,r) * pfob(c,s,r)}}
        - sum{m,MARG, VTMD(m,r) * pt(m)}
        - SAVE(r) * psave(r)]

    : Changes in population
        + 0.01 * INCOMEEV(r) * pop(r);
```

The left hand side of this expression is given the name *EV_ALT*. This is because it is a distinct variable from *EV*. The latter is computed from the utility function, whereas *EV_ALT* is computed indirectly. However, if the theory is correctly implemented, these two variables should be the same—subject to computational accuracy. So this is another check on model implementation (along with Walras' Law).

The first term on the right hand side of this equation captures the impact of changing preferences on welfare. Normally this should be zero, as it doesn't really make sense to measure welfare using a utility function which is changing over the course of the simulation. However, as noted above, there are some instances where policy analysts wish to alter the rate of saving or government spending, and this is done by endogenizing these shift parameters. By isolating the effect of these utility function changes, we can still use the other parts of the welfare decomposition.

The next term in Listing 23 is a scaling factor. Since all the remaining variables are expressed in percentage change terms (i.e., proportion change * 100%), we must divide them all by 100 (or multiply by 0.01). Furthermore, due to the non-homotheticity in final demand, the value of additional income falls as the income level rises. For this reason, we must also apply a scale factor, $EVSCALFACT_r$, which depends on changes in the marginal utility of income (see McDougall (2003) for more details).

The next block of terms in the welfare decomposition measure changes in allocative efficiency in the economy. These arise from the interplay between economic distortions and the reallocation of resources in the economy. To gain insight into the nature of these expressions, turn to Figure 4 which depicts what happens when we eliminate the bilateral tariff on imports into region *d* of commodity *c* from trading partner *s*. In the initial equilibrium, the *ad valorem* power of the tariff is given by τ so that the size of the distortion is $\tau \cdot PCIF$. This explains the difference between the initial CIF price, $PCIF_0$, and the initial domestic price for this imported good, $PMDS_0$. The resulting cost to the economy is captured by the shaded triangle representing the area between consumer's willingness to pay for this imported good and the marginal cost of supply. When the tariff is eliminated, we move to a new equilibrium wherein $PCIF_0 = PMDS_0$. So, absent shifts in the supply and demand curves, the allocative efficiency effect represents the increased welfare associated with this tariff reform.

We capture this allocative efficiency gain from tariff reform in the final term of the allocative efficiency block in Listing 23. Picking out only the term associated with this particular tariff on commodity *c* shipped from source *s* to destination *d*, we can write the allocative efficiency expression as follows:

$$EV_d\left(\tau_{Mc,s,d}\right) = \psi_d\left(\tau_{Mc,s,d} PCIF_{c,s,d} dQXS_{c,s,d}\right) \tag{98}$$

As in Figure 4, $\left(\tau_{Mc,s,d} PCIF_{c,s,d}\right)$ is the per unit tariff revenue on imports of good *c* from *s* into *d*, associated with the *ad valorem* tariff rate $\tau_{Mc,s,d}d$. This is multiplied by the change in the volume of imports of *c* from *s* into *d*: $dQXS_{c,s,d}$. In order to evaluate the area of this "Harberger triangle" as the tariff is eliminated, we must consider both the "base" $\left(\tau_{Mc,s,d} PCIF_{c,s,d}\right)$ and the "height" ($dQXS_{c,s,d}$).[60] By continually reevaluating the base of this triangle as the tariff is eliminated, we track the diminishing gap between $PCIF_{c,s,d}$ and $PMDS_{c,s,d}$. In this way, we are able to accu-

---

[60] For those accustomed to computing "Harberger triangles" as ½ *base* × *height*, it may appear that we need a ½ pre-multiplying the right hand side of equation (98). However, this is not required. The numerical integration procedure facilitated by GEMPACK continually re-evaluates the base of the "triangle" Harrison and Pearson (2002).

rately measure its area, which is then added to the aggregate welfare measure, subject to application of the appropriate scaling factor, ψ_d. In order to properly perform this numerical integration, the welfare decomposition equations must be solved in conjunction with the CGE model, using appropriate solution procedures. We use the *GEMPACK* software suite developed by Harrison and Pearson (1996) which is ideally suited to this problem, as it solves the non-linear CGE model using a linearized version of the behavioral equations, coupled with updating equations that link the change, in this case $dQXS_{c,s,d}$, with the levels variables, $QXS_{c,s,d}$. Standard extrapolation techniques can be used to obtain arbitrarily accurate solutions to any well-posed non-linear problem (Harrison and Pearson, 1996). To relate these terms back to the expression in Listing 23, note that $MTAX_{c,s,d} = \tau_{Mc,s,d} PCIF_{c,s,d} QXS_{c,s,d}$ and $100 \times dQXS_{c,s,d}/QXS_{c,s,d} = qxs_{c,s,d}$. Note that the allocative efficiency effects also pick up the interplay between partial policy reforms and pre-existing distortions which are unchanging. This is important and highlights the value of CGE analysis in evaluating the welfare impacts of potentially second-best reforms.

The next two terms in Listing 23 capture the impact of changing endowments on regional welfare. Obviously any exogenous increase in labor or land will improve regional welfare—at least assuming no change in the regional terms of trade. Note that any shock to capital stock will increase not only capital services, but also depreciation and this is factored into the welfare calculation.

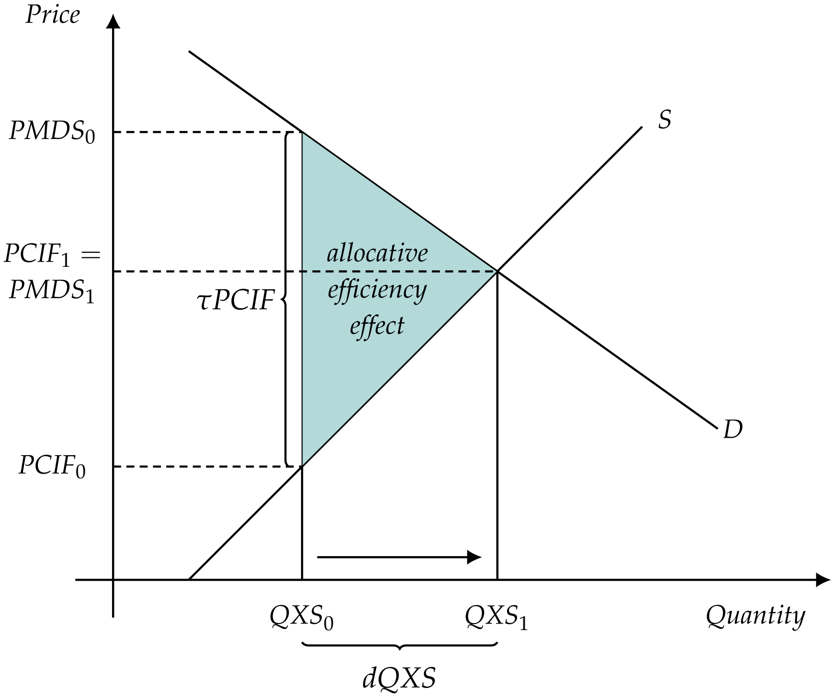

Figure 4. Computing allocative efficiency effects

The next block of terms in the welfare decomposition relates to changes in technology. This is straightforward. Enhanced efficiency in the economy will boost welfare, again holding the terms of trade constant.

And finally, turning to the terms of trade—these are captured in six terms at the bottom of Listing 23. The first of these captures the fact that higher export prices boost regional welfare, with the impact depending on the amount of exports sold into world markets. However, it is not just normal goods and services which are exported by a given region in the model. There are also international trade and transport services sales to the global transport sector which must be accounted for. Hence the term involving $VST_{m,r}$. Finally, regions also 'export' the sale of investment services to satisfy the demands of savers worldwide. This, too, has the potential to influence a region's terms of trade.

On the import side of the terms of trade, higher prices diminish welfare, as do higher prices for the margins services implicitly imported along with merchandise goods. For this reason, imports are valued at their FOB prices, and the component of CIF prices reflected by the international trade and transportation services is separately modeled. Since the transactions with the global bank must be squared as well, purchases of saving by the regional household show up as a potential source of terms of trade changes. However, the price of saving is defined in a way which tends to minimize these effects.

The final term in Listing 23 relates to the change in population. Since EV refers to total regional welfare, and up to this point we have been working on a per capita basis, we must now bring in population as an explanatory variable. If population doubles, we would roughly expect that regional welfare would double.

In summary, it is little surprise that the welfare decomposition tool in GTAP is among the most widely used analysis tools in trade policy analysis. By providing a comprehensive decomposition of sources of welfare change from a given policy, it prevents confabulation and enriches the presentation of findings to policy makers, thereby enhancing the credibility of the policy analyst.

3.12 Bridging to the 'classic' model

Table 1 provides a bridge from v6.2 of the 'classic' GTAP model to the new standard GTAP v7 model described in this section. It begins by noting differences in sets. The most important change is the separation of activities from commodities, thereby permitting both multi-product sectors and multiple sectors producing the same commodity. This necessitates introduction of the 'make' matrix (section 5 of the table). Having separate variables to describe investment demand is an important pedagogical change (see section 8 of Table 1). The other changes are more minor but they are nonetheless important for those familiar with the prior model. Appendix tables A.1 to A.4 provide a detailed mapping between sets, parameters, data and variables in the two models.

Table 1. Old vs. new notation

	GTAPv6.2	**GTAPv7**	**Notes**

(1) Sets and indices	`REG(r)`
	`TRAD_COMM(i)`
	`MARG_COMM(m)`
	`NMRG_COMM(n)`
	`ENDW_COMM(e)`
	`DEMD_COMM(i)`
	`ENDWC_COMM(i)`
	`ENDWM_COMM(i)`
	`ENDWS_COMM(i)`
	`PROD_COMM(j)`
		`REG(r)`
		`COMM(c)`
		`MARG(m)`
		`NMRG(n)`
		`ENDW(e)`
		`DEMD(d)`
		`ENDWC(e)`
		`ENDWM(e)`
		`ENDWS(e)`
		`ACTS(a)`
		`ENDWF(e)`
		`ENDWMS(e)`
			•`ACTS` defines production activities and excludes production of investment goods
			•`ENDWF` defines sector-specific (natural resource) endowment
			•`ENDWMS` defines mobile and sluggish endowments
			•`CGDS_COMM` and `NSAV_COMM` are no longer used in GTAPv7

(2) Top level production nest (see Section 3.1.1 for details)	$qo_{i,r}$, $ps_{i,r}$, $pm_{i,r}$ defined over `NSAV_COMM`	$qo_{a,r}$, $po_{a,r}$, $pb_{a,r}$, $pds_{c,r}$, $ps_{c,a,r}$	•$qo_{a,r}$ is output of activity *a* •$po_{a,r}$ is unit cost of activity *a* •$pb_{a,r}$ is activity tax-inclusive price, which is equal to $pds_{c,r}$ if each activity produces just one commodity (i.e., diagonal '*make*' matrix) •$pds_{c,r}$ is basic price of domestic commodity (i.e., *includes* activity tax but *excludes* user-specific consumption tax). $pds_{c,r}$ replaces $pm_{i,r}$. •$ps_{c,a,r}$ is supply (i.e., production tax *exclusive*) price of commodity produced by each activity
(3) Intermediate demand bundle (see Section 3.1.2 for details)	N/A	$ESUBC_{a,r}$ $qint_{a,r}$, $aint_{a,r}$, $pint_{a,r}$	•CES intermediate demand bundle for each activity •$ESUBC_{a,r}$ is CES substitution parameter with a default value of 0.0 (i.e., Leontief)
(4) Commodity sourcing by firms (see Section 3.1.3 for details)	$ESUBD_c$ $qf_{i,j,r}$, $pf_{i,j,r}$	$ESUBD_{c,r}$ $qfa_{c,a,r}$, $pfa_{c,a,r}$	•Armington parameter $ESUBD_{c,r}$ is now commodity- and region-specific •Change in variable names with 'a' as shorthand for 'agent'

Table 1. Old vs. new notation, ctd.

	GTAPv6.2	**GTAPv7**	**Notes**
(5) *MAKE* transformation for commodity supply and demand (see Section 3.2 for details)	N/A	$ETRAQ_{a,r}$ $ESUBQ_{c,r}$ $ps_{c,a,r}$, $qca_{c,a,r}$, $pca_{c,a,r}$, $qc_{c,r}$	•Possibility for each activity to produce more than one commodity; and for commodities to be the aggregation of output by one or more activitiess •CET for supply of commodities by activities with transformation parameter $ETRAQ_{a,r}$ •CES for sourcing of commodities by activities with substitution parameter $ESUBQ_{c,r}$ representing the inverse of CES substitution elasticity—i.e., $ESUBQ = 1/\sigma$ with a default value of 0 implying perfect substitution •$qc_{c,r}$ is total supply of commodities and replaces $qo_{i,r}$
(6) Private demand (see Section 3.5.1 for details)	$qp_{i,r}$, $pp_{i,r}$	$qpa_{c,r}$, $ppa_{c,r}$	•Change in variable names with 'a' as shorthand for 'agent'
(7) Government demand (see Section 3.5.2 for details)	Cobb-Douglas $qg_{i,r}$, $pg_{i,r}$	CES function with parameter $ESUBG_r$ $qga_{c,r}$, $pga_{c,r}$	•CES specification for flexibility in government demand •$ESUBG_r$ is region-specific substitution parameter with default value of 1.0 (i.e., Cobb-Douglas) •Change in variable names with 'a' as shorthand for 'agent'
(8) Investment demand (see Section 3.5.3 for details)	Investment is part of `PROD_COMM` $qo_{cgds,r}$, $qf_{i,cgds,r}$, $qfd_{i,cgds,r}$, $qfm_{i,cgds,r}$ $pm_{cgds,r}$, $pf_{i,cgds,r}$, $pfd_{i,cgds,r}$, $pfm_{i,cgds,r}$ $tfd_{i,cgds,r}$, $tfm_{i,cgds,r}$	Explicit investment agent $qinv_r$, $qia_{c,r}$, $qid_{c,r}$, $qim_{c,r}$ $pinv_r$, $pia_{c,r}$, $pid_{c,r}$, $pim_{c,r}$ $tid_{c,r}$, $tim_{c,r}$	•Defines an investment agent with nested Leontief-CES demand function •$qinv_r$ and $pinv_r$ are aggregate demand and price index for investment •$qia_{c,r}$ and $pia_{c,r}$ are demand for and price of composite investment commodity •$qid_{c,r}$ and $pid_{c,r}$ are demand for and price of domestic investment commodity •$qim_{c,r}$ and $pim_{c,r}$ are demand for and price of imported investment commodity •$tid_{c,r}$ and $tim_{c,r}$ are powers of consumption tax on domestic and imported investment commodities

Table 1. Old vs. new notation, ctd.

	GTAPv6.2	**GTAPv7**	**Notes**
(9) Sourcing of imports (see Section 3.6.1 for details)	$ESUBM_i$, $qim_{i,r}$ and $pim_{i,r}$ $qxs_{i,s,r}$ and $pms_{i,s,r}$	$ESUBM_{c,r}$, $qms_{c,r}$ and $pms_{c,r}$ $qxs_{c,s,d}$ and $pmds_{c,s,d}$	•Armington parameter $ESUBM_c$ is now commodity- and region-specific •Change in variable names •Change in exporter and importer indices where '*s*' represents the source region and '*d*' for region of destination
(10) International trade and transport margins (see Section 3.6.2 for details)	Cobb-Douglas function	CES function with parameter $ESUBS_m$	•CES specification for flexibility in global demand for margin services •$ESUBS_m$ is the margin-specific substitution parameter with a default value of 1.0 (i.e., Cobb-Douglas)
(11) Market clearing for commodities (see Section 3.6.4 for details)	$qo_{i,r}$ defined over `TRAD_COMM`	$qc_{c,r}$	•$qc_{c,r}$ is total commodity supply for domestic and exports sales
(12) Income distribution and factor market equilibrium (see Sections 3.3 and 3.7 for details)	$qo_{i,r}$, $ps_{i,r}$, $to_{i,r}$, $pm_{i,r}$ defined over `ENDW_COMM` $pmes_{i,r}$, $qoes_{i,r}$ and $ETRAE_i$ defined over `ENDWS_COMM`	$qe_{e,r}$, $pe_{e,r}$ defined over `ENDWMS` $qes_{e,a,r}$, $pes_{e,a,r}$, $tinc_{e,a,r}$ and $peb_{e,a,r}$ defined over `ENDW` $ETRAE_{e,r}$ defined over `ENDWS`	•Factor remuneration is now evaluated at the activity level, partly to reflect that natural resources is now sector-specific •$qe_{e,r}$ and $pe_{e,r}$ are aggregate supply and price of non-sector-specific endowments •$qes_{e,a,r}$ and $pes_{e,a,r}$ are endowment supply and supplier's price at the activity level •$tinc_{e,a,r}$ is power of income tax on endowment use at the activity level. $tinc_{e,a,r}$ replaces previous $to_{i,r}$ for endowments. •$peb_{e,a,r}$ is basic price of endowment at the activity level. •$ETRAE_{e,r}$ is now sluggish endowment- and region-specific

4. Extensions of the 'standard' GTAP model

As noted in the introduction, one of the key roles of the standard GTAP model is to provide a foundation upon which more sophisticated extensions can be built. While there will never be 'one global CGE model to rule them all', having a relatively generic, modularized global model which can be readily modified has been a productive approach. In keeping with this philosophy, we have sought to offer a menu of models, each aimed at a different policy issue, or a different economic and/or biophysical dimension of the global economy. This section is organized around three broad types of extensions. The first is extensions of the economic theory underlying the standard model. This is followed by extensions which improve the policy resolution/relevance of model results. Finally, we consider extensions which permit closer engagement with interdisciplinary work, by incorporating richer engineering or biophysical data.

4.1 Extensions to the theory

The standard GTAP model embodies the workhorse assumptions of perfect competition and constant returns to scale which ensure robust numerical behavior and 'no surprises'. In addition, these assumptions are appropriate for medium run sector level modeling in many industries. As has been shown by Diewert (1981), regardless of the nature of technology exhibited by individual firms (e.g, increasing or decreasing returns to scale), in a sector characterized by a large number of firms, absent barriers to entry and exit, in equilibrium the sector-level production function will tend to exhibit constant returns to scale. However, with increasing concentration of firms in key industries, and with the dominance of much of international trade by a small number of large firms, there is growing demand for more flexibility in how firm behavior is modeled. So it is hardly surprising that this is the area in which most of the theoretical extensions to GTAP have arisen.

The first of these extensions, authored by Joseph Francois, and identified as 'GTAP-IRTS' (Francois, 1998) in Table 2, is a classic extension of the standard model. By simply adding a few equations to the standard model, providing a few additional parameters, and altering the model closure (e.g., endogenizing total factor productivity as well as a mark-up mimicking endogenous output tax), Francois allows users to introduce a variety of market structures, including monopolistic competition and Cournot oligopoly with scale economies. These extensions have permitted users to provide a much richer analysis of a variety of key policy issues, including Eastward Enlargement of the EU (Baldwin et al., 1997) and multilateral trade policy reform under the WTO (Francois et al., 2003). In so doing, it exemplifies what we have in mind for GTAP model extensions, i.e., readily bringing richer theory to bear when, and where, empirical support exists and there is a need for these extensions to shed light on critical policy questions.

Over the past two decades, there has been a surge of interest in incorporating firm level data into analyses of trade policy (Bernard et al., 2007). The path-breaking work of Marc Melitz (Melitz, 2002) opened the door to incorporation of firm heterogeneity into multi-sector models such as GTAP. This has led to a growing number of applications of Melitz-type models in the global CGE context (Zhai, 2008; Balistreri et al., 2007; Dixon et al., 2016). These implementations vary importantly in their assumptions. For example Zhai assumes no entry/exit from the industry in order to simplify his implementation. Balistreri et al. focus on disaggregation of a single sector within a larger CGE model. Dixon et al. follow Francois in assuming that the composition of fixed costs and variable costs are the same so that, like Francois, they can scale output to account for added variety in the marketplace, rather than modeling love of variety on the consumer side. None of these models build on the standard GTAP framework in a way that makes them readily accessible to users. This gap motivated Akgul et al. (2016) to develop the GTAP-HET (Table 2), a variant of the standard model that incorporates firm heterogeneity without the restrictive assumptions embedded in the earlier work.

One thing that has become increasingly clear about the Melitz-style applications is the key role of the parameterization of the firm heterogeneity component. In addition to the product elasticities of substitution—which now have a different interpretation and require a different estimation approach (Akgul et al., 2015)—information about the distribution of firm productivities within the industry is also required. Such estimates are only beginning to appear in the literature (Spearot, 2016) and they often violate the regularity conditions dictated by the Melitz theory. In short, while the tools are now available to bring firm heterogeneity theory to bear in global general equilibrium analysis, the parameters, as well as the practical experience required to deliver meaningful results at broad scale, are still evolving.

There is one last extension of the standard model which should be noted in this section, and that is GDyn—the recursive dynamic version of GTAP. In a clever extension of the standard model, Ianchovichina and McDougall (2000) introduce time as a continuous variable in the GTAP model which, when shocked, moves the economy forward in time. This model has been primarily used to develop future projections of the global economy (Ianchovichina and Walmsley, 2012). The most important theoretical extension in this model involves the treatment of international capital mobility and the foreign ownership of capital. It has also proven useful in evaluating the long run, regional impact of trade reforms (Ianchovichina and Walmsley, 2005), as well as the impact of choosing different time paths for these reforms (Walmsley and Hertel, 2000).

4.2 Policy-oriented extensions

Given the growing use of applied general equilibrium analysis across a wide range of policy questions, it is not surprising that the standard model has been frequently modified to make it better suited to policy analysis. Some of these are ‘one-off’ extensions undertaken by individual authors to address a specific issue, but others have been documented and published, along with the computer files necessary for replication and extension. It is these extensions which we consider here.

In light of the strong interest in agricultural policy reform under the Doha Development Agenda of the WTO, GTAP-AGR (Keeney and Hertel, 2005) was created (Table 2). This implementation alters the standard model in a few key ways to allow more useful and accurate assessment of changes in farm support and related trade policies. Central to such analysis is the segmentation of factor markets which permits examination of the differential impacts of trade reform on farm and non-farm factor returns. In order to evaluate the consequences of such reforms for poverty in the world's poorest countries, a further extension of GTAP-AGR, nick-named GTAP-POV (Hertel et al., 2011), introduces household strata drawn from household survey data. These strata are based on the primary earnings sources of the households and comprise seven different household types (including households relying on agriculture for their primary income). Together, these seven household groups exhaust poverty in each focus country. As factor and commodity prices change, real income for each stratum changes and this translates into an increase/decrease in the stratum and national poverty rates in that country.

In many policy applications the interest in distributional impacts goes beyond poverty, in which case there is a need to break out households in other ways. There is an extension of the standard model, nick-named 'MyGTAP' which facilitates such household disaggregation for individual regions within the global model (Walmsley and Minor, 2013). It also enables more detailed analysis of government payments, remittances, foreign aid and income transfers. This extension of the standard model is accompanied by a set of data programs which facilitate the creation of the necessary social accounts for focus regions in MyGTAP (Minor and Walmsley, 2013).

Quantitative analysis of most bilateral and multilateral trade negotiations typically follow a pattern in which initial analyses are rather stylized and focus on the overall impact on the national economy, employment and trade. This is where GTAP-type analyses thrive, and the resulting numbers are often used to either 'sell' or 'derail' trade agreements. As the negotiations progress, 'the devil lurks in the details' and analysis moves to the level of individual tariff lines. Here, the standard GTAP commodity aggregation quickly falls short of the mark. To take but one example, one of the individual GTAP sectors with the most sizable protection is dairy products. However, within the dairy sector there are nearly two dozen tariff lines. And, in the case of the United States in 2004, average tariffs ranged from 34% for milk and cream to 0% for casein (Grant et al., 2009). To make things worse the nature of the protection varied widely within this sector, with some commodities subject to Tariff Rate Quotas (TRQs) and some free of TRQs. Moreover, the TRQs themselves were binding in some cases, but not in others. Short of disaggregating the model to the tariff line, there is little that can be done to inform detailed negotiations using the standard GTAP model. Therefore, Grant et al. (Grant et al.,

2009) nested, within the GTAP model, a detailed, partial equilibrium model of the dairy sector which allows for sophisticated analysis of tariff-line TRQ expansion. Subsequently, this idea of nested PE-GE analysis has been implemented as a modular extension of the standard GTAP model and is available for download from the GTAP web site (Narayanan et al., 2010). It permits trade policy analysts to remain within the overall GTAP framework, even as they dive into the 'nitty-gritty' of specific sectoral negotiations.

One of the most important areas of work in the GTAP community over the past two decades has related to climate mitigation policy, and energy-economy interactions. This derived demand for energy-related analysis led to the creation of GTAP-E (Burniaux and Truong, 2002), perhaps the most widely cited extension of the standard model (Table 2). The core idea behind the GTAP-E extension is to more accurately represent energy demands across the economy, and to tie these to greenhouse gas (GHG) emissions. Thus GTAP-E has a different production and consumption structure which focuses on the potential for substituting amongst fossil fuels (the main source of CO_2 emissions) and between those fuels and other sources of energy as well as capital/labor. It also requires a database on energy consumption, expressed in volume terms, along with the GHG emissions factors dictating how fuel combustion translates into CO_2 releases into the atmosphere. By including additional equations in the model, GTAP-E also permits implementation of cap-and-trade policies for controlling emissions. As interest in climate policy has deepened, other versions of this model have been developed. These have typically required more extensive inter-disciplinary collaboration and are therefore covered in the next sub-section.

A final variant of the standard GTAP model which emerged in response to a key policy question is the GMig model (Table 2). GMig (Walmsley et al., 2007) adds to the standard database bilateral information on migrants (i.e., where they are working and where they have migrated from) and remittances from these migrants to their home country. Model implementation requires estimates of the relative productivity of migrant and resident workers. Variants of this model have been used to examine the national and global impacts of relaxing global restrictions on migration (Walmsley et al., 2007) as well as in a regional context, including Mexico-US migration (Aguiar and Walmsley, 2014).

4.3 Inter-disciplinary extensions

The last group of model extensions covered in Table 2 draw primarily on data and expertise supplied by other disciplines and have served to foment interdisciplinary collaboration between those in the GTAP community and those in the sciences and engineering. The first of these is GTAP-AEZ, where AEZ stands for 'Agro-Ecological Zones'—a concept with its roots at the Food and Agricultural Organization of the United Nations (FAO), but which was used in the context of the GTAP-based FARM model developed at the Economic Research Services of the

United States Department of Agriculture (ERS/USDA) in the 1990's (Darwin et al., 1995). The basic idea is to allow for heterogeneous land endowments in each region. In GTAP-AEZ, the AEZs are defined in terms of 60 day-long length-of-growing-periods, of which there are six, each differentiated by climatic zone (tropical, temperate and boreal), giving a total of $3 \times 6 = 18$ AEZs. The initial AEZ boundaries were developed by Navin Ramankutty, a geographer/ecologist who works on global land use issues. These AEZs were populated with crops and forests based on the work of Monfreda et al. (2009) and Sohngen et al. (2009). The GTAP-AEZ model was initially developed to look at land-based climate mitigation and for that purpose was merged with data on non-CO_2 GHG emissions which are dominated by farming activity (Hertel et al., 2008). However, use of GTAP-AEZ first really took off in the context of the debate over induced land use change from biofuels (Hertel et al., 2010b).

The demand for analysis of the global land use impacts of biofuels led naturally to versions of the model which disaggregate biofuel production (GTAP-BIO). The first of these was developed by Birur et al. (2008). The use of GTAP in biofuel regulatory analysis gave rise to a long line of refinements, illustrating a natural evolution of the model from a tool for obtaining policy insights, into one which has been used for regulatory analysis by the California Air Resources Board (Taheripour and Tyner, 2013; Tyner et al., 2011). An important part of these enhancements included explicitly modeling the by-products from biofuel production—which can account for a significant portion of total revenue, and which substitute in livestock feeding for the feedstocks being consumed. Studies which ignored this aspect greatly overstated the induced land use change (**?**).

A natural progression in this line of work involved the incorporation of water as a potentially limiting resource on cropland expansion. In an initial application of the water-augmented version of GTAP, Taheripour et al. (2013b) show that factoring in physical constraints on irrigation expansion boosts carbon emissions from biofuels by 25%, as it results in more land conversion in more carbon-rich, rain-fed areas. As with the GTAP-AEZ database, the water database is developed from gridded information which is subsequently aggregated to the river basin level (Haqiqi et al., 2016). Most of the applications of this model have focused on the impacts of future water scarcity (Liu et al., 2014, 2016). And the resulting model, GTAP-BIO-W was developed as an additional component of the biofuels implementation (Taheripour et al., 2013a).

The final inter-disciplinary extension of the standard model discussed here is GTAP-POWER. Authored by Jeffrey Peters (Peters, 2016a), this runs in conjunction with the GTAP-POWER database (Peters, 2016b). The latter is an important new community-funded resource which differentiates electric power generation by major type (e.g., coal, nuclear, hydroelectric, etc.), as well as differentiating base vs. peak load capacities. Development of this database was supported by members of the GTAP Consortium undertaking analysis of power sector regulation as well as climate policy. They use it in their own CGE models, which themselves build on the GTAP Data Base. By supporting this activity at the GTAP Center, they shared the cost of disaggregating the electric power sector, and have subsequently obtained a high quality, replicable data product, which allows more readily for cross-model comparisons. This illustrates once again the tremendous gains from sharing in the development of public goods used in global economic modeling.

Table 2. Extensions to the standard GTAP model

Model	Issues/Policies	Additional Theory	Additional Data	Extent of Usage^{a,b}
Extensions to the Theory				
(0) GTAP 'classic' model	• Trade policy	• N/A	• N/A	• 3,643 citations (Hertel, 1997) • 442 citations and 12,041 web page visits
(1) GTAP-IRTS	• Economies of scale and product differentiation	• Increasing returns to scale, markup pricing and market elasticity of demand	• Standard GTAP Data Base (but parameter $ESUBD = ESUBM$) • Scale elasticities and markup estimates	• 181 citations and 7,988 web page visits (Francois, 1998)
(2) GTAP-HET	• Gains from trade reform, firm productivity, fixed trade costs • Intra-industry reallocation effects associated with international trade	• Firm heterogeneity based on Melitz—endogenous sector productivity • Monopolistic competition (mark-up pricing, fixed and variable costs) • Dixit-Stiglitz-CES Love-of-variety utility function for firms, private households, and government • Pareto distribution for firm productivity	• Uses MRIO GTAP Data Base with import-sourcing by agent • Structural estimates of sectoral shape parameter for productivity distribution using GTAP trade flows and tariff data	• 6 citations (Akgul et al., 2016) • 211 web page visits and 135 article downloads (www.jgea.org)

Table 2. Extensions to the standard GTAP model, ctd.

(3) GDYN	• Economic growth (e.g., changes in economic policies, technology, population and factor endowments over time) • International capital mobility, capital accumulation and impacts at the regional and global level • Timing of policies	• Adaptive expectations in investment behavior • Capital accumulation • International capital mobility and accounting relations to keep track of foreign ownership of capital (i.e., asset ownership and claims to physical capital)	• GDyn Data Base (includes foreign income flow, convergence parameters)	• 223 citations and 12,245 web page visits (Ianchovichina and McDougall, 2000) • 54 citations and 7,877 web page visits (Ianchovichina and Walmsley, 2005)
Policy-oriented Extensions				
(4) GTAP-AGR	• Agricultural trade, markets and supply response • Domestic support policies (e.g., output subsidies, input subsidies, land-based and capital-based subsidy payments)	• Production structure that allows feedstuff substitution in the livestock sector • Agr/Nonagr factor market segmentation • Separability of food and non-food commodities in consumer demand	• CET elasticity parameters for land, labor and capital supplied to agriculture	• 144 citations and 5,407 web page visits (Keeney and Hertel, 2005) • 188 citations (Hertel et al., 2009a)
(5) GTAP-POV	• Poverty impact of climate, trade and domestic policies (e.g., Linking global economic and environmental policies to national poverty impacts)	• AIDADS utility function (incorporates subsistence levels and marginal expenditure shares at low and high incomes) • Labor and Capital market segmentation via a CET function; mapping of income sources to household strata • Top-down macro and microsimulation procedure	• GTAP-POV Data Base (roughly 25 countries with household disaggregated by earnings strata) • Parameter estimates for AIDADS demand function • Income shares by factor and strata • Poverty shares	• 17 citations and 3,486 web page visits (Hertel et al., 2011) • 260 citations (Hertel et al., 2010a) • 52 citations (Hertel et al., 2009a) • 161 citations (Ahmed et al., 2009) • 13 citations (Hussein et al., 2013) • 26 citations (Stone et al., 2010) • 8 citations (Verma et al., 2011)

Table 2. Extensions to the standard GTAP model, ctd.

(6) MyGTAP	• Household-level analysis (e.g., linking global economic impacts to different household types)	• Multiple private households (CDE or LES) specification • Provision for additional factors of production • Explicit government budget accounts (income, expenditure and household transfers) • Inter-regional transfers (e.g., remittances, returns from foreign capital and foreign aid)	• Facility to incorporate user provided household consumption and factor income shares; and if available, household income from domestic and foreign sources for country of interest • Remittances, foreign capital income and foreign aid • Frisch parameter (if LES function is used)	• 11 citations and 17,354 web page visits (Walmsley and Minor, 2013) • 30 citations (Siddig et al., 2014)
(7) GTAP-PE-GE	• Impacts of tariff changes at disaggregated (HS or tariff lines) level	• PE model (appended at the bottom of GTAP model code) • Equations and variables to facilitate linkage between PE and GTAP models	• Standard GTAP Data Base • Trade and tariff data at the HS level (from MAcMap data or TASTE software)	• 42 citations and 1,711 web page visits (Narayanan et al., 2010)
(8) GTAP-E	• Energy-economy linkages • Carbon mitigation policies (e.g., carbon tax, emissions trading, marginal cost of CO_2 abatement and abatement responses at the sectoral, regional and global level)	• Carbon accounting and emissions trading • Production structure with energy (inter-fuel and fuel-factor) substitution • Separability of energy and non-energy commodities in private household and government demands	• GTAP energy volume and CO_2 data	• 559 citations and 19,423 web page visits (Burniaux and Truong, 2002) • 91 citations and 7,684 page visits (McDougall and Golub, 2007)

Table 2. Extensions to the standard GTAP model, ctd.

(9) GMig-2	• Policies associated with labor migration and migrant remittances • Mobility of natural persons (WTO Mode 4 Services Trade)	• International labor mobility: changes in migrant quotas or migrants respond to real wage differences between their host and home countries	• GMig2 Data Base (includes bilateral stocks of labor/migrants and remittances sent by migrant workers) • Parameters governing relative productivity of migrant and resident workers	• 45 citations and 5,156 web page visits (Walmsley et al., 2007) • 14 citations (Walmsley et al., 2011) • 13 citations (Ahmed and Walmsley, 2009) • Aguiar and Walmsley (2014)
(10) GTAP-SC	• Supply chain analyses • Trade in value-added	• Sources imports by use: intermediate, investment and consumption	• GTAP-MRIO Data Base (Forthcoming)	• 4 citations (Walmsley et al., 2014) • 2 citations and 746 web page (Hertel et al., 2014) • 213 web page visits (Carrico, 2016)
(11) GTAP-M	• Analysis of domestic marketing issues	• Disaggregation of domestic wholesale, retail and transport margins	• Margins Data Base (forthcoming, GTAP Data Base version 10)	• 6 citations and 3,269 web page visits (Peterson, 2006) • Corong (2017)

Table 2. Extensions to the standard GTAP model, ctd.

Interdisciplinary Extensions				
(12a) GTAP-AEZ (focus on land use)	• Opportunity costs of alternative land uses (e.g., agricultural land vs. forestry)	• Climate mitigation policies: tax GHG emissions/sequestration subsidies • Nested production structure for agriculture and forestry sectors with: – Composite output and non-CO_2 emissions – Forest carbon management	• GTAP-AEZ Data Base (forest and agricultural land classified by 18 Agro Ecological Zones (AEZ)	• 115 citations and 18,666 web page visits (Lee et al., 2005) • 86 citations and 2,749 web page visits (Lee et al., 2009) • 82 citations (Hertel et al., 2008) • 59 citations Sohngen et al. (2009)
(12b) GTAP-AEZ (focus on GHG emissions)	• Land-based mitigation strategies for terrestrial carbon and non-CO_2 greenhouse gas (GHG) emissions	• Same as (12a)	• Terrestrial carbon stocks • GTAP non-CO_2 emissions Data Base and marginal abatement costs of mitigation	• 21 citations and 19,688 web page visits (Hertel et al., 2009b) • 75 citations Monfreda et al. (2009) • 42 citations (Rose and Lee, 2008) • 5 citations and 1,966 web page visits (Gibbs et al., 2014) • 7 citations and 1,966 web page visits (Plevin et al., 2014)
(13) GTAP-Bio	• Biofuel policies at the regional and global level • Food-energy nexus • Carbon emissions	• Modified GTAP-E production and demand structure to allow for substitution between fossil fuel and biofuel commodities	• GTAP Data Base with bio fuels data • Elasticity parameters (substitution between fossil fuel and biofuel commodities)	• 183 citations and 21,927 web page visits (Birur et al., 2008) • 207 citations (Hertel et al., 2010b) • 276 citations and 4,675 web page visits (Taheripour et al., 2008)

Table 2. Extensions to the standard GTAP model, ctd.

(14) GTAP-Bio-W	• Water policy issues at the regional and global level • Water-food nexus	• Production function that: – distinguishes between irrigated and rainfed agriculture – Similar to GTAP-AEZ: accounts for land heterogeneity across AEZ and models land competition among crops, livestock and forestry sectors	• GTAP-Water Data Base (distinguishes rainfed and irrigated cropping) and water resource (raw data based on water supply by river basin within each country/region)	• 9 citations and 17,658 web page visits (Taheripour et al., 2013b) • 32 citations and 687 web page visits (Liu et al., 2014) • 161 web page visits and 87 article downloads (Haqiqi et al., 2016) (www.jgea.org)
(15) GTAP-Power	• Electricity generation planning and policies • Electricity sector abatement responses associated with carbon mitigation policies	• Electricity input substitution—by technology—in production and demand structure of GTAP-E • Additive Constant Elasticity of Substitution (ACES) function for electricity inputs	• GTAP-Power Data Base (electricity classified by transmission & distribution, and 11 generation technologies)	• Data Base: 279 web page visits and 119 article downloads (Peters, 2016b) (www.jgea.org) • Model: 178 web page visits and 92 article downloads (Peters, 2016a) (www.jgea.org)

Notes: N/A—not applicable; (a) Based on Google citation of GTAP Technical paper, *Journal of Global Economic Analysis* (JGEA) article or book whenever applicable. (b) Based on GTAP Technical paper's web page visits or access statistics from JGEA web page if indicated (as of March 3, 2017).

5. Conclusions and future directions

It has been twenty years since the publication of the 'GTAP book', including documentation of the model and version 2 of the database. Since that time, there have been many applications of this model, many more modifications and extensions. The database has also evolved, with the most recent public release referring to version 9 (Aguiar et al., 2016). Clearly it was time to provide new documentation, designed first of all to 'catch up' to changes to the model made since 1997, as well as to provide a 'facelift' by building in important new flexibility as well as modifying notation to enhance the usability of this model. The GTAP model documented in this paper will be referred to as the current 'standard' model and is given the identifier: *version 7*, to reflect its historical positioning. In the future, this documentation will be dynamically updated, and the version identifier incremented, as model changes arise. In this way, we plan to keep the documentation current over time.

Looking back over the quarter-century since the founding of the GTAP project in 1992, we are struck by the remarkable, and rather unexpected, success of this venture. Prior to the arrival of GTAP, construction of global general equilibrium models was a laborious process requiring several years of staff time. This greatly limited the potential to respond with 'real time' analysis of contemporary policy issues. Such projects were also subject to premature termination when results did not support the position of decision makers (Powell, 2007). By facilitating international collaboration across dozens of institutions and individuals from more than a hundred countries, GTAP has created a common language for global CGE analysis. The model documented here represents the core syntax for this common language and it is from this role that it derives its primary value.

By providing a common platform, the standard GTAP model facilitates development of many different extensions. Section 4 of this paper provides an overview of some of these extensions, many of which are quite innovative. They serve to bring in new theory, as well as making the tool itself more relevant to policy makers. And, over the past decade, new extensions of the GTAP model have shown that it has value beyond the economics profession. Collaborations with geographers, ecologists, engineers and climate scientists are creating new frontiers for research with the GTAP framework documented in this paper, as well as for CGE analyses more generally (Adams and Parmenter, 2013).

Acknowledgements

We thank Mary Burfisher and Glyn Wittwer for valuable suggestions on an earlier draft of this paper. Mark Horridge provided detailed suggestions on Tablo coding in GEMPACK, while also enhancing the model condensation strategy. We also thank Angel Aguiar who proofread the paper with considerable care.

References

Adams, P.D., and B.R. Parmenter. 2013. "Chapter 9 - Computable General Equilibrium Modeling of Environmental Issues in Australia: Economic Impact of an Emissions Trading Scheme." In *Handbook of Computable General Equilibrium Modeling SET, Vols. 1A and 1B*, edited by P. B. Dixon and D. W. Jorgenson. Elsevier, vol. 1A of *Handbook of Computable General Equilibrium Modeling*, pp. 553–657. doi:10.1016/B978-0-444-59568-3.00009-2.

Aguiar, A., B. Narayanan, and R. McDougall. 2016. "An Overview of the GTAP 9 Data Base." *Journal of Global Economic Analysis*, 1(1): 181–208. doi:10.21642/JGEA.010103AF.

Aguiar, A.H., and T.L. Walmsley. 2014. "The importance of timing in the U.S. response to undocumented immigrants: A recursive dynamic approach." *Economic Modelling*, 41: 253 – 262. doi:10.1016/j.econmod.2014.05.025.

Ahmed, S.A., N.S. Diffenbaugh, and T.W. Hertel. 2009. "Climate volatility deepens poverty vulnerability in developing countries." *Environmental Research Letters*, 4(3): 034004. doi:10.1088/1748-9326/4/3/034004.

Ahmed, S.A., and T.L. Walmsley. 2009. "Gains from the Liberalization of Temporary Migration: The Case of India." *South Asia Economic Journal*, 10(1): 61–80. doi:10.1177/139156140901000103.

Akgul, Z., N. Villoria, and T. Hertel. 2015. "Theoretically-Consistent Parameterization of a Multi-sector Global Model with Heterogeneous Firms." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, Presented at the 18th Annual Conference on Global Economic Analysis, Melbourne, Australia. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4731.

Akgul, Z., N.B. Villoria, and T.W. Hertel. 2016. "GTAP-HET: Introducing Firm Heterogeneity into the GTAP Model." *Journal of Global Economic Analysis*, 1(1): 111–180. doi:10.21642/JGEA.010102AF.

Armington, P.S. 1969. "A Theory of Demand for Products Distinguished by Place of Production." *IMF Staff Papers*, 16(1): 159–178.

Baldwin, R.E., J.F. Francois, and R. Portes. 1997. "The costs and benefits of eastern enlargement: the impact on the EU and central Europe." *Economic Policy*, 12(24): 125. doi:10.1111/1468-0327.00018.

Baldwin, R.E., and A.J. Venables. 1995. "Chapter 31 Regional Economic Integration." *Handbook of International Economics*, 3: 1597–1644. doi:10.1016/S1573-4404(05)80011-5.

Balistreri, E.J., R.H. Hillberry, and T.F. Rutherford. 2007. "Structural Estimation and Solution of International Trade Models with Heterogeneous Firms." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, Presented at the 10th Annual Conference on Global Economic Analysis, Purdue University, USA. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2356.

Bernard, A.B., J.B. Jensen, S.J. Redding, and P.K. Schott. 2007. “Firms in International Trade.” *Journal of Economic Perspectives*, 21(3): 105–130. doi:10.1257/jep.21.3.105.

Birur, D.K., T.W. Hertel, and W.E. Tyner. 2008. “Impact of Biofuel Production on World Agricultural Markets: A Computable General Equilibrium Analysis.” Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 53. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2413.

Brockmeier, M. 2001. “A Graphical Exposition of the GTAP Model, 2001 Revision.” Global Trade Analysis Project (GTAP), Purdue University, West Lafayette, IN, GTAP Technical Paper No. 8. https://www.gtap.agecon.purdue.edu/resources/download/181.pdf.

Burniaux, J.M., F. Delorme, I. Lienert, and J.P. Martin. 1990. “WALRAS–A Multi-Sector, Multi-Country Applied General Equilibrium Model for Quantifying the Economy-wide Effects of Agricultural Polices.” *OECD Economic Studies*, , (13)pp. 69–102.

Burniaux, J.M., and T.P. Truong. 2002. “GTAP-E: An Energy-Environmental Version of the GTAP Model.” Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 16. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=923.

Carrico, C. 2016. “Improving Global Value Chain Analysis: Introducing Tariff Revenues into a GTAP-Based MRIO Framework.” http://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=5089.

Corong, E. 2017. “Incorporating Domestic Margins into GTAP Data Base and Model.” Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, Paper prepared for the 20th Annual Conference on Global Economic Analysis, West Lafayette, IN, USA.

Darwin, R., M. Tsigas, J. Lewandrowski, and A. Raneses. 1995. “World Agriculture and Climate Change: Economic Adaptations.” Natural Resources and Environment Division, Economic Research Service, U.S. Department of Agriculture, Agricultural Economic Report No. 703. https://www.ers.usda.gov/publications/pub-details/?pubid=40598.

Diewert, W.E. 1981. “The Comparative Statics of Industry Long-Run Equilibrium.” *The Canadian Journal of Economics / Revue canadienne d'Economique*, 14(1): 78–92. doi:10.2307/134841.

Dixon, P.B., M. Jerie, and M.T. Rimmer. 2016. “Modern Trade Theory for CGE Modelling: The Armington, Krugman and Melitz Models.” *Journal of Global Economic Analysis*, 1(1): 1–110. doi:10.21642/JGEA.010101AF.

Dixon, P.B., B.R. Parmenter, J. Sutton, and D.P. Vincent. 1982. *ORANI: A Multisectoral Model of the Australian Economy*. Amsterdam: North-Holland.

Dixon, P.B., and M.T. Rimmer. 2002. *Dynamic general equilibrium modelling for fore-*

casting and policy: a practical guide and documentation of MONASH, 1st ed., vol. 256. Amsterdam: Elsevier.

Fischer, G., K. Frohberg, M.A. Keyzer, and K.S. Parikh. 1988. *Linked National Models: A Tool for International Food Policy Analysis*. Dordrecht, the Netherlands: International Institute for Applied Systems Analysis and Kluwer Academic Publishers.

Francois, J., H. van Meijl, and F. van Tongeren. 2003. "A Forward Looking Analysis of the Doha Round." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, Presented at the 6th Annual Conference on Global Economic Analysis, The Hague, The Netherlands. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=1244.

Francois, J.F. 1998. "Scale Economies and Imperfect Competition in the GTAP Model." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 14. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=317.

Gibbs, H., S. Yui, and R. Plevin. 2014. "New Estimates of Soil and Biomass Carbon Stocks for Global Economic Models." http://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4344.

Ginsburgh, V.A., and J.L. Waelbroeck. 1981. *Activity Analysis and General Equilibrium Modelling*. Amsterdam, Netherlands: North Holland.

Grant, J.H., T.W. Hertel, and T.F. Rutherford. 2009. "Dairy Tariff-Quota Liberalization: Contrasting Bilateral and Most Favored Nation Reform Options." *American Journal of Agricultural Economics*, 91(3): 673. doi:10.1111/j.1467-8276.2009.01263.x.

Hanoch, G. 1975. "Production and Demand Models with Direct or Indirect Implicit Additivity." *Econometrica*, 43(3): 395–419. http://www.jstor.org/stable/1914273.

Haqiqi, I., F. Taheripour, J. Liu, and D. van der Mensbrugghe. 2016. "Introducing Irrigation Water into GTAP Data Base Version 9." *Journal of Global Economic Analysis*, 1(2): 116–155. doi:10.21642/JGEA.010203AF.

Harrison, W., and K.R. Pearson. 1996. "Computing Solutions for Large General Equilibrium Models Using GEMPACK." *Computational Economics*, 9(2): 83–127. doi:10.1007/BF00123638.

Harrison, W., and K.R. Pearson. 2002. "GEMPACK User Documentation Release 8." Centre of Policy Studies and IMPACT Project, Monash University, Report. https://www.copsmodels.com/ftp/gpdoc/rel80/gpd1.pdf.

Hertel, T. 2013. "Chapter 12 - Global Applied General Equilibrium Analysis Using the Global Trade Analysis Project Framework." In *Handbook of Computable General Equilibrium Modeling SET, Vols. 1A and 1B*, edited by P. B. Dixon and D. W. Jorgenson. Elsevier, vol. 1B of *Handbook of Computable General Equilibrium Modeling*, pp. 815–876. doi:10.1016/B978-0-444-59568-3.00012-2.

Hertel, T., M. Burke, and D. Lobell. 2010a. "The Poverty Implications of Climate-

Induced Crop Yield Changes by 2030." Center for Global Trade Analysis, Department of Agricultural Economics, Purdue University, Report. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=3731.

Hertel, T., K. Itakura, and R. McDougall. 2000. "GTAP Model Version 5.0." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, Center for Global Trade Analysis. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=575.

Hertel, T., M. Verma, M. Ivanic, E. Magalhaes, C. Ludena, and A.R. Rios. 2011. "GTAP-POV: A Framework for Assessing the National Poverty Impacts of Global Economic and Environmental Policies." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 31. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=3731.

Hertel, T.W., ed. 1997. *Global Trade Analysis: Modeling and Applications*. New York, NY: Cambridge University Press.

Hertel, T.W., A.A. Golub, A.D. Jones, M. O'Hare, R.J. Plevin, and D.M. Kammen. 2010b. "Effects of US Maize Ethanol on Global Land Use and Greenhouse Gas Emissions: Estimating Market-mediated Responses." *BioScience*, 60(3): 223. doi:10.1525/bio.2010.60.3.8.

Hertel, T.W., J.M. Horridge, and K. Pearson. 1992. "Mending the family tree a reconciliation of the linearization and levels schools of AGE modelling." *Economic Modelling*, 9(4): 385 – 407. doi:10.1016/0264-9993(92)90020-3.

Hertel, T.W., D. Hummels, and T.L. Walmsley. 2014. "Developing a GTAP-based multi-region, input-output framework for supply chain analysis." In *Asia and Global Production Networks: Implications for Trades, Incomes and Vulnerability*. Edward Elgar Publishing.

Hertel, T.W., R. Keeney, M. Ivanic, and L.A. Winters. 2009a. "Why Isn't the Doha Development Agenda more Poverty Friendly?" *Review of Development Economics*, 13(4): 543–559. doi:10.1111/j.1467-9361.2008.00483.x.

Hertel, T.W., H. Lee, S. Rose, and B. Sohngen. 2008. "Modeling Land-use Related Greenhouse Gas Sources and Sinks and their Mitigation Potential." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 44. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2605.

Hertel, T.W., E. Peterson, Y. Surry, P.V. Preckel, and M.E. Tsigas. 1991. "Implicit Additivity as a Strategy for Restricting the Parameter Space in Computable General Equilibrium Models." *Economic and Financial Computing*, 1: 265–289.

Hertel, T.W., S. Rose, and R. Tol. 2009b. "Land Use In Computable General Equilibrium Models: An Overview." In *Economic Analysis of Land Use in Global Climate Change Policy*. United Kingdom: Routledge, Routledge Explorations in Environmental Economics.

Horridge, M. 2014. "Making CET quantities add up." Center of Policy Studies

(CoPS), Melbourne, Australia, Report, September. https://www.copsmodels.com/archivep/tpmh0143.zip.

Horridge, M., A. Meeraus, K. Pearson, and T.F. Rutherford. 2013. "Chapter 20 - Solution Software for Computable General Equilibrium Modeling." In *Handbook of Computable General Equilibrium Modeling SET, Vols. 1A and 1B*, edited by P. B. Dixon and D. W. Jorgenson. Elsevier, vol. 1 of *Handbook of Computable General Equilibrium Modeling*, pp. 1331–1381. doi:10.1016/B978-0-444-59568-3.00020-1.

Howe, H. 1975. "Development of the extended linear expenditure system from simple saving assumptions." *European Economic Review*, 6(3): 305–310. doi:10.1016/0014-2921(75)90014-8.

Huff, K.M., and T.W. Hertel. 2001. "Decomposing Welfare Changes in GTAP." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 5. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=308.

Hussein, Z., T. Hertel, and A. Golub. 2013. "Climate change mitigation policies and poverty in developing countries." *Environmental Research Letters*, 8(3): 035009. doi:10.1088/1748-9326/8/3/035009.

Ianchovichina, E., and R. McDougall. 2000. "Theoretical Structure of Dynamic GTAP." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 17. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=480.

Ianchovichina, E., and T. Walmsley. 2005. "Impact of China's WTO Accession on East Asia." *Contemporary Economic Policy*, 23(2): 261–277. doi:10.1093/cep/byi020.

Ianchovichina, E., and T.L. Walmsley, eds. 2012. *Dynamic Modeling and Applications for Global Economic Analysis*. New York, NY: Cambridge University Press. http://www.cambridge.org/gb/academic/subjects/economics/macroeconomics-and-monetary-economics/dynamic-modeling-and-applications-global-economic-analysis?format=PB&isbn=9781107002432.

Johansen, L. 1960. *A Multi-sectoral Study of Economic Growth*. Amsterdam: North-Holland.

Jomini, P., R. McDougall, G. Watts, and P.S. Dee. 1994. "The SALTER Model of the World Economy: Model Structure, Database and Parameters." Industry Commission, Canberra, Australia, Working Paper No. 24. http://www.pc.gov.au/research/supporting/salter-model/workingpaper24/workingpaper24.pdf.

Keeney, R., and T.W. Hertel. 2005. "GTAP-AGR: A Framework for Assessing the Implications of Multilateral Changes in Agricultural Policies." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 24. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=1869.

Keller, W.J. 1980. *Tax Incidence : a General Equilibrium Approach*. Amsterdam: North-Holland Publishing Co.

Koopman, R., Z. Wang, and S. Wei. 2014. "Tracing Value-Added and Double Counting in Gross Exports." *American Economic Review*, 104(2): 459–94. doi:10.1257/aer.104.2.459.

Lee, H., T. Hertel, S. Rose, and M. Avetsiyan. 2009. "Economic Analysis of Land Use in Global Climate Change Policy." In *An Integrated Land Use Data Base for CGE Analysis of Climate Policy Options*. Abingdon, UK: Routledge Press, pp. 72–88.

Lee, H.L., T.W. Hertel, B. Sohngen, and N. Ramankutty. 2005. "Towards and Integrated Land Use Data Base for Assessing the Potential for Greenhouse Gas Mitigation." Purdue University, West Lafayette, IN, Report. https://www.gtap.agecon.purdue.edu/resources/download/2375.pdf.

Liu, J., T. Hertel, and F. Taheripour. 2016. "Analyzing Future Water Scarcity in Computable General Equilibrium Models." *Water Economics and Policy*, 2(4): 1650006. doi:10.1142/S2382624X16500065.

Liu, J., T.W. Hertel, F. Taheripour, T. Zhu, and C. Ringler. 2014. "International trade buffers the impact of future irrigation shortfalls." *Global Environmental Change*, 29: 22–31. doi:10.1016/j.gloenvcha.2014.07.010.

Lluch, C. 1973. "The extended linear expenditure system." *European Economic Review*, 4(1): 21–32. doi:10.1016/0014-2921(73)90028-7.

Malcolm, G. 1998. "Adjusting Tax Rates in the GTAP Data Base." Global Trade Analysis Project (GTAP), Purdue University, West Lafayette, IN, GTAP Technical Paper No. 12. https://www.gtap.agecon.purdue.edu/resources/download/580.pdf.

McDonald, S., and K. Thierfelder. 2004. "Deriving a Global Social Accounting Matrix from GTAP Versions 5 and 6 Data." Global Trade Analysis Project (GTAP), Purdue University, West Lafayette, IN, GTAP Technical Paper No. 22. https://www.gtap.agecon.purdue.edu/resources/download/2012.pdf.

McDonald, S., K. Thierfelder, and S. Robinson. 2007. "Globe: A SAM Based Global CGE Model using GTAP Data." Department of Economics, U.S. Naval Academy, Annapolis, MD, USNA Economics Department Working Paper No. 15. http://www.cgemod.org.uk/usnawp14.pdf.

McDougall, R. 2003. "A New Regional Household Demand System for GTAP (Revision 1)." Global Trade Analysis Project (GTAP), Purdue University, West Lafayette, IN, GTAP Technical Paper No. 20. https://www.gtap.agecon.purdue.edu/resources/download/1593.pdf.

McDougall, R., and A. Golub. 2007. "GTAP-E Release 6: A Revised Energy-Environmental Version of the GTAP Model." Purdue University, West Lafayette, In, Report. https://www.gtap.agecon.purdue.edu/resources/download/4212.pdf.

Melitz, M.J. 2002. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." National Bureau of Economic Research, Working

Paper No. 8881, April. doi:10.3386/w8881.

Minor, P., and T. Walmsley. 2013. "MyGTAP: A Program for Customizing and Extending the GTAP Database for Multiple Households, Split Factors, Remittances, Foreign Aid and Transfers." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 79. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4321.

Monfreda, C., N. Ramankutty, and T.W. Hertel. 2009. "Global Agricultural Land Use Data for Climate Change Analysis." In *Economic Analysis of Land Use in Global Climate Change Policy*, edited by T. W. Hertel, S. K. Rose, and R. S. J. Tol. Oxon, UK: Routledge, chap. 2, pp. 33–48.

Narayanan, B., T.W. Hertel, and M. Horridge. 2010. "Linking Partial and General Equilibrium Models: A GTAP Application Using TASTE." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 29. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=3192.

Peters, J.C. 2016a. "GTAP-E-Power: An Electricity-detailed Economy-wide Model." *Journal of Global Economic Analysis*, 1(2): 156–187. doi:10.21642/JGEA.010204AF.

Peters, J.C. 2016b. "The GTAP-Power Data Base: Disaggregating the Electricity Sector in the GTAP Data Base." *Journal of Global Economic Analysis*, 1(1): 209–250. doi:10.21642/JGEA.010104AF.

Peterson, E.B. 2006. "GTAP-M: A GTAP Model and Data Base that Incorporates Domestic Margins." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Technical Paper No. 26. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2200.

Plevin, R., H. Gibbs, J. Duffy, S. Yui, and S. Yeh. 2014. "Agro-ecological Zone Emission Factor (AEZ-EF) Model (v47)." http://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4346.

Powell, A.A. 2007. "Why How and When did GTAP Happen? What has it Achieved? Where is it Heading?" Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 38. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2467.

Preckel, P.V., J.A.L. Cranfield, and T.W. Hertel. 2010. "A modified, implicit, directly additive demand system." *Applied Economics*, 42(2): 143–155. doi:10.1080/00036840701591361.

Pyatt, G., and J.I. Round, eds. 1985. *Social Accounting Matrices: A Basis for Planning*. Washington, DC: The World Bank. http://www-wds.worldbank.org/external/default/WDSContentServer/WDSP/IB/1999/12/02/000178830_98101901520228/Rendered/PDF/multi_page.pdf.

Reinert, K.A., and D.W. Roland-Holst. 1997. "Social Accounting Matrices." In *Ap-*

plied Methods for Trade Policy Analysis: A Handbook, edited by J. F. Francois and K. A. Reinert. Cambridge, UK: Cambridge University Press, chap. 4, pp. 94–121.

Rimmer, M.T., and A.A. Powell. 1992. “An Implicitly Directly Additive Demand System: Estimates for Australia.” Centre of Policy Studies and the Impact Project, Monash University, Preliminary Working Paper No. OP-73. http://www.copsmodels.com/ftp/workpapr/op-73.pdf.

Rose, S., and H.L. Lee. 2008. “Non-CO2 Greenhouse Gas Emissions Data for Climate Change Economic Analysis.” http://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2604.

Shoven, J.B., and J. Whalley. 1992. *Applying General Equilibrium*. Cambridge Surveys of Economic Literature, Cambridge UK and New York, NY: Cambridge University Press. http://www.cambridge.org/us/academic/subjects/economics/microeconomics/applying-general-equilibrium.

Siddig, K., A. Aguiar, H. Grethe, P. Minor, and T. Walmsley. 2014. “Impacts of removing fuel import subsidies in Nigeria on poverty.” *Energy Policy*, 69: 165–178. doi:10.1016/j.enpol.2014.02.006. http://linkinghub.elsevier.com/retrieve/pii/S0301421514000950.

Sohngen, B., C. Tennity, M. Hnytka, , and K. Meeusen. 2009. “Global Forestry Data for the Economic Modelling of Land Use.” In *Economic Analysis of Land Use in Global Climate Change Policy*, edited by T. W. Hertel, S. K. Rose, and R. S. J. Tol. Oxon, UK: Routledge, chap. 3, pp. 49–71.

Spearot, A. 2016. “Unpacking the Long-Run Effects of Tariff Shocks: New Structural Implications from Firm Heterogeneity Models.” *American Economic Journal: Microeconomics*, 8(2): 128–67. doi:10.1257/mic.20140015.

Stone, S., T.W. Hertel, and A. Strutt. 2010. “Assessing Socioeconomic Impacts of Transport Infrastructure Projects in the Greater Mekong Subregion.” Asian Development Bank Institute, Report. https://ideas.repec.org/p/eab/macroe/22819.html.

Surry, Y. 1993. “The Constant Difference of Elasticities Function with Applications to the EC Animal Feed Sector.” *Journal of Agricultural Economics*, 44(1): 110–125. doi:10.1111/j.1477-9552.1993.tb00254.x.

Taheripour, F., T.W. Hertel, and J. Liu. 2013a. “Introducing water by river basin into the GTAP-BIO model: GTAP-BIO-W.” Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 77. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4304.

Taheripour, F., T.W. Hertel, and J. Liu. 2013b. “The role of irrigation in determining the global land use impacts of biofuels.” *Energy, Sustainability and Society*, 3(1): 4. doi:10.1186/2192-0567-3-4.

Taheripour, F., T.W. Hertel, W.E. Tyner, J.F. Beckman, and D.K. Birur. 2008. “Biofuels, Agriculture And Climate Change.” Orlando, Florida, USA.

Taheripour, F., T.W. Hertel, W.E. Tyner, J.F. Beckman, and D.K. Birur. 2010. “Biofuels and their by-products: Global economic and environmental implications." *Biomass and Bioenergy*, 34(3): 278 – 289. doi:10.1016/j.biombioe.2009.10.017.

Taheripour, F., and W.E. Tyner. 2013. "Biofuels and Land Use Change: Applying Recent Evidence to Model Estimates." *Applied Sciences*, 3(1): 14–38. doi:10.3390/app3010014.

Tyner, W.E., F. Taheripour, and A. Golub. 2011. "Calculation of Indirect Land Use Change (ILUC) Values for Low Carbon Fuel Standard (LCFS) Fuel Pathways, Interim Report to the California Air Resources Board." California Air Resources Board (CARB), Report. https://www.gtap.agecon.purdue.edu/resources/download/5629.pdf.

van der Mensbrugghe, D. 2013. "Chapter 14 - Modeling the Global Economy—Forward-looking Scenarios for Agriculture." In *Handbook of Computable General Equilibrium Modeling SET, Vols. 1A and 1B*, edited by P. B. Dixon and D. W. Jorgenson. Elsevier, vol. 1B of *Handbook of Computable General Equilibrium Modeling*, pp. 933–994. doi:10.1016/B978-0-444-59568-3.00014-6.

Varian, H.R. 1992. *Microeconomic Analysis: Third Edition*. W. W. Norton & Company, Inc. http://books.wwnorton.com/books/detail.aspx?ID=11247.

Verma, M., T.W. Hertel, and P.V. Preckel. 2011. "Predicting within country household food expenditure variation using international cross-section estimates." *Economics Letters*, 113(3): 218–220. http://econpapers.repec.org/article/eeeecolet/v_3a113_3ay_3a2011_3ai_3a3_3ap_3a218-220.htm.

Walmsley, T., and T. Hertel. 2000. "China's Accession to the WTO: Timing is Everything." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 13. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=403.

Walmsley, T., and P. Minor. 2013. "MyGTAP Model: A Model for Employing Data from the MyGTAP Data Application—Multiple Households, Split Factors, Remittances, Foreign Aid and Transfers." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 78. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=4320.

Walmsley, T.L., T.W. Hertel, and D. Hummels. 2014. "Developing a GTAP-based multi-region, input-output framework for supply chain analysis." In *Asia and Global Production Networks: Implications for Trades, Incomes and Vulnerability*. Edward Elgar Publishing.

Walmsley, T.L., A. Winters, and S.A. Ahmed. 2011. "The Impact of the Movement of Labour: Results from a Model of Bilateral Migration Flows." *Global Economy Journal*, 11(4). doi:10.2202/1524-5861.1738.

Walmsley, T.L., L.A. Winters, and S.A. Ahmed. 2007. "Measuring the Impact of the Movement of Labor Using a Model of Bilateral Migration Flows." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue

University, West Lafayette, IN, GTAP Technical Paper No. 28. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2529.

Whalley, J. 1984. *Trade Liberalization among Major World Trading Areas*. Cambridge, MA: MIT Press.

Zhai, F. 2008. "Armington Meets Melitz: Introducing Firm Heterogeneity in a Global CGE Model of Trade." *Journal of Economic Integration*, 23(3): 575–604. doi:10.11130/jei.2008.23.3.575.

Appendix A. Summary of old vs. new sets, parameters, data flows and variables in the model

Table A.1. Set definition

No.	**GTAPv6.2**	**Header**[a]	**Description**	**No.**	**GTAPv7**	**Header**[a]	**Description**
1	`REG`	`H1`	regions in the model	1	`REG`	`REG`	regions
2	`TRAD_COMM`	`H2`	traded commodities	2	`COMM`	`COMM`	commodities
3	`MARG_COMM`	`MARG`	margin commodities	3	`MARG`	`MARG`	margin commodities
4	`NMRG_COMM`		non-margin commodities	4	`NMRG`		non-margin commodities
5	`CGDS_COMM`	`H9`	capital goods commodities				
6	`PROD_COMM`		produced commodities	5	`ACTS`	`ACTS`	Activities
7	`ENDW_COMM`	`H6`	endowment commodities	6	`ENDW`	`ENDW`	endowments
8	`DEMD_COMM`		demanded commodities	7	`DEMD`		commodities and endowments
9	`ENDWS_COMM`		sluggish endowment commodities	8	`ENDWS`	`ENDS`	sluggish endowment
10	`ENDWM_COMM`		mobile endowment commodities	9	`ENDWM`	`ENDM`	mobile endowments
11	`ENDWC_COMM`		capital endowment commodity	10	`ENDWC`		capital endowment
12	`NSAV_COMM`		non-savings commodities	11	`ENDWF`		Sector-specific endowment
				12	`ENDWMS`		mobile and sluggish endowments

Notes: (a) Header name if read from sets input file (sets.har or gsdset.har)

Table A.2. Parameter input file[a]

No.	GTAPv6.2	Set	Description	No.	GTAPv7	Set	Description
1	ESBD	TRAD_COMM	Armington CES for domestic/imported allocation	1	ESBD	COMM*REG	Armington CES for domestic/imported allocation
2	ESBM	TRAD_COMM	Armington CES for regional allocation of imports	2	ESBM	COMM*REG	Armington CES for regional allocation of imports
3	ESBT	PROD_COMM	Elasticity of intermediate input substitution	3	ESBT	ACTS*REG	CES between primary factors and intermediate inputs
4	ESBV	PROD_COMM	CES between primary factors in production	4	ESBV	ACTS*REG	CES between primary factors in production
5	ETRE	ENDW_COMM	CET between sectors for sluggish primary factors	5	ETRE	ENDW*REG	CET between sectors for sluggish primary factors
6	INCP	TRAD_COMM*REG	CDE expansion parameter	6	INCP	COMM*REG	CDE expansion parameter
7	SUBP	TRAD_COMM*REG	CDE substitution parameter	7	SUBP	COMM*REG	CDE substitution parameter
8	RDLT		Investment allocation binary coefficient	8	RDLT		Investment allocation binary coefficient
9	RFLX	REG	Expected rate of return flexibility parameter	9	RFLX	REG	Expected rate of return flexibility parameter
10	SLUG		Binary parameter for factor mobility: 1=sluggish 0=mobile	10	ESBG	REG	CES elasticity of substitution for government demands
				11	ETRQ	ACTS*REG	CET elasticity of transformation for commodities produced by industries
				12	ESBS	MARG	CES elasticity of substitution for international transport margin services
				13	ESBC	ACTS*REG	CES elasticity of substitution for intermediate inputs
				14	ESBQ	COMM*REG	1/CES elasticity for commodity sourcing

Notes: (a) Based on parameter input file (default.prm or gsdpar.har)

Table A.3. GTAP Data Base input file[a]

No.	GTAPv6.2	Description	No.	GTAPv7	Description
1	`ADRV`	anti-dumping duty	1	`ADRV`	anti-dumping duty
2	`EVFA`	primary factor purchases, at agents' prices	2	`EVFP`	primary factor purchases, at producer prices
3	`EVOA`	primary factor sales, at agents' prices	3	`EVOS`	primary factor sales, at supply (post-income tax) prices
4	`FBEP`	gross factor-based subsidies	4	`FBEP`	gross factor-based subsidies
5	`FTRV`	gross factor employment tax revenue	5	`FTRV`	gross factor employment tax revenue
6	`ISEP`	net intermediate input subsidies	6	`ISEP`	net investment input subsidies
7	`MFRV`	export tax equivalent of MFA quota premia	7	`MFRV`	export tax equivalent of MFA quota premia
8	`OSEP`	net ordinary output subsidy	8	`OSEP`	net ordinary output subsidy
9	`POP`	population	9	`POP`	population
10	`PURV`	export tax equivalent of price undertakings	10	`PURV`	export tax equivalent of price undertakings
11	`SAVE`	net saving, by region	11	`SAVE`	net saving, by region
12	`TFRV`	ordinary import duty	12	`TFRV`	ordinary import duty
13	`TVOM`	sales of domestic product, at market prices	13	`VOSB`	sales of domestic product, at basic prices
14	`VDEP`	capital depreciation	14	`VDEP`	capital depreciation
15	`VDFA`	domestic purchases, by firms, at agents' prices	15	`VDFP`	domestic purchases, by firms, at producer prices
16	`VDFM`	domestic purchases, by firms, at market prices	16	`VDFB`	domestic purchases, by firms, at basic prices
17	`VDGA`	domestic purchases, by government, at agents' prices	17	`VDGP`	domestic purchases, by government, at producer prices
18	`VDGM`	domestic purchases, by government, at market prices	18	`VDGB`	domestic purchases, by government, at basic prices
19	`VDPA`	domestic purchases, by households, at agents' prices	19	`VDPP`	domestic purchases, by households, at producer prices
20	`VDPM`	domestic purchases, by households, at market prices	20	`VDPB`	domestic purchases, by households, at basic prices
21	`VFM`	primary factor purchases, by firms, at market prices	21	`EVFB`	primary factor purchases, by firms, at basic prices

Notes: (a) Based on data input file (basedata.har or gsddat.har)

Table A.3. GTAP Data Base input file, ctd.

No.	GTAPv6.2	Description	No.	GTAPv7	Description
22	`VIFA`	import purchases, by firms, at agents' prices	22	`VMFP`	import purchases, by firms, at producer prices
23	`VIFM`	import purchases, by firms, at market prices	23	`VMFB`	import purchases, by firms, at basic prices
24	`VIGA`	import purchases, by government, at agents' prices	24	`VMGP`	import purchases, by government, at producer prices
25	`VIGM`	import purchases, by government, at market prices	25	`VMGB`	import purchases, by government, at basic prices
26	`VIMS`	imports, at market prices	26	`VMSB`	imports, at basic prices
27	`VIPA`	import purchases, by households, at agents' prices	27	`VMPP`	import purchases, by households, at producer prices
28	`VIPM`	import purchases, by households, at market prices	28	`VMPB`	import purchases, by households, at basic prices
29	`VIWS`	imports, at world prices	29	`VCIF`	imports, at CIF prices
30	`VKB`	capital stock	30	`VKB`	capital stock
31	`VRRV`	export tax equivalent of voluntary export restraints	31	`VRRV`	export tax equivalent of voluntary export restraints
32	`VST`	margin exports	32	`VST`	margin exports
33	`VTSS`	Import tariff Rev by type of tariffs paid	33	`VMTS`	Import tariff Rev by type of tariffs paid
34	`VTWR`	margins by margin commodity	34	`VTWR`	margins by margin commodity
35	`VXMD`	non-margin exports, at market prices	35	`VXSB`	non-margin exports, at basic prices
36	`VXWD`	non-margin exports, at world prices	36	`VFOB`	non-margin exports, at FOB prices
37	`XTRV`	ordinary export tax	37	`XTRV`	ordinary export tax
			38	`VDIB`	domestic purchases, by investment, at basic prices
			39	`VDIP`	domestic purchases, by investment, at producer prices
			40	`VMIB`	import purchases, by investment, at basic prices
			41	`VMIP`	import purchases, by investment, at producer prices
			42	`CSEP`	net intermediate input subsidies
			43	`MAKS`	multi-production ('make') matrix at supply prices
			44	`MAKB`	multi-production ('make') matrix at basic prices

Table A.4. Variable List

No.	GTAPv6.2	Set	No.	GTAPv7	Set
1	*psave*	REG	1	*psave*	REG
2	*qsave*	REG	2	*qsave*	REG
3	*pgd*	TRAD_COMM*REG	3	*pgd*	COMM*REG
4	*qgd*	TRAD_COMM*REG	4	*qgd*	COMM*REG
5	*pm*	NSAV_COMM*REG	5	*pds*	COMM*REG
6	*pgm*	TRAD_COMM*REG	6	*pgm*	COMM*REG
7	*qgm*	TRAD_COMM*REG	7	*qgm*	COMM*REG
8	*pim*	TRAD_COMM*REG	8	*pms*	COMM*REG
9	*ppd*	TRAD_COMM*REG	9	*ppd*	COMM*REG
10	*qpd*	TRAD_COMM*REG	10	*qpd*	COMM*REG
11	*ppm*	TRAD_COMM*REG	11	*ppm*	COMM*REG
12	*qpm*	TRAD_COMM*REG	12	*qpm*	COMM*REG
13	*ps*	NSAV_COMM*REG	13	*po*	ACTS*REG
14	*qo*	NSAV_COMM*REG	14	*qo*	ACTS*REG
15	*pfe*	ENDW*PROD_COMM*REG	15	*pfe*	ENDW*ACTS*REG
16	*qfe*	ENDW*PROD_COMM*REG	16	*qfe*	ENDW*ACTS*REG
17	*pfd*	TRAD_COMM*PROD_COMM*REG	17	*pfd*	COMM*ACTS*REG
18	*qfd*	TRAD_COMM*PROD_COMM*REG	18	*qfd*	COMM*ACTS*REG
19	*pfm*	TRAD_COMM*PROD_COMM*REG	19	*pfm*	COMM*ACTS*REG
20	*qfm*	TRAD_COMM*PROD_COMM*REG	20	*qfm*	COMM*ACTS*REG
21	*pmes*	ENDWS_COMM*PROD_COMM*REG	21	*peb*	ENDW*ACTS*REG
22	*kb*	REG	22	*kb*	REG
23	*pcgds*	REG	23	*pinv*	REG
24	*pms*	TRAD_COMM*REG*REG	24	*pmds*	COMM*REG*REG
25	*qxs*	TRAD_COMM*REG*REG	25	*qxs*	COMM*REG*REG
26	*pcif*	TRAD_COMM*REG*REG	26	*pcif*	COMM*REG*REG
27	*pfob*	TRAD_COMM*REG*REG	27	*pfob*	COMM*REG*REG
28	*qst*	MARG_COMM*REG	28	*qst*	MARG*REG

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
29	*y*	REG	29	*y*	REG
30	*pop*	REG	30	*pop*	REG
31	*qoes*	ENDWS_COMM*PROD_COMM*REG			
32	*endwslack*	ENDW_COMM*REG	31	*endwslack*	ENDW*REG
33	*pgov*	REG	32	*pgov*	REG
34	*yg*	REG	33	*yg*	REG
35	*ug*	REG	34	*ug*	REG
36	*ppriv*	REG	35	*ppriv*	REG
37	*uepriv*	REG	36	*uepriv*	REG
38	*yp*	REG	37	*yp*	REG
39	*up*	REG	38	*up*	REG
40	*to*	PROD_COMM*REG	39	*to*	COMM*ACTS*REG
41	*qim*	TRAD_COMM*REG	40	*qms*	COMM*REG
42	*globalcgds*		41	*globalcgds*	
43	*pcgdswld*		42	*pcgdswld*	
44	*del_taxrgc*	REG	43	*del_taxrgc*	REG
45	*del_taxrpc*	REG	44	*del_taxrpc*	REG
46	*del_taxriu*	REG	45	*del_taxriu*	REG
47	*del_taxrfu*	REG	46	*del_taxrfu*	REG
48	*del_taxrout*	REG	47	*del_taxrout*	REG
49	*del_taxrexp*	REG	48	*del_taxrexp*	REG
50	*del_taxrimp*	REG	49	*del_taxrimp*	REG
51	*del_taxrinc*	REG	50	*del_taxrinc*	REG
52	*pg*	TRAD_COMM*REG	51	*pga*	COMM*REG
53	*qg*	TRAD_COMM*REG	52	*qga*	COMM*REG
54	*tgd*	TRAD_COMM*REG	53	*tgd*	COMM*REG
55	*tgm*	TRAD_COMM*REG	54	*tgm*	COMM*REG
56	*pp*	TRAD_COMM*REG	55	*ppa*	COMM*REG

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
57	*qp*	`TRAD_COMM*REG`	56	*qpa*	`COMM*REG`
58	*tp*	`TRAD_COMM*REG`	57	*tp*	`REG`
59	*tpd*	`TRAD_COMM*REG`	58	*tpd*	`COMM*REG`
60	*atpd*	`REG`	59	*atpd*	`COMM*REG`
61	*tpm*	`TRAD_COMM*REG`	60	*atpm*	`COMM*REG`
62	*atpm*	`TRAD_COMM*REG`	61	*tpm*	`COMM*REG`
63	*pva*	`PROD_COMM*REG`	62	*pva*	`ACTS*REG`
64	*qva*	`PROD_COMM*REG`	63	*qva*	`ACTS*REG`
65	*pf*	`TRAD_COMM*PROD_COMM*REG`	64	*pfa*	`COMM*ACTS*REG`
66	*qf*	`TRAD_COMM*PROD_COMM*REG`	65	*qfa*	`COMM*ACTS*REG`
67	*ao*	`PROD_COMM*REG`	66	*ao*	`ACTS*REG`
68	*ava*	`PROD_COMM*REG`	67	*ava*	`ACTS*REG`
69	*af*	`TRAD_COMM*PROD_COMM*REG`	68	*afa*	`COMM*ACTS*REG`
70	*afe*	`ENDW*PROD_COMM*REG`	69	*afe*	`ENDW*ACTS*REG`
71	*ams*	`TRAD_COMM*REG*REG`	70	*ams*	`COMM*REG*REG`
72	*aosec*	`PROD_COMM`	71	*aosec*	`ACTS`
73	*aoreg*	`REG`	72	*aoreg*	`REG`
74	*aoall*	`PROD_COMM*REG`	73	*aoall*	`ACTS*REG`
75	*avasec*	`PROD_COMM`	74	*avasec*	`ACTS`
76	*avareg*	`REG`	75	*avareg*	`REG`
77	*avaall*	`PROD_COMM*REG`	76	*avaall*	`ACTS*REG`
78	*afcom*	`TRAD_COMM`	77	*afcom*	`COMM`
79	*afsec*	`PROD_COMM`	78	*afsec*	`ACTS`
80	*afreg*	`REG`	79	*afreg*	`REG`
81	*afall*	`TRAD_COMM*PROD_COMM*REG`	80	*afall*	`COMM*ACTS*REG`
82	*tfd*	`TRAD_COMM*PROD_COMM*REG`	81	*tfd*	`COMM*ACTS*REG`
83	*tfm*	`TRAD_COMM*PROD_COMM*REG`	82	*tfm*	`COMM*ACTS*REG`
84	*tf*	`ENDW*PROD_COMM*REG`	83	*tfe*	`ENDW*ACTS*REG`

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
85	*afecom*	ENDW	84	*afecom*	ENDW
86	*afesec*	PROD_COMM	85	*afesec*	ACTS
87	*afereg*	REG	86	*afereg*	REG
88	*afeall*	ENDW*PROD_COMM*REG	87	*afeall*	ENDW*ACTS*REG
89	*profitslack*	PROD_COMM*REG	88	*profitslack*	ACTS*REG
90	*rental*	REG	89	*rental*	REG
91	*ke*	REG	90	*ke*	REG
92	*rore*	REG	91	*rore*	REG
93	*rorc*	REG	92	*rorc*	REG
94	*qcgds*	REG	93	*qinv*	REG
95	*ksvces*	REG			
96	*EXPAND*	ENDWC*REG	94	*expand*	ENDWC*REG
97	*rorg*		95	*rorg*	
98	*cgdslack*	REG	96	*cgdslack*	REG
99	*psaveslack*	REG	97	*psaveslack*	REG
100	*tx*	TRAD_COMM*REG	98	*tx*	COMM*REG
101	*txs*	TRAD_COMM*REG*REG	99	*txs*	COMM*REG*REG
102	*tm*	TRAD_COMM*REG	100	*tm*	COMM*REG
103	*tms*	TRAD_COMM*REG*REG	101	*tms*	COMM*REG*REG
104	*pr*	TRAD_COMM*REG	102	*pr*	COMM*REG
105	*qtmfsd*	MARG_COMM*TRAD_COMM*REG*REG	103	*qtmfsd*	MARGc*COMM*REG*REG
106	*atmfsd*	MARG_COMM*TRAD_COMM*REG*REG	104	*atmfsd*	MARGc*COMM*REG*REG
107	*atm*	TRAD_COMM	105	*atm*	COMM
108	*atf*	TRAD_COMM	106	*atf*	COMM
109	*ats*	REG	107	*ats*	REG
110	*atd*	REG	108	*atd*	REG
111	*atall*	MARG_COMM*TRAD_COMM*REG*REG	109	*atall*	MARGc*COMM*REG*REG
112	*ptrans*	TRAD_COMM*REG*REG	110	*ptrans*	COMM*REG*REG

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
113	*qtm*	MARG_COMM	111	*qtm*	MARG
114	*pt*	MARG_COMM	112	*pt*	MARG
115	*uelas*	REG	113	*uelas*	REG
116	*dppriv*	REG	114	*dppriv*	REG
117	*dpgov*	REG	115	*dpgov*	REG
118	*dpsave*	REG	116	*dpsave*	REG
119	*fincome*	REG	117	*fincome*	REG
120	*del_indtaxr*	REG	118	*del_indtaxr*	REG
121	*del_ttaxr*	REG	119	*del_ttaxr*	REG
122	*incomeslack*	REG	120	*incomeslack*	REG
123	*dpav*	REG	121	*dpav*	REG
124	*p*	REG	122	*p*	REG
125	*au*	REG	123	*au*	REG
126	*dpsum*	REG	124	*dpsum*	REG
127	*u*	REG	125	*u*	REG
128	*qds*	TRAD_COMM*REG	126	*qds*	COMM*REG
129	*tradslack*	TRAD_COMM*REG	127	*tradslack*	COMM*REG
130	*walras_sup*		128	*walras_sup*	
131	*walras_dem*		129	*walras_dem*	
132	*walraslack*		130	*walraslack*	
133	*vxwfob*	TRAD_COMM*REG	131	*vxwfob*	COMM*REG
134	*viwcif*	TRAD_COMM*REG	132	*vmwcif*	COMM*REG
135	*vxwreg*	REG	133	*vxwreg*	REG
136	*viwreg*	REG	134	*vmwreg*	REG
137	*pfactreal*	ENDW*REG	135	*pfactreal*	ENDW*ACTS*REG
138	*pfactor*	REG	136	*pfactor*	REG
139	*pfactwld*		137	*pfactwld*	
140	*psw*	REG	138	*psw*	REG

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
141	*pdw*	REG	139	*pdw*	REG
142	*tot*	REG	140	*tot*	REG
143	*vgdp*	REG	141	*vgdp*	REG
144	*pgdp*	REG	142	*pgdp*	REG
145	*qgdp*	REG	143	*qgdp*	REG
146	*compvalad*	PROD_COMM*REG	144	*compvalad*	ACTS*REG
147	*pxw*	TRAD_COMM*REG	145	*pxw*	COMM*REG
148	*vxwcom*	TRAD_COMM	146	*vxwcom*	COMM
149	*vxwwld*		147	*vxwwld*	
150	*viwcom*	TRAD_COMM	148	*vmwcom*	COMM
151	*valuew*	TRAD_COMM	149	*valuew*	COMM
152	*valuewu*	TRAD_COMM	150	*valuewu*	COMM
153	*pxwreg*	REG	151	*pxwreg*	REG
154	*pxwcom*	TRAD_COMM	152	*pxwcom*	COMM
155	*pxwwld*		153	*pxwwld*	
156	*piw*	TRAD_COMM*REG	154	*pmw*	COMM*REG
157	*piwreg*	REG	155	*pmwreg*	REG
158	*piwcom*	TRAD_COMM	156	*pmwcom*	COMM
159	*pw*	TRAD_COMM	157	*pw*	COMM
160	*pwu*	TRAD_COMM	158	*pwu*	COMM
161	*qxw*	TRAD_COMM*REG	159	*qxw*	COMM*REG
162	*qxwreg*	REG	160	*qxwreg*	REG
163	*qxwcom*	TRAD_COMM	161	*qxwcom*	COMM
164	*qxwwld*		162	*qxwwld*	
165	*qiw*	TRAD_COMM*REG	163	*qmw*	COMM*REG
166	*qiwreg*	REG	164	*qmwreg*	REG
167	*qiwcom*	TRAD_COMM	165	*qmwcom*	COMM
168	*qow*	TRAD_COMM	166	*qow*	COMM

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
169	*qowu*	TRAD_COMM	167	*qowu*	COMM
170	*DTBALi*	TRAD_COMM*REG	168	*del_tbalc*	COMM*REG
171	*DTBAL*	REG	169	*del_tbal*	REG
172	*DTBALR*	REG	170	*del_tbalry*	REG
173	*uelasev*	REG	171	*uelasev*	REG
174	*ueprivev*	REG	172	*ueprivev*	REG
175	*ugev*	REG	173	*ugev*	REG
176	*upev*	REG	174	*upev*	REG
177	*qsaveev*	REG	175	*qsaveev*	REG
178	*yev*	REG	176	*yev*	REG
179	*ypev*	REG	177	*ypev*	REG
180	*ygev*	REG	178	*ygev*	REG
181	*qpev*	TRAD_COMM*REG	179	*qpev*	COMM*REG
182	*ysaveev*	REG	180	*ysaveev*	REG
183	*dpavev*	REG	181	*dpavev*	REG
184	*EV*	REG	182	*EV*	REG
185	*WEV*	1	183	*WEV*	
186	*EV_ALT*	REG	184	*EV_ALT*	REG
187	*WEV_ALT*	1	185	*WEV_ALT*	
188	*CNTdpar*	REG	186	*CNTdpar*	REG
189	*CNTpopr*	REG	187	*CNTpop*	REG
190	*CNTqor*	REG	188	*CNTqor*	REG
191	*CNTqoir*	PROD_COMM*REG	189	*CNTqo*	ACTS*REG
192	*CNTqfer*	REG	190	*CNTqfer*	REG
193	*CNTqfeir*	ENDW*REG	191	*CNTqfeer*	ENDW*REG
194	*CNTqfeijr*	ENDW*PROD_COMM*REG	192	*CNTqfe*	ENDW*ACTS*REG
195	*CNTqfmr*	REG*DIR	193	*CNTqfr*	REG*DIR
196	*CNTqfdr*				

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
197	*CNTqfmir*				
198	*CNTqfdir*				
199	*CNTqfmijr*	TRAD_COMM⋆PROD_COMM⋆REG	194	*CNTqfm*	COMM⋆ACTS⋆REG
200	*CNTqfdijr*	TRAD_COMM⋆PROD_COMM⋆REG	195	*CNTqfd*	COMM⋆ACTS⋆REG
201	*CNTqpmr*	REG	196	*CNTqpr*	REG⋆DIR
203	*CNTqpdr*	REG			
202	*CNTqpmir*	TRAD_COMM⋆REG	197	*CNTqpm*	COMM⋆REG
204	*CNTqpdir*	TRAD_COMM⋆REG	198	*CNTqpd*	COMM⋆REG
205	*CNTqgmr*	REG	199	*CNTqgr*	REG⋆DIR
206	*CNTqgdr*	REG			
207	*CNTqgmir*	TRAD_COMM⋆REG	200	*CNTqgm*	COMM⋆REG
208	*CNTqgdir*	TRAD_COMM⋆REG	201	*CNTqgd*	COMM⋆REG
209	*CNTqxsr*	REG	203	*CNTqxsr*	REG
210	*CNTqxsirs*	TRAD_COMM⋆REG⋆REG	202	*CNTqxs*	COMM⋆REG⋆REG
211	*CNTqimr*	REG	204	*CNTqmsr*	REG
212	*CNTqimisr*	TRAD_COMM⋆REG⋆REG	205	*CNTqms*	COMM⋆REG⋆REG
213	*CNTalleffr*	REG⋆CTAX	206	*CNTalleffr*	REGt⋆CTAX
214	*CNTalleffir*	TRAD_COMM⋆REG	207	*CNTalleffcr*	COMM⋆REG
215	*CNTtotr*	REG	208	*CNTtotr*	REG
216	*CNTcgdsr*	REG	209	*CNTpinv*	REG
217	*CNTendwr*	REG	210	*CNTendwr*	REG
218	*CNTendwir*	ENDW⋆REG	211	*CNTendw*	ENDW⋆REG
219	*CNTtechr*	REG	212	*CNTtechr*	TECHTYPE⋆REG
220	*CNTtech_aor*	REG			
221	*CNTtech_afer*	REG			
222	*CNTtech_avajr*	REG			
223	*CNTtech_afijr*	REG			
224	*CNTtech_atrr*	REG			

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
225	*CNTtech_amsr*	REG			
226	*CNTtech_aoir*	PROD_COMM*REG	213	*CNTtech_ao*	ACTS*REG
227	*CNTtech_afeijr*	ENDW*PROD_COMM*REG	214	*CNTtech_afe*	ENDW*ACTS*REG
228	*CNTtech_avar*	PROD_COMM*REG	215	*CNTtech_ava*	ACTS*REG
229	*CNTtech_afr*	TRAD_COMM*PROD_COMM*REG	216	*CNTtech_af*	COMM*ACTS*REG
230	*CNTtech_afmfdsd*	MARG_COMM*TRAD_COMM*REG*REG	217	*CNTtech_atmfsd*	MARGc*COMM*REG*REG
231	*CNTtech_amsirs*	TRAD_COMM*REG*REG	218	*CNTtech_ams*	COMM*REG*REG
232	*CNTkbr*	REG	219	*CNTkb*	REG
233	*pm_ir*	TRAD_COMM*REG	220	*pm_cr*	COMM*REG
234	*px_ir*	TRAD_COMM*REG			
235	*px_i*	TRAD_COMM			
236	*px_*				
237	*c1_ir*	TRAD_COMM*REG	221	*c1_cr*	COMM*REG
238	*c2_ir*	TRAD_COMM*REG	222	*c2_cr*	COMM*REG
239	*c3_ir*	TRAD_COMM*REG	223	*c3_cr*	COMM*REG
240	*c1_r*	REG	224	*c1_r*	REG
241	*c2_r*	REG	225	*c2_r*	REG
242	*c3_r*	REG	226	*c3_r*	REG
243	*tot2*	REG	227	*tot2*	REG
			228	*pb*	ACTS*REG
			229	*pca*	COMM*ACTS*REG
			230	*qca*	COMM*ACTS*REG
			231	*qc*	COMM*REG
			232	*pe*	ENDWMS*REG
			233	*qe*	ENDWMS*REG
			234	*qes*	ENDW*ACTS*REG
			235	*pes*	ENDW*ACTS*REG
			236	*tinc*	ENDW*ACTS*REG

Table A.4. Variable List, ctd.

No.	GTAPv6.2	Set	No.	GTAPv7	Set
			237	*pint*	`ACTS⋆REG`
			238	*qint*	`ACTS⋆REG`
			239	*aint*	`ACTS⋆REG`
			240	*aintsec*	`ACTS`
			241	*aintreg*	`REG`
			242	*aintall*	`ACTS⋆REG`
			243	*pia*	`COMM⋆REG`
			244	*qia*	`COMM⋆REG`
			245	*tid*	`COMM⋆REG`
			246	*tim*	`COMM⋆REG`
			247	*pid*	`COMM⋆REG`
			248	*qid*	`COMM⋆REG`
			249	*pim*	`COMM⋆REG`
			250	*qim*	`COMM⋆REG`
			251	*del_taxric*	`REG`
			252	*ps*	`COMM⋆ACTS⋆REG`
			253	*qesf*	`ENDWF⋆ACTS⋆REG`
			254	*ywld*	
			255	*CNTqca*	`COMM⋆ACTS⋆REG`
			256	*CNTqe*	`ENDW⋆REG`
			257	*CNTqia*	`REG⋆DIR`
			258	*CNTqim*	`COMM⋆REG`
			259	*CNTqid*	`COMM⋆REG`
			260	*CNTtech_aint*	`ACTS⋆REG`

Appendix B. Mathematical appendix

We derive here some of the key mathematical features of the GTAP model. The first section introduces some of the key concepts underpinning the implementation of the model in the *GEMPACK* software, i.e., the log-linearization of the model equations. The second section expands on the implementation of the tax revenue streams in the GTAP model and highlights additional details regarding the log-linearization of the model. There are two key features of the GTAP model that differentiate it with respect to many other CGE models—the top-level utility function of the representative household and the Constant-Differences-in-Elasticity (CDE) sub-utility function for the allocation of private expenditures. Few models have any explicit top-level utility function for the allocation of regional income to private, public and saving expenditures. And many models use some variant of the Linear Expenditure System (LES) for allocating private expenditures. These special features of GTAP are developed in Sections B.3 and B.4.

B.1 Log linearization

The GTAP model derives from the Johansen/ORANI/MONASH tradition (Johansen (1960), Dixon et al. (1982), Dixon and Rimmer (2002)). As such, all of its equations in levels are converted to percent change form using log-linearization.[61]

The conversion of a model in levels into a model in percentage change form relies on three rules described in Table B.1.[62] To illustrate the mechanics of the conversion, and its usefulness for providing economic insights, we will derive the log-linearization of a generic CES function.

	Representation in:		
	levels		percentage changes
Multiplication rule	$X = YZ$	$\Rightarrow$	$x = y + z$
Power rule	$X = Y^{\alpha}$	$\Rightarrow$	$x = \alpha y$
Addition rules	$X = Y + Z$	$\Rightarrow$	$Xx = Yy + Zz$
			or $x = S_y y + S_z z$

X, Y and Z are levels of variables, x, y and z are percentage changes, α is a parameter and S_y and S_z are shares evaluated at the current solution. In the first step of a Johansen/Euler computation, the current solution is the initial solution. Hence $S_y = Y_0/X_0$ and $S_z = Z_0/X_0$. In subsequent steps, S_y and S_z are recomputed as X, Y and Z move away from their initial values.

Table B.1. Rules for deriving a percentage-change version of a model

The most generic form of the CES function, for example in terms of a single

---

[61] There are some rare exceptions to deal with variables that could take zero as a value in levels (for example tax revenues) in which case the equations are specified as ordinary changes.

[62] Reprised from Dixon and Rimmer (2002).

nested production function, can be written as follows in levels form:

$$\min C = \sum_i P_i X_i$$

$$\text{subject to:}$$

$$Q = A\left[\sum_i a_i\left(\lambda_i X_i\right)^{\rho}\right]^{1/\rho}$$

where C is total cost ($P \cdot Q$), X_i are the input components with a cost of P_i, Q is output, with a price of P, a_i are the so-called primal share parameters, ρ is the CES exponent, A is a neutral technology shifter, and the λ_i parameters are input-specific technology shifters (or biased technology shifters).

It is easy to show that the first order conditions of the CES leads to the following set of equations in levels:

$$X_i = \alpha_i\left(\lambda_i A\right)^{\sigma-1}\left(\frac{P}{P_i}\right)^{\sigma} Q$$

$$P = \frac{1}{A}\left[\sum_i \alpha_i\left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \iff P \cdot Q = \sum_i P_i X_i$$

where the primal and dual parameters are linked by $\sigma = 1/(1-\rho)$ and $\alpha_i = a_i^{\sigma}$. The parameter $\sigma > 0$ is the so-called CES substitution elasticity. The implementation of the CES can typically dispense with the primal parameters. The levels form requires calibrating the α parameters using base year data.

Using the formulas from Table B.1, the CES demand function and its accompanying price expression are readily converted to percent change form:

$$\dot{x}_i = \dot{q} - \dot{a} - \dot{\lambda}_i - \sigma\left(\dot{p}_i - \dot{\lambda}_i - \dot{p} - \dot{a}\right)$$

$$\dot{p} = -\dot{a} + \sum_i S_i\left(\dot{p}_i - \dot{\lambda}_i\right) \qquad \text{and } S_i = \frac{P_i X_i}{P \cdot Q}$$

where we are using a dot over the lower case variable to designate the variable in percentage change form. The first equation is intuitively appealing (ignoring the technical change coefficents for a moment). The percentage change in demand for component i changes in proportion with overall output, $\dot{q}$, adjusted for any change in the price P_i relative to the aggregate price, P, where σ determines the level of adjustment. At one extreme, when σ is zero, demand changes in exact proportion with output—the so-called Leontief technology. As σ increases, the price adjustments become more prominent. The price formula is also intuitive. A first order approximation of an increase in the price of P_i on aggregate cost is the percentage change in P_i adjusted by its cost share.

As we can see from this example, the implementation of the CES in percentage change form does not require calibrating the CES share parameters, though it does require specifying an updating formula for the cost shares in the price expression.

In the model write-up, we have used the percentage change formulation for the behavioral equations where there appeal to intuition is most clear. However, many of the accounting identities and equilibrium conditions have been left in level form to improve exposition.

B.2 Revenue tax streams in the GTAP model

A generic form of a tax stream in levels form can be expressed as:

$$REV = \sum_i \left(T_i - 1\right) PB_i X_i$$

where T is the power of the tax, PB is the base price of a good (or factor), and X is the quantity being taxed. In fact, in the implementation of the GTAP model, the equations are formulated as the ratio of the tax revenue and income, and thus the relevant variable is $R = REV/Y$.

Since taxes and thus tax revenues, can be zero, positive or negative, there is a danger to using the standard percentage change form for the equation, and instead, the tax revenue equations are expressed as ordinary changes. Thus we have:

$$Y \cdot \Delta R + R \cdot \Delta Y = \sum_i \left[\Delta T_i PB_i X_i + \left(T_i - 1\right) \Delta PB_i X_i + \left(T_i - 1\right) PB_i \Delta X_i\right]$$

The relevant variables can be converted to percentage change form:

$$Y \cdot \Delta R + R \cdot Y \cdot \dot{y} = \sum_i \left[T_i PB_i X_i \dot{t}_i + \left(T_i - 1\right) PB_i X_i \left(\dot{p}_i + \dot{x}_i\right)\right]$$

In the final step, we replace the variables in levels with GTAP coefficients that are updated at each iteration (replacing $R \cdot Y$ with REV) and multiplying ΔR by 100 to convert it to a change in percent[63]:

$$100 \cdot Y \cdot \Delta R + REV \cdot \dot{y} = \sum_i \left[VXP_i \dot{t}_i + VTAX_i \left(\dot{pb}_i + \dot{x}_i\right)\right]$$

where VXP is the value of X at producer prices (PP) and $VTAX$ is the value of the tax revenue (for each commodity i). Thus we have the following equations and updating formulas:

---

[63] All the indirect tax ratios are evaluated as decimal changes, thus there use in percentage change formulas requires multiplying by 100

$$\begin{aligned}
\dot{pp}_i &= \dot{pb}_i + \dot{t}_i \\
VXP_i &= \dot{pp}_i \dot{x}_i \\
VXB_i &= \dot{pb}_i \dot{x}_i \\
VTAX_i &= VXP_i - VXB_i \\
REV &= \textstyle\sum_i VTAX_i
\end{aligned}$$

where VXB is the value of X at basic prices. The first expression is a model equation. The remaining four represent model coefficients that are updated at each iteration. The updating expressions for VXP and VXB are interpreted by GEMPACK to mean to apply the percentage change of the relevant variables to their product:

$$\begin{aligned}
VXP_{i,n} &= PP_{i,n}X_{i,n} = PP_{i,n-1}(1+0.01\dot{pp}_{i,n})X_{i,n-1}(1+0.01\dot{x}_{i,n}) \\
VXB_{i,n} &= PB_{i,n}X_{i,n} = PB_{i,n-1}(1+0.01\dot{pb}_{i,n})X_{i,n-1}(1+0.01\dot{x}_{i,n})
\end{aligned}$$

where n represents an iteration counter. For the first iteration, the relevant base is the initial solution.

The listing below shows the relevant equations and formulas for the import tax revenues and they can be readily identified with the equations and formulas above. The two key differences are that the sums are over commodities and regions of origin (s) and that the import tariff is composed of two components—a bilateral tariff, initially calibrated to base year levels, and a shifter, uniform across source regions, that is set to 0 in the initial database.

Listing B.1. GEMPACK equations for Import tax revenues and related formulas

```
Update (all,c,COMM)(all,s,REG)(all,d,REG)
   VMSB(c,s,d) = pmds(c,s,d) * qxs(c,s,d);
Update (all,c,COMM)(all,s,REG)(all,d,REG)
   VCIF(c,s,d) = pcif(c,s,d) * qxs(c,s,d);

Formula (all,c,COMM)(all,s,REG)(all,d,REG)
   MTAX(c,s,d) = VMSB(c,s,d) - VCIF(c,s,d);
Formula (all,d,REG)
   TMTAX(d) = sum{c,COMM, sum{s,REG, MTAX(c,s,d)}};

Equation E_pmds
# links basic domestic import prices and CIF import prices #
(all,c,COMM)(all,s,REG)(all,d,REG)
   pmds(c,s,d) = pcif(c,s,d) + tm(c,d) + tms(c,s,d);

Equation E_del_taxrimp
# change in ratio of import tax payments to regional income #
(all,d,REG)
   100.0 * INCOME(d) * del_taxrimp(d) + TMTAX(d) * y(d)
      = sum{c,COMM, sum{s,REG, VMSB(c,s,d) * [tm(c,d) + tms(c,s,d)]
      + MTAX(c,s,d) * [pcif(c,s,d) + qxs(c,s,d)]}};
```

The equation for regional income levels is given by:

$$Y = FY + RT \cdot Y$$

where RT is the share of all indirect taxes in income and FY is aggregate factor income (including income taxes). Converting this into ordinary change, we have:

$$\Delta Y = \Delta FY + Y \cdot \Delta RT + RT \cdot \Delta Y$$

which can be converted to variables in percent differences:

$$Y \cdot \dot{y} = FY \cdot \dot{fy} + 100 \cdot Y \cdot \Delta RT + REVT \cdot \dot{y}$$

where $REVT = RT \cdot Y$ is total revenues from indirect taxes. Note again that the variable ΔRT is multiplied by 100 to convert to a percent. See equation `E_y` in the TABLO code.

B.3 Top level utility function of the representative household

Regional income is given by Y in nominal terms.[64] Income is allocated across i expenditure components such as to maximize total utility:

$$\max_{U_i} U = A \prod_i U_i^{\beta_i} \text{ subject to } \sum_i Y_i(U_i) = Y \tag{B.1}$$

where U_i represents the sub-utility derived from expenditure Y_i on expenditure component i. The β_i parameters are the Cobb-Douglas preference parameters and A is a scale parameter. Solving this for Y_i, yields the following equation:

$$Y_i = \frac{\beta_i}{\varphi_i} \frac{U}{\lambda} \tag{B.2}$$

where λ is the Lagrangian multiplier from the constrained optimization and φ_i is the elasticity of expenditure on commodity i with respect to sub-utility i:

$$\varphi_i = \frac{\partial Y_i}{\partial U_i} \frac{U_i}{Y_i} \tag{B.3}$$

Taking the sum of equation (B.2) and isolating λ, we have:

$$\lambda = \frac{U}{Y} \sum_j \frac{\beta_j}{\varphi_j}$$

---

[64] This section has been adapted from McDougall (2003).

and re-inserting back into equation (B.2) we derive:

$$Y_i = \frac{\beta_i}{\varphi_i} \frac{Y}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{B.4}$$

or

$$S_i = \frac{Y_i}{Y} = \frac{\beta_i}{\varphi_i} \frac{1}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{B.5}$$

where S_i is the expenditure share of component i.

We can also derive a relation between the elasticity of total expenditure to total utility, as a function of the sub-utility expenditure elasticities.

$$\Phi = \frac{\partial Y}{\partial U} \frac{U}{Y} = \frac{U}{Y} \sum_i \frac{\partial Y_i}{\partial U_i} \frac{\partial U_i}{\partial U} \tag{B.6}$$

This can be shown to be:[65]

$$\Phi = \frac{1}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{B.7}$$

Inserting this expression back into equation (B.5) we get another expression for the budget shares:

$$S_i = \frac{Y_i}{Y} = \frac{\beta_i}{\varphi_i} \Phi \tag{B.8}$$

In log-differentiated form, this yields:

$$\dot{y}_i - \dot{y} = \dot{\Phi} + \dot{\beta}_i - \dot{\varphi}_i \tag{B.9}$$

Equation (B.7) can also be log-differentiated to yield:

$$\dot{\Phi} = \sum_i S_i \dot{\varphi}_i - \sum_i S_i \dot{\beta}_i \tag{B.10}$$

The second term on the right-hand side is the weighted sum of the percent change in the preference parameters. In most simulations preferences will be unchanged and thus this expression is typically zero. In the model implementation it is replaced by a variable, say χ, and thus we write:

---

[65] See McDougall (2003).

$$\dot{\Phi} = \sum_i S_i \dot{\varphi}_i - \chi \tag{B.11}$$

and add an equation for χ:[66]

$$\chi = \sum_i S_i \dot{\beta}_i \tag{B.12}$$

In the GTAP model, there are three broad expenditure components at the top-level—private (P), public (or government, G) and saving (S). Public expenditures and saving are associated with a generic CES sub-utility function.[67] We know that for a CES utility function, the expenditure function can be written as $Y = U \times P$, where P is the aggregate price index. Thus clearly the expenditure elasticity is equal to 1, and in percentage change terms it is 0. We thus need to derive an expression for the elasticity of private expenditure with respect to private sub-utility. Hanoch shows that in the case of the CDE, the expression has the following form:

$$\varphi_p = \sum_c S_c^p e_c$$

where S_c^p represents the budget share of commodity c and e_c is the so-called expansion parameter of the CDE utility function (see following section). Log-differentiating this expression leads to:

$$\dot{\varphi}_p = \frac{\sum_c \dot{s}_c^p S_c^p e_c}{\varphi_p}$$

The differentiated share variable can be replaced by its components to yield:

$$\dot{\varphi}_p = \sum_c S_c^e \left(\dot{q}_c + \dot{p}_c - \dot{y}\right) \tag{B.13}$$

where q_c is private purchases of commodity c and p_c is its price. The 'share' parameter s^e is given by the following expression:

$$S_c^e = \frac{S_c^p e_c}{\varphi_p}$$

We finally add two more results that are required for implementation. By log-differentiating the top level utility expression, equation (B.1), we derive the follow-

---

[66] Note that χ is not a percent change in a specific variable, it simply represents the weighted average of the percent change in the preference parameters.

[67] Saving almost trivially since it is a scalar.

ing expression:

$$\dot{u} = \dot{a} + \sum_i \beta_i \dot{u}_i + \sum_i \beta_i \ln\left(U_i\right) \dot{\beta}_i \tag{B.14}$$

It can be shown that the following holds:

$$\sum_i \beta_i \dot{u}_i = \frac{\dot{y} - \dot{p}}{\Phi}$$

We start with the following formula that expresses the change in sub-utility as a function in the change in expenditure:[68]

$$\dot{u}_i = \frac{\dot{y}_i - \dot{p}_i}{\varphi_i}$$

in which we replace $\dot{y}_i$ with expression (B.9), thus:

$$\begin{aligned}
\sum_i \beta_i \dot{u}_i &= \sum_i \frac{\beta_i}{\varphi_i}\left(\dot{y} + \dot{\Phi} + \dot{\beta}_i - \dot{\varphi}_i - \dot{p}_i\right) \\
&= \frac{\dot{y}}{\Phi} + \frac{\dot{\Phi}}{\Phi} + \sum_i \frac{\beta_i}{\varphi_i}\left(\dot{\beta}_i - \dot{\varphi}_i - \dot{p}_i\right) \\
&= \frac{\dot{y}}{\Phi} + \frac{\dot{\Phi}}{\Phi} - \sum_i \frac{S_i}{\Phi}\left(\dot{\varphi}_i - \dot{\beta}_i\right) - \sum_i \frac{S_i}{\Phi}\dot{p}_i \\
&= \frac{\dot{y} - \dot{p}}{\Phi}
\end{aligned}$$

The second line uses the expression for Φ, the third line replaces the ratio of β/φ with expression (B.8) and the fourth line uses the definition of $\dot{\Phi}$ to cancel out the middle terms.

With this substitution, we get the final expression for the top-level utility function:

$$\dot{u} = \dot{a} + \frac{\dot{y} - \dot{p}}{\Phi} + \sum_i \beta_i \ln\left(U_i\right) \dot{\beta}_i \tag{B.15}$$

where we define the aggregate price index—in percent change form—using the following expression.

$$\dot{p} = \sum_i S_i \dot{p}_i \tag{B.16}$$

Thus implementation of the top-level utility function relies on equations (B.9),

---

[68] See equations (32) and (38) and a similar expression can be formulated for the utility of saving.

(B.11), (B.11), (B.12), (B.15) and (B.16). Listing these out, we have the following set of equations, where we use the fact that $\dot{\varphi}_G = \dot{\varphi}_S = 0$:[69]

$$\begin{gathered}
\dot{y}_P = \dot{y} + \dot{\Phi} + \dot{\beta}_P - \dot{\varphi}_P \\
\dot{y}_G = \dot{y} + \dot{\Phi} + \dot{\beta}_G \\
\dot{y}_S = \dot{y} + \dot{\Phi} + \dot{\beta}_S \\
\dot{\Phi} = S_P \dot{\varphi}_P - \chi \\
\chi = S_P \dot{\beta}_P + S_G \dot{\beta}_G + S_S \dot{\beta}_S \\
\dot{\varphi}_P = \sum_c S_c^e \left(\dot{q}_c + \dot{p}_c - \dot{y}_P \right) \\
\dot{u} = \dot{a} + \frac{\dot{y} - \dot{p}}{\Phi} + \sum_i \beta_i \ln \left(U_i \right) \dot{\beta}_i \\
\dot{p} = S_P \dot{P}_P + S_G \dot{P}_G + S_S \dot{P}_S
\end{gathered}$$

In terms of the model implementation, Table B.2 provides a link between the generic variables and coefficients described above and those implemented in the model. The remaining details of the implementation in the model, such as the updating formulas for the coefficients are available in the TABLO code.[70]

B.4 Constant Difference of Elasticities

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods and for non-homotheticity.[71] The starting point is an implicitly additive indirect utility function (see Hanoch (1975)) from which we can derive demand using Roy's identity (and the implicit function theorem).

General form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$V(P, U, Y) = \sum_{i=1}^{n} \alpha_i U^{e_i b_i} \left(\frac{P_i}{Y} \right)^{b_i} \equiv 1 \tag{B.17}$$

where P is the vector of commodity prices, U is (per capita) utility and Y is (per capita) income. Using Roy's identity and the implicit function theorem[72] we can derive demand, X, where v is the indirect utility function (defined implicitly):

---

[69] The formulas simplify considerably in the case of fixed preference parameters.

[70] This refers specifically to updating of the share coefficients, the sub-utilities in levels and the Φ coefficient.

[71] More detailed descriptions of the CDE can be found in Hertel et al. (1991), Surry (1993) and Hertel (1997).

[72] See Varian (1992), p. 106.

Table B.2. Main variables and coefficients in utility module

Generic	Model	Description
$\dot{y}_P$	`yp`	Aggregate private expenditures
$\dot{y}_G$	`yg`	Aggregate government expenditures
$\dot{y}_S$	`qsave,psave`	Aggregate savings
$\dot{\Phi}$	`uelas`	Elasticity of income wrt to utility
χ	`dpav`	Average shift in preference parameters
$\dot{\varphi}_P$	`uepriv`	Elasticity of private expenditures wrt to private utility
$\dot{u}$	`u`	Aggregate utility
$\dot{p}$	`p`	Aggregate price index
$\dot{a}$	`au`	Uniform shifter in utility function
$\dot{\beta}_P$	`dppriv`	Private consumption distribution parameter
$\dot{\beta}_G$	`dpgov`	Public consumption distribution parameter
$\dot{\beta}_S$	`dpsave`	Savings distribution parameter
Φ	`UTILELAS`	Elasticity of income wrt to utility
β_P	`DPARPRIV`	Private consumption distribution parameter
β_G	`DPARGOV`	Public consumption distribution parameter
β_S	`DPARSAVE`	Saving distribution parameter
S_P	`XSHRPRIV`	Private consumption budget share
S_G	`XSHRGOV`	Public consumption budget share
S_S	`XSHRSAVE`	Saving budget share

$$X_i = -\frac{\partial v}{\partial P_i} \Big/ \frac{\partial v}{\partial Y} = -\left(\frac{\partial V}{\partial P_i} \Big/ \frac{\partial V}{\partial U}\right) \Big/ \left(\frac{\partial V}{\partial Y} \Big/ \frac{\partial V}{\partial U}\right) = -\left(\frac{\partial V}{\partial P_i} \Big/ \frac{\partial V}{\partial Y}\right) \tag{B.18}$$

This then leads to the following demand function:

$$X_i = \frac{\alpha_i b_i U^{e_i b_i} \left(\dfrac{P_i}{Y}\right)^{b_i - 1}}{\displaystyle\sum_j \alpha_j b_j U^{e_j b_j} \left(\dfrac{P_j}{Y}\right)^{b_j}} \tag{B.19}$$

Implementation is easier if we define the following variable:

$$Z_i = \alpha_i b_i U^{e_i b_i} \left(\frac{P_i}{Y}\right)^{b_i} \tag{B.20}$$

Then the budget shares can be expressed as:

$$S_i = \frac{P_i X_i}{Y} = \frac{Z_i}{\sum_j Z_j} \tag{B.21}$$

and the demand expression is:

$$X_i = \frac{S_i}{P_i} Y \tag{B.22}$$

Implementation also requires evaluating U. This can be done by implementing equation (B.17) and inserting the expression for Z:

$$\sum_{i=1}^{n} \frac{Z_i}{b_i} \equiv 1 \tag{B.23}$$

Elasticities

In this section we derive the price and income elasticities for the CDE function. These formulas will be needed to implement the CDE in percentage change form.[73]

The own-price elasticity is given by the following:

$$\varepsilon_i = \frac{\partial X_i}{\partial P_i} \frac{P_i}{X_i} = b_i \left(1 - S_i\right) - 1 - S_i \left[e_i b_i - \sum_j S_j e_j b_j\right] \Big/ \sum_j S_j b_j \tag{B.24}$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

$$\frac{\partial U}{\partial P_i} \frac{P_i}{U} = -\frac{P_i}{U} \left(\frac{\partial V}{\partial P_i}\right) \Big/ \left(\frac{\partial V}{\partial U}\right) = -\frac{S_i}{\sum\limits_j S_j e_j} \tag{B.25}$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters, e, is close to unity.

Letting $\sigma_i = 1 - b_i$ (or $b_i = 1 - \sigma_i$), we can also write:

$$\varepsilon_i = S_i \left[\sigma_i - \frac{e_i \left(1 - \sigma_i\right)}{\sum\limits_j S_j e_j} - \frac{\sum\limits_j S_j e_j \sigma_j}{\sum\limits_j S_j e_j}\right] - \sigma_i \tag{B.26}$$

The derivation of the cross elasticities is almost identical and is not carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes the following form where we use the Kronecker product, δ:[74]

---

[73] The formulas can also be used to calibrate the CDE function in levels given a set of initial budget shares and income and price elasticities.

[74] δ takes the value of 1 along the diagonal (i.e., when $i = j$) and the value 0 off-diagonal (i.e., when

$$\varepsilon_{ij} = S_j \left[-b_j - \frac{e_i b_i}{\sum\limits_k S_k e_k} + \frac{\sum\limits_j S_k e_k b_k}{\sum\limits_k S_k e_k} \right] + \delta_{ij} \left(b_i - 1 \right) \tag{B.27}$$

Again, we replace b by $1 - \sigma$, to get:

$$\varepsilon_{ij} = S_j \left[\sigma_j - \frac{e_i \left(1 - \sigma_i \right)}{\sum\limits_k S_k e_k} - \frac{\sum\limits_j S_k e_k \sigma_k}{\sum\limits_k S_k e_k} \right] - \delta_{ij} \sigma_i \tag{B.28}$$

The income elasticities are derived in a similar fashion:

$$\eta_i = \frac{\partial X_i}{\partial Y} \frac{Y}{X_i} = \frac{1}{\sum\limits_k S_k e_k} \left[e_i b_i - \sum_k S_k e_k b_k \right] - \left(b_i - 1 \right) + \sum_k S_k b_k \tag{B.29}$$

For this, we need the elasticity of utility with respect to income:

$$\frac{\partial U}{\partial U} \frac{Y}{U} = -\frac{Y}{U} \left(\frac{\partial V}{\partial Y} \right) \Big/ \left(\frac{\partial V}{\partial U} \right) = \frac{1}{\sum\limits_k S_k e_k} \tag{B.30}$$

Replacing b with $1 - \sigma$, equation (B.29) can be re-written to be:

$$\eta_i = \frac{1}{\sum\limits_k S_k e_k} \left[e_i \left(1 - \sigma_i \right) + \sum_k S_k e_k b_k \right] + \sigma_i - \sum_k S_k \sigma_k \tag{B.31}$$

From the Slutsky equation, we can calculate the compensated demand elasticities:

$$\xi_{ij} = \varepsilon_{ij} + S_j \eta_i = -\delta_{ij} \sigma_i + S_j \left[\sigma_j + \sigma_i - \sum_k S_k \sigma_k \right] \tag{B.32}$$

The cross-Allen partial elasticities are equal to the compensated demand elasticities divided by the share:

$$\sigma^a_{ij} = \sigma_j + \sigma_i - \sum_k S_k \sigma_k - \delta_{ij} \sigma_i / S_j \tag{B.33}$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities*.

$i \neq j$).

$$\sigma^a_{ij} - \sigma^a_{il} = \sigma_j - \sigma_l \tag{B.34}$$

CDE in first differences

It is useful to decompose changes in demand using a linearized version of the demand function, and that which is used in the standard GEMPACK version of the CDE function. We begin with differentiating the expression for Z_i above, this will prove useful below:

$$\dot{z}_i = e_i b_i \dot{u} + b_i \dot{p}_i - b_i \dot{y} \iff \sum_k \frac{S_k}{b_k} \dot{z}_k = \dot{u} \sum_k S_k e_k + \sum_k S_k \dot{p}_k - \dot{y}$$

We can also differentiate the implicit utility function defined in terms of Z_i:

$$\sum_j \Delta Z_j / b_j = \sum_j Z_j \dot{z}_j / b_j = \sum_j \left[\sum_k Z_k \right] S_j \dot{z}_j / b_j \equiv 0 \iff \sum_j S_j \dot{z}_j / b_j \equiv 0$$

Merging the two expressions above we can derive an expression for $\dot{u}$ in terms of $\dot{y}$ and $\dot{p}_i$:

$$\dot{u} = \left[\dot{y} - \sum_k S_k \dot{p}_k \right] / \sum_k S_k e_k$$

We now proceed to insert the expression for $\dot{z}$ from above into the percent change for the budget shares and finally into the percent change for the demand function:

$$\dot{s}_i = \dot{z}_i - \sum_j S_j \dot{z}_j \Rightarrow \dot{x}_i = \dot{z}_i - \sum_j S_j \dot{z}_j + \dot{y} - \dot{p}_i$$

After substitution we derive:

$$\dot{x}_i = e_i b_i \dot{u} + b_i \dot{p}_i - b_i \dot{y} - \sum_j \left[S_j e_j b_j \dot{u} + b_j \dot{p}_j - b_j \dot{y} \right] + \dot{y} - \dot{p}_i$$

We then insert the expression for $\dot{u}$ from above and collect terms to get:

$$\begin{aligned}\dot{x}_i &= \dot{y}\left[1-b_i+\sum_k S_k b_k + \frac{e_i b_i - \sum\limits_k S_k e_k b_k}{\sum\limits_k S_k e_k b_k}\right]\\ &+ \sum_j \left\{ S_j \left[-b_j - \frac{e_i b_i}{\sum\limits_k S_k e_k} + \frac{\sum\limits_k S_k e_k b_k}{\sum\limits_k S_k e_k}\right]\dot{p}_j\right\} + (b_i-1)\dot{p}_i\end{aligned}$$

We can identity the terms for $\dot{y}$ and $\dot{p}$ as the income and price elasticities respectively and thus we have:

$$\dot{x}_i = \eta_i \dot{y} + \sum_j \varepsilon_{ij}\dot{p}_j \tag{B.35}$$

Implementation in GTAP

Equation B.35 is implemented in the GEMPACK version of GTAP as the following:

$$qpa_{c,r} - pop_r = \sum_k EP_{c,k,r} ppa_{k,r} + EY_{c,r}\left(yp_r - pop_r\right)$$

where *qpa* is aggregate private demand (hence the population correction), $ppa_{k,r}$ is the vector of consumer prices and $yp_r - pop_r$ represents per capita total expenditure. The coefficients *EP* and *EY* represent respectively the price and income elasticities. The latter are updated at each iteration using the formulas from above and the intermediate values of the budget shares. Instead of using directly the price elasticity formula, the implementation uses the simpler expression for the Allen partial elasticity that is held in the coefficent *APE*. The implementation then uses the coefficients *EY* and *APE* to calculate the matrix of price elasticities using $\varepsilon_{ij} = S_j(\sigma^a_{ij} - \eta_i)$. The current implementation does not use Kronecker's δ and instead the formula for *APE* is specified twice—once for all (c,k) combinations and a second time for (c,c). In the formulas, the b parameters are represented by *SUBPAR* and the e parameters are represented by the *INCPAR*. Finally, the elasticity formulas are expressed in terms of the σ parameter from above that is represented by the coefficient *ALPHA*, where $ALPHA = 1 - SUBPAR$.

The model implementation also requires an expression for utility. Above we derived a key relation between utility, income and prices:

$$\dot{u} = \left[\dot{y} - \sum_k S_k \dot{p}_k\right] / \sum_k S_k e_k$$

This is implemented in the model where the expression $\sum_k S_k \dot{p}_k$ is replaced by the variable *ppriv* that represents the percent change in the consumer price index, and $\dot{y}$ is replaced with per capita expenditure, i.e., $yp_r - pop_r$. Finally, the denominator, that represents the share weighted sum of the expansion parameters, is updated using the coefficient $UELASPRIV$.

Appendix C. Accounting relations

C.1 Introduction

This appendix outlines the key accounting identities in the GTAP database and its correspondence in the GTAP model. The organizing framework is a Social Accounting Matrix—an international standard for preparing and presenting national accounts.[75] The following list describes some of the key accounting extensions relative to the 'classic' version of the GTAP model:

- The new standard version of the model has the capability of having a non-diagonal 'make' matrix. Among other things, this implies that there is no longer a one-to-one correspondence between activities and commodities.
- The production tax vector has been converted to an $a \times c$ matrix and the tax is applied at the individual product level—not the aggregate output level. Note that the data is still only being compiled at the activity level and thus the rates will be uniform across commodities supplied by the same activity, though not necessarily after an aggregation of the database.
- Output and export taxes are now evaluated relative to basic prices and not so-called market prices. Thus tax rates will be positive and subsidy rates are negative.
- The income tax vector has been converted to an $a \times e$ matrix, thus taxes on factor remuneration are activity as well as endowment specific. For the moment, the tax rates are uniform across activities, though not necessarily after an aggregation of the database.
- Investment expenditures (or the formation of capital) has been extracted from 'production' activities. Thus variables such as *QFD* no longer have a 'CGDS' activity, which is now included as a separate final demand account with names such as *QID*.
- Separate variable names have been introduced for endowments (or factors of production). In GTAP 'classic', variables such as *PM* and *QO* were defined for both produced commodities (including 'CGDS') and endowments.
- All endowments in the 'classic' version had an aggregate level and price. Only sluggish variables had an activity specific return (*PMES*). To facilitate a number of new expressions, all endowments have an activity specific return—in the case of mobile factors, percent changes in the activity specific return will equal the percent change in the aggregate return. The new model also introduces an activity-specific endowment (natural resources). This implies that we no longer necessarily have aggregate endowment lev-

---

[75] See Pyatt and Round (1985) and Reinert and Roland-Holst (1997). McDonald and Thierfelder (2004) provided an earlier exposition of the GTAP accounting framework in a SAM format.

els and prices—an additional reason to treat endowments as activity specific in most expressions.

C.2 Accounting relations in new Standard Model

This section describes the accounting framework for the new standard version of the GTAP database and model. Table C.1 shows the entire SAM using the header arrays from the GTAP database and thus provides an overall view of the accounts for each region/country. Below are some additional notes on the SAM accounting framework:

- The SAM deviates somewhat from a standard SAM. The number of columns is not equal to the number of rows. The indirect and direct tax columns have been collapsed into a single column, labeled *TTAX*, that represents all of the tax revenues collected (and provided to the regional household).
- The database hews to the previous practice of providing all value flows as pre- and post-tax matrices, i.e. the revenues derived from taxes (and subsidies) are derived via residual. The new standard database also introduces a new harmonization rule and that is that all taxes are assessed relative to what is now called 'basic' prices and thus tax revenues will be evaluated as positive flows (and subsidies as negative flows). This affects only two of the 'classic' taxes—taxes on domestic output and taxes on exports.
- The value of output at suppliers' prices is given by *(VOS)*, formerly *VOA*.
- The domestic supply column ('DOM') has the possibility of having a non-diagonal 'make' matrix—the cell identified as `MAKES`, the 'make' matrix at suppliers' prices. In fact, the 'make' matrix is no longer required to be square, it has the dimensions of $a \times c$.
- The `PTAX` matrix has the same dimensionality as the `MAKE` matrix, i.e. taxes on domestic supply are both activity- and commodity-specific. As well, since all taxes are evaluated relative to basic prices, the `PTAX` matrix will be positive for taxes. In the 'classic' SAM, the `PTAX` matrix was represented by the vector `OSEP`, which would enter as `−OSEP` instead of the current `PTAX` matrix.
- Export taxes (`XTRV`), derived from the difference between exports at border prices and basic prices is now a positive matrix. Note that the relevant basic price is *PDS*, which by assumption is uniform across destination markets. The export tax is bilateral and hence the border price, *PFOB*, is bilateral.
- The parameter `VOSB` corresponds to the parameter `VOM` from GTAP 'classic', i.e. the value of output gross of the output tax (i.e. `VOM` = `VOA` - `OSEP`).
- Income taxes are now both activity- and endowment-specific. In the SAM they are captured as a single row with one column for each of the endowments. The formula for each cell of the row vector is therefore:

$$\sum_a \left(TINC_{e,a} - 1\right) PES_{e,a} QFE_{e,a}$$

and the term `EVFB-EVOS` is short-hand for the sum over activities, as the individual matrices are indexed by *a* and *e*.

- `TTAX` represents the sum of taxes on domestic sales (`DTAX`), taxes on import sales (`ITAX`), taxes on factor-use (`ETAX`), taxes on domestic output (`PTAX`), import tariffs (`MTAX`), export taxes (`XTAX`) and income taxes (`YTAX`).
- Household income is derived from the sum of after tax factor remuneration (`EVOS`), net of depreciation, and total taxes (`TTAX`). It is allocated across three categories of expenditures: private expenditures (`PRIVEXP`), public expenditures (`GOVEXP`) and `SAVE`. The latter is part of the database. The former are derived from the expenditure vectors `VDPP`, `VMPP`, `VDGP` and `VMGP`. Thus one test of data consistency is to assess the consistency of regional household income with its disposition.
- The investment expenditure vectors `VDIB`, `VMIB`, `VDIP`, `VMIP` are new to the database. Investment was derived from the 'CGDS' column of firms' expenditures in GTAP 'classic'.
- The balance of payments account is another measure of the consistency of the database. The scalar `SAVF` is not part of the database, but can be caclulated by residual, i.e. the difference between total exports, including margin exports, and total imports. At the global level, the sum of `SAVF` across regions should be zero.

Table C.1. Social accounting scheme for GTAP Data Base

	ACT	DOM	IMP	VA	TTAX	REGH	PRIV	GOV	INV	DEPR	ITT	TRADE	ROW	Total
Activities		*MAKES*												*VOS*
Domestic commodities	*VDFB*						*VDPB*	*VDGB*	*VDIB*		*VST*	*VFOB*		*VOSB+XTAX*
Imported commodities	*VMFB*						*VMPB*	*VMGB*	*VMIB*					*VMSB*
Value added	*EVFB*													*EVFB*
Tax on dom. commodities	*VDFP-VDFB*						*VDPP-VDPB*	*VDGP-VDGB*	*VDIP-VDIB*					*DTAX*
Tax on imp. commodities	*VMFP-VMFB*						*VMPP-VMPB*	*VMGP-VMGB*	*VMIP-VMIB*					*ITAX*
Tax on factor use	*EVFP-EVFB*													*ETAX*
Production tax		*MAKEB-MAKES*												*PTAX*
Import tax			*VMSB-VCIF*											*MTAX*
Export tax		*VFOB-VXSB*												*XTAX*
Income tax				*EVFB-EVOS*										*YTAX*
Regional household				*EVOS*	*TTAX*									*REGY*
Private expenditures						*PRIVEXP*								*PRIVEXP*
Public expenditures						*GOVEXP*								*GOVEXP*
Investment expenditures						*SAVE*				*VDEP*			*SAVF*	*EINV*
Depreciation allowance						*VDEP*								*VDEP*
Margin exports													*VST*	*VST*
Trade			*VCIF*										*VFOB-VCIF*	*VFOB*
Balance of payments														0
Total	*VOS*	*VOSB+XTAX*	*VMSB*	*EVFB*	*TTAX*	*REGY*	*PRIVEXP*	*GOVEXP*	*EINV*	*VDEP*	*VST*	*VFOB*	0	

Tables C.2 and C.3 provide a concordance between the matrices and vectors in the database (the so-called header arrays) and the corresponding expressions in levels using the model variables. They can also be linked to the entries in the SAM, thus there is a three-way correspondence between the header arrays in the database, the entries in the SAM, and the model variables. It would be possible to re-construct the SAM post-simulation using the formulas in these tables.[76]

We conclude this section with additional figures highlighting the linkages in the model. The price linkages were described in section 3 and depicted in Figure 2. Below are two additional figures, one for the quantity linkages (Figure C.1) and the other for the value linkages (Figure C.2).

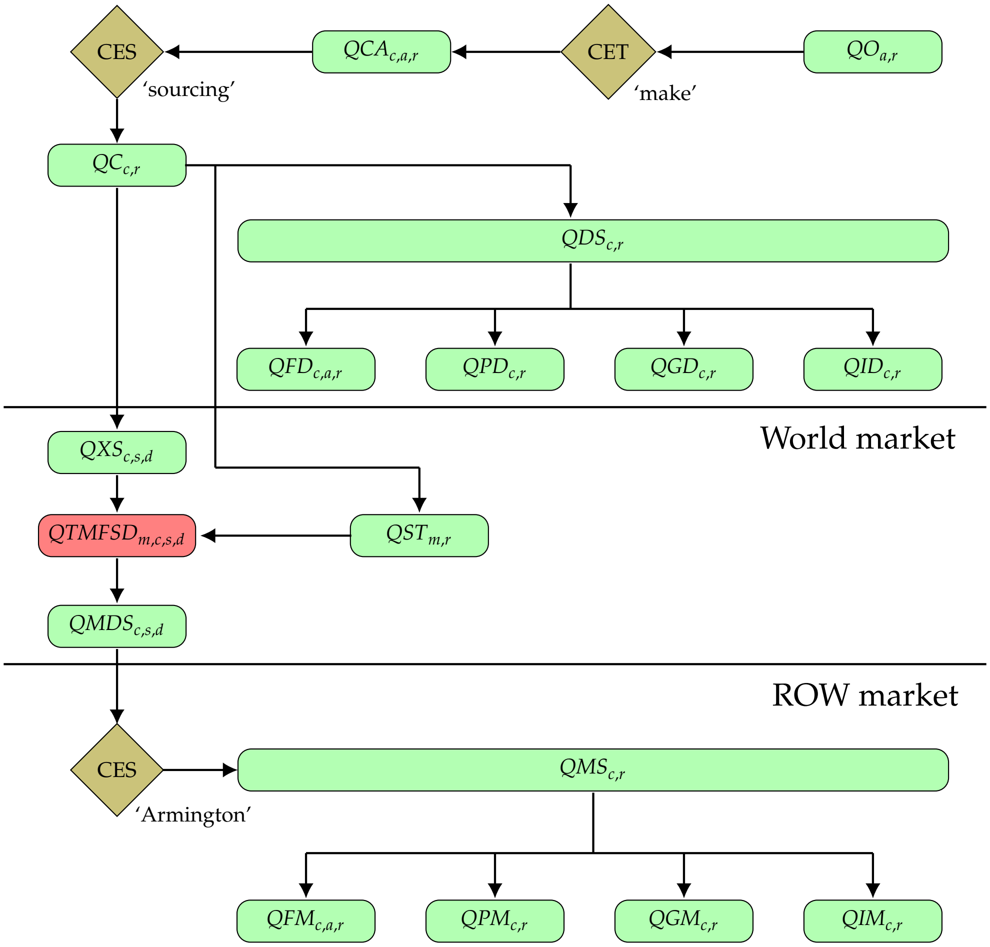

Figure C.1. Quantity linkages in the model

---

[76] The foreign saving component can be derived from the `TBAL` coefficient.

Table C.2. Supply relationships

Row description	*Data table name*	*Expression*
Firm demand for dom. commodities at basic prices	`VDFB(COMM,ACTS)`	$PDS_{c,r}QFD_{c,a,r}$
Firm demand for imp. commodities at basic prices	`VMFB(COMM,ACTS)`	$PMS_{c,r}QFM_{c,a,r}$
Factor payments at basic prices	`EVFB(ENDW,ACTS)`	$PEB_{e,a,r}QFE_{e,a,r}$
Firm demand for dom. commodities at producer prices	`VDFP(COMM,ACTS)`	$TFD_{c,a,r}PDS_{c,r}QFD_{c,a,r} = PFD_{c,a,r}QFD_{c,a,r}$
Firm demand for imp. commodities at producer prices	`VMFP(COMM,ACTS)`	$TFM_{c,a,r}PMS_{c,r}QFM_{c,a,r} = PFM_{c,a,r}QFM_{c,a,r}$
Factor payments at producer prices	`EVFP(ENDW,ACTS)`	$TFE_{e,a,r}PEB_{e,a,r}QFE_{e,a,r} = PFE_{e,a,r}QFE_{e,a,r}$
Value of output at suppliers' prices	`VOS(ACTS) = sum(COMM,VDFP+VMFP) + sum(ENDW,EVFP)`	$PO_{a,r}QO_{a,r} = \sum_c PFD_{c,a,r}QFD_{c,a,r} + \sum_c PFM_{c,a,r}QFM_{c,a,r} + \sum_e PFE_{e,a,r}QFE_{e,a,r}$
Value of domestic supply at suppliers' prices	`MAKES(ACTS,COMM)`	$PS_{c,a,r}QCA_{c,a,r}$
Value of domestic supply at basic prices	`MAKEB(ACTS,COMM)= MAKES(ACTS,COMM) + PTAX(ACTS,COMM)`	$PCA_{c,a,r}QCA_{c,a,r} = TO_{c,a,r}PS_{c,a,r}QCA_{c,a,r}$
Value of exports at basic prices	`VXSB(COMM,DEST)`	$PDS_{c,s}QXS_{c,s,d}$
Value of exports at world prices	`VFOB(COMM,DEST) = VXSB(COMM,DEST) + XTRV(COMM,DEST)`	$TXS_{c,s,d}PDS_{c,s}QXS_{c,s,d} = PFOB_{c,s,d}QXS_{c,s,d}$
Value of imports at world prices	`VCIF(COMM,SOURCE)`	$PCIF_{c,s,d}QXS_{c,s,d}$
Value of imports at domestic basic prices	`VMSB(COMM,SOURCE)= VCIF(COMM,SOURCE) + TFRV(COMM,SOURCE)`	$PMDS_{c,s,d}QXS_{c,s,d} = TMS_{c,s,d}PCIF_{c,s,d}QXS_{c,s,d}$

Table C.3. Income and demand relations

Row description	*Data table name*	*Expression*
Household factor income	`EVOS(ENDW,ACTS)`	$PES_{e,a,r}QFD_{e,a,r}$
Household expenditures	`PRIVEXP`	YP_r
Government expenditures	`GOVEXP`	YG_r
Domestic savings	`SAVE`	$SAVE_r = PSAVE_rQSAVE_r$
Depreciation allowance	`VDEP`	$\delta_rPINV_rKB_r$
Household demand for dom. commodities at basic prices	`VDPB(COMM)`	$PDS_{c,r}QPD_{c,r}$
Household demand for imp. commodities at basic prices	`VMPB(COMM)`	$PMS_{c,r}QPM_{c,r}$
Household demand for dom. commodities at producer prices	`VDPP(COMM)`	$TPD_{c,r}PDS_{c,r}QPD_{c,r} = PPD_{c,r}QPD_{c,r}$
Household demand for imp. commodities at producer prices	`VMPP(COMM)`	$TPM_{c,r}PMS_{c,r}QPM_{c,r} = PPM_{c,r}QPM_{c,r}$
Gov. demand for dom. commodities at basic prices	`VDGB(COMM)`	$PDS_{c,r}QGD_{c,r}$
Gov. demand for imp. commodities at basic prices	`VMGB(COMM)`	$PMS_{c,r}QGM_{c,r}$
Gov. demand for dom. commodities at producer prices	`VDGP(COMM)`	$TGD_{c,r}PDS_{c,r}QGD_{c,r} = PGD_{c,r}QGD_{c,r}$
Gov. demand for imp. commodities at producer prices	`VMGP(COMM)`	$TGM_{c,r}PMS_{c,r}QGM_{c,r} = PGM_{c,r}QGM_{c,r}$
Inv. demand for dom. commodities at basic prices	`VDIB(COMM)`	$PDS_{c,r}QID_{c,r}$
Inv. demand for imp. commodities at basic prices	`VMIB(COMM)`	$PMS_{c,r}QIM_{c,r}$
Inv. demand for dom. commodities at producer prices	`VDIP(COMM)`	$TID_{c,r}PDS_{c,r}QID_{c,r} = PID_{c,r}QID_{c,r}$
Inv. demand for imp. commodities at producer prices	`VMIP(COMM)`	$TIM_{c,r}PMS_{c,r}QIM_{c,r} = PIM_{c,r}QIM_{c,r}$
Demand for margins exports	`VST(MARG)`	$PDS_{m,r}QST_{m,r}$

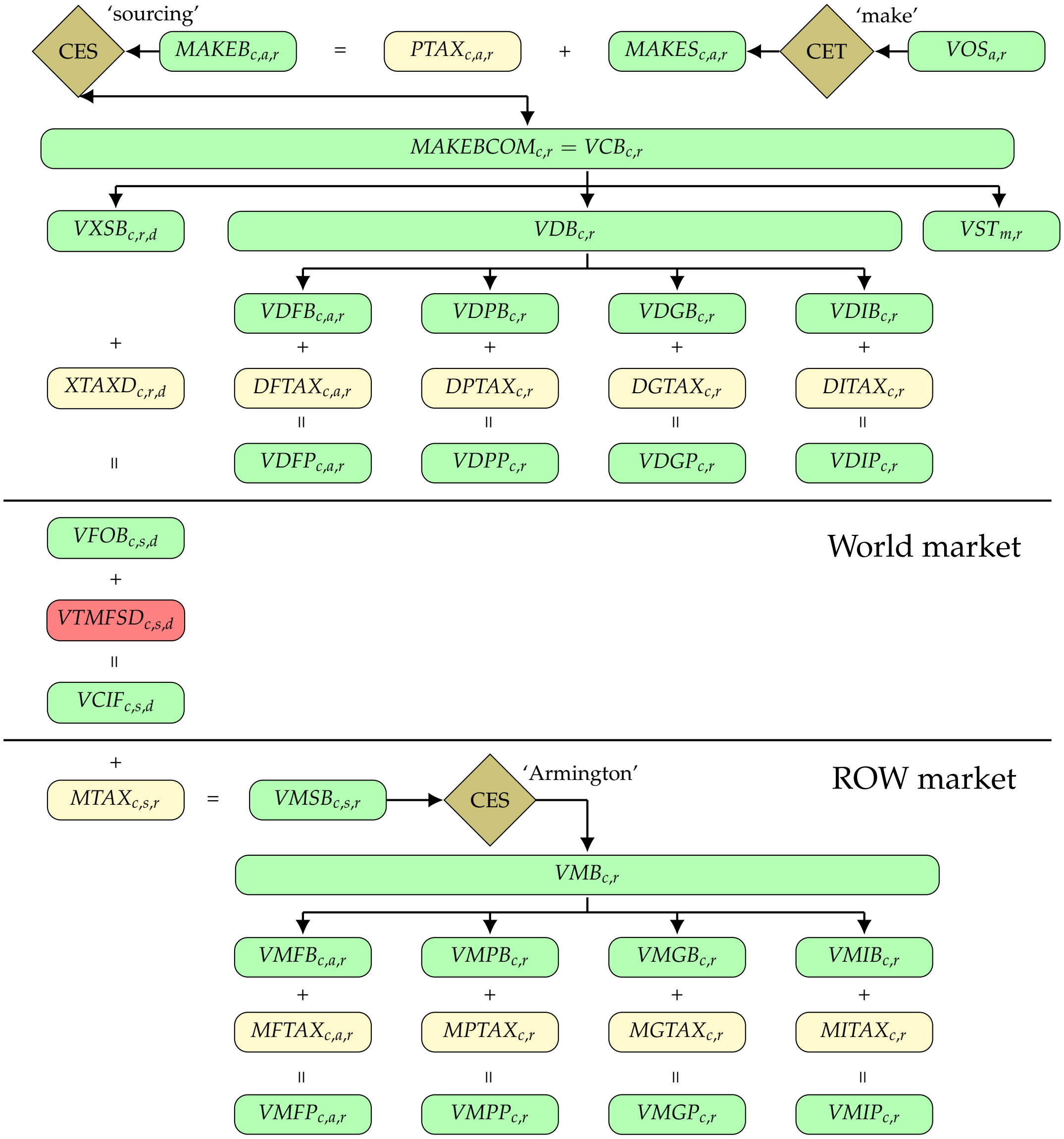

Figure C.2. Value linkages in the model